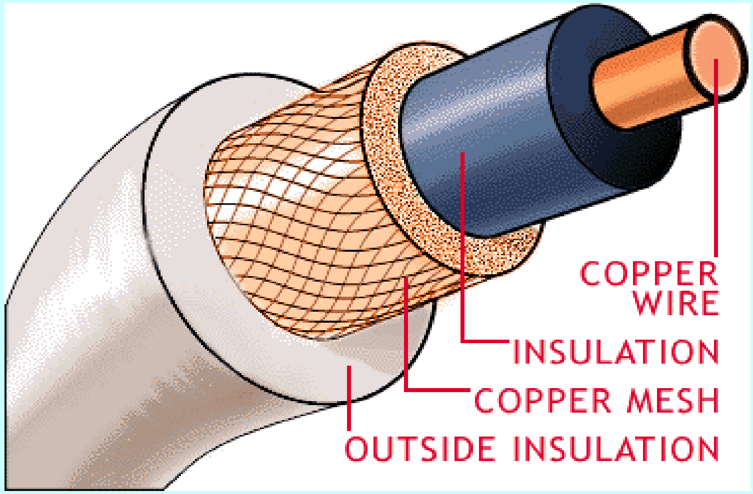
Cylindrical cable

- Simplest design - long wire (r=a), with surrounding conducting cylinder (r=b)

  this is easy to calculate

  λ = charge per unit length of wire



- Gauss' law

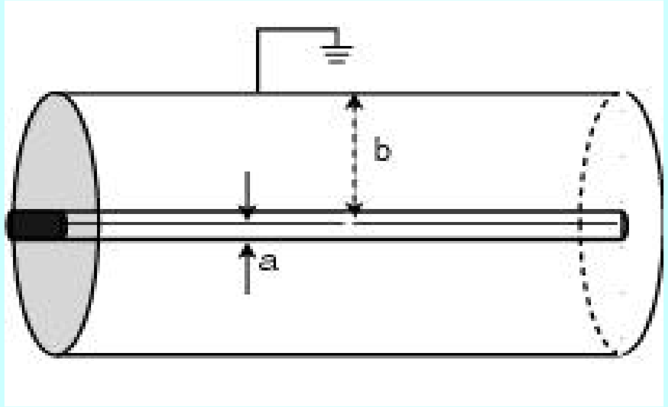
  $\int \mathbf{E.n}.dS = \lambda/\varepsilon$

  $E(r) = \lambda / 2\pi\varepsilon r = -d\varphi/dr$

  apply boundary conditions

  $V = (\lambda/2\pi\varepsilon)\ln(b/a)$

  $C = Q/V = 2\pi\varepsilon/\ln(b/a)$ = capacitance per unit length



  eg. $\varepsilon \approx 2.3\varepsilon_0$ a = 0.5mm b = 2mm $C \approx 1$ pF.cm^{-1}

Inductance

- arises from magnetic effects, also giving rise to frequency dependent impedance

- Definition   Faraday's law

  EMF = potential difference = -d(flux)/d(time)

  magnetic flux = $\Phi = \int_S \mathbf{B.n}\, dS$

- Inductance defined L = d(flux) /d(current) = $d\Phi/dI$

  so L is a measure of change of magnetic flux in response to change in current

  $$d\Phi = LdI$$

  real systems likely to involve flux changes which depend on several components in the system

  except for transformers less likely to concern us

Inductance (2)

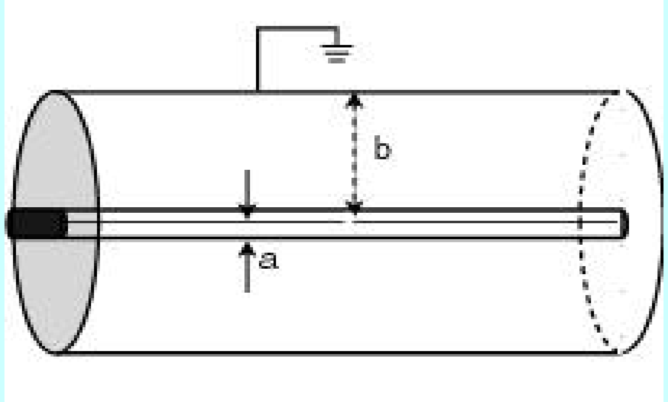
- Example thin wire with current I

Ampere's law

$\int_C (B.dl) = \mu_0 I = B.2\pi r$ around circuit

Φ : consider area at r, dimensions z x dr

$d\Phi = B.z.dr = \mu_0 I z.dr / 2\pi r$



- integrate, with limits a < r < b

$d\Phi = B.z.dr = (\mu_0 I z / 2\pi) \ln(b/a)$

$L = (\mu_0 z / 2\pi) \ln(b/a)$

eg. a = 0.5mm b = 2mm $L \approx 2.8$ nH.cm^{-1}

Inductance (3)

•why is it important? $V = -L\, dI/dt$

transient effects

sudden change in current in a long cable could generate large voltage spike

time varying signals $I = I_0 e^{j\omega t}$ $\omega = 2\pi f$

$V = j\omega L.I = Z_{L.} I$

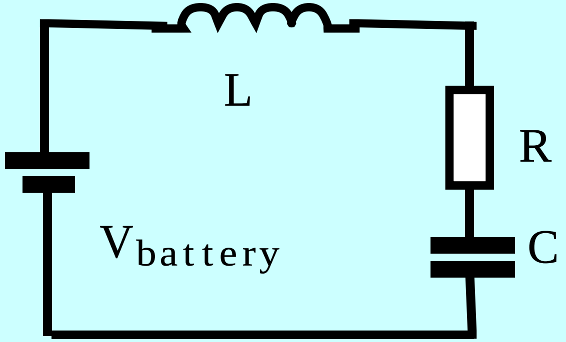
impedance $Z = j\omega L$

NB sign positive because applied voltage moved to right side of equation

$V_{battery} - L\, dI/dt = IR + \int I.\, dt/C$

$V_{applied} = IR + \int I.\, dt/C + L\, dI/dt$

$v = iR + i/j\omega C + j\omega L.i$ $v = V_0 e^{j\omega t}$



Equivalent circuits

- Real instruments and devices quite complex

  but usually need to simplify to understand behaviour & influence

  Equivalent circuit represents major properties

  often useful to consider voltage generator or source + series impedance

  or current source + parallel impedance

  Impedances not usually simple resistances - frequency dependent

- Calculate using two important theorems

Thevenin	voltage
Norton	current

- first recall how to analyse circuits...

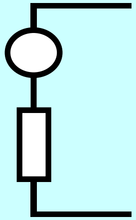
Kirchoff's laws

- sum of currents into point = sum of currents out
- sum of voltage drops around closed circuit = 0

- Thevenin's theorem

any two terminal network or resistors and voltage sources is equivalent to single resistor R_{th} in series with voltage source V_{th}

$R_{th} = V_{open} / I_{short}$ $V_{th} = V_{open}$



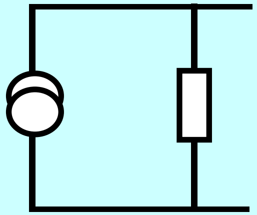
eg voltage divider

can generalise to more complex impedances

- Norton's theorem

...network equivalent to ... R_n in parallel with current source I_n

$I_n = I_{short}$ $R_n = V_{open} / I_{short}$



Earth or ground

- Very important concept in instrumentation applications

represents infinite reservoir of electric charge, always capable of receiving electric flux (field) form any charged body

"earth" - best earth connection is solid & substantial connection to ground

deep Cu stake embedded in moist soil

In practice few grounds so substantial

eg consider path to ground pin on oscilloscope (or kettle!)
tenuous path with current carrying cables nearby
plenty of chance for induced currents

- Important consequences to be considered later...

Jargon, names & concepts

•Linear systems *will be a frequent assumption*

input signal = f(t) output = g(t)

expect output to vary with input as A f(t) -> A g(t)

not always the case, eg amplifiers frequently exhibit saturation

can arise for several reasons

constraints in amplifier design

if 0-5V power, don't expect output signals over full 5V (and none >5V!)

deliberate design choice to measure signals with precision depending on size

eg relative precision often required

•Superposition

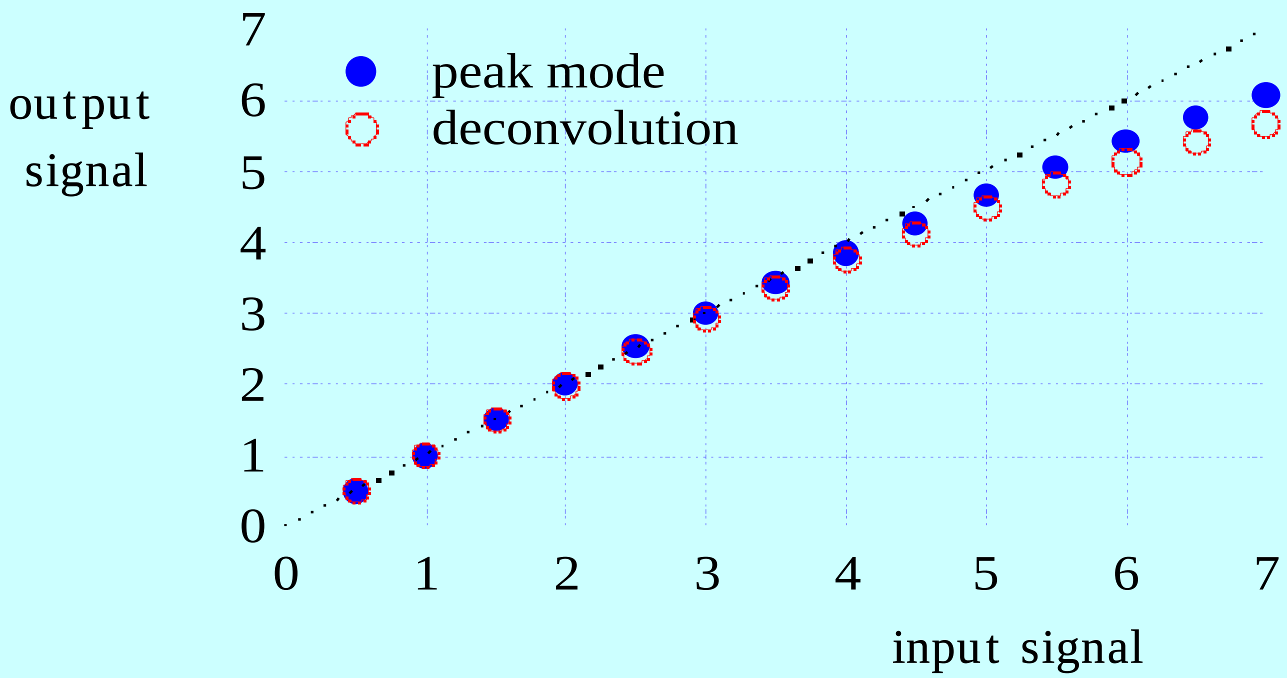
important principle in many areas of physics & mathematical physics

If $f_1(t) \to g_1(t)$ and $f_2(t) \to g_2(t)$

then $a f_1(t) + b f_2(t) \to a g_1(t) + b g_2(t)$

Saturation example

- An example of a real amplifier



Decibels

- decibels (dB)

  signal magnitudes cover wide range so frequently prefer logarithmic scale

  Number of dB = $10\log_{10}(P_2/P_1)$

  often measuring voltages in system: dB =$20 \log_{10}(V_2/V_1)$

- Not an absolute unit and sometimes encounter variants

  dBm: dB with P_{in} = 1mW

Dynamic range

- In most systems there will be a smallest <u>measurable</u> signal

  if there is noise present, it is most likely to be related to the smallest signal distinguishable from noise

  3 x rms noise?   5 x rms noise?

  or quantisation unit in measurement

- and a largest <u>measurable</u> signal

  most likely set by apparatus or instrument, eg saturation

- Dynamic range = ratio of largest to smallest signal

  often expressed in dB or bits

  eg 8 bits = dynamic range is 256_{10}

  = 48dB   (if signal is voltage)

Precision

- many measurements involve detection of particle or radiation quantum (photon)

  simple presence or absence sometimes sufficient = binary (0 or 1)

  other measurements are of energy

- why do we need such observations?

  primary measurement may be energy

  eg medical imaging using gammas or high energy x-rays, astro-particle physics

  extra information to improve data quality

  removes experimental background, eg Compton scattered photons mistaken for real signal

  optical communications - constant pressure to increase “bandwidth” - eg number of telephone calls carried per optical fibre

  wavelength division multiplexing - several “colours” or wavelengths in same fibre simultaneously

  require wavelength sensitive sensor to distinguish different signals

- what is ultimate limit to precision?

Statistical limit to energy measurement

- Assume no limit from anything other than sensor

  often not realistic assumption, but best possible case

N_{quanta} observed = E/ε

$$= \frac{\text{energy deposited by radiation}}{\text{energy required to generate quantum of measurement}}$$

examples

semiconductor: energy for electron-hole pair ~ few eV
gaseous ionisation detector: energy for electron-ion ~ few x 10 eV
scintillation sensor: energy per photon of scintillation light ~ 100 eV

- Basic Poisson statistics

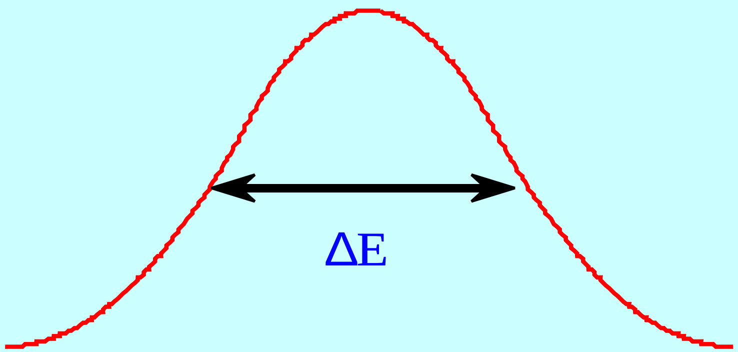
$E_{meas} \sim N_q$

$\sigma^2(N_q) = N_q$

$\sigma(E)/E = \sigma(N_q)/N_q = 1/\sqrt{N_q}$

expect gaussian distribution of N_q for large N_q

advantage in sensor with small ε



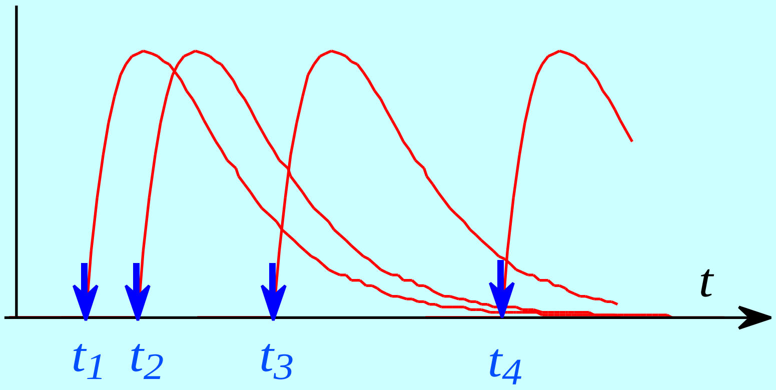
$\Delta E_{FWHM} = 2.35\sigma(E)$

Time invariant systems

•many systems respond so that output depends only on time of application

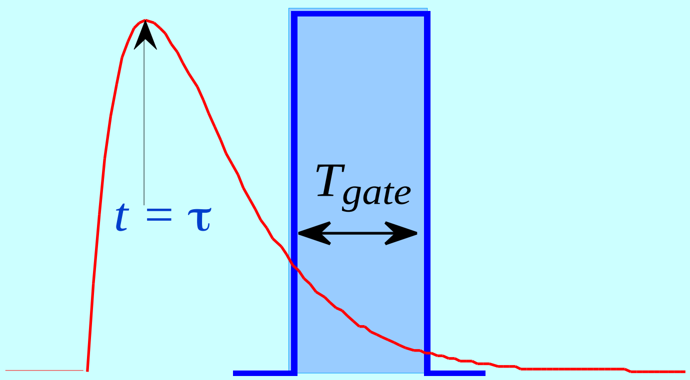
ie later application of same signal results in delayed, but identical, output

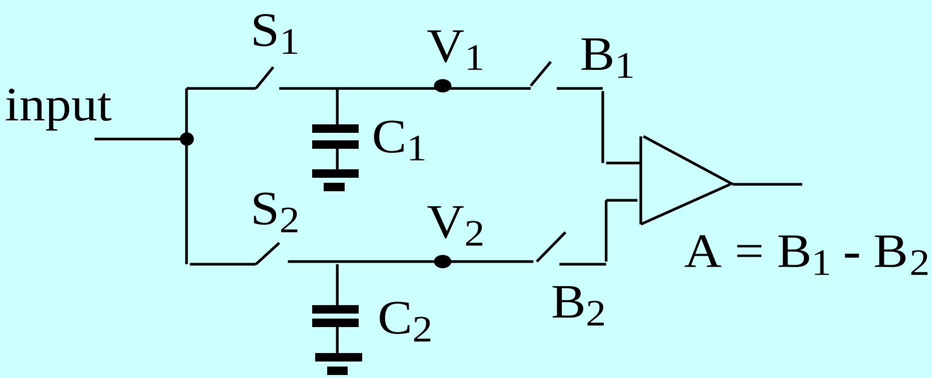
If f(t) -> g(t) then f(t+t_0) -> g(t +t_0)



seems trivial but many counter examples

e.g sampled signals



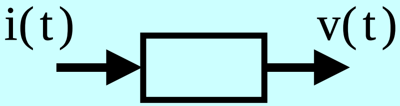


examples of time variant systems

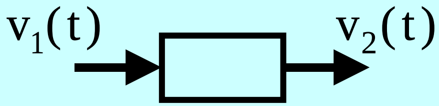
Filters

- Device or components to transform electrical signals from one form to another

eg

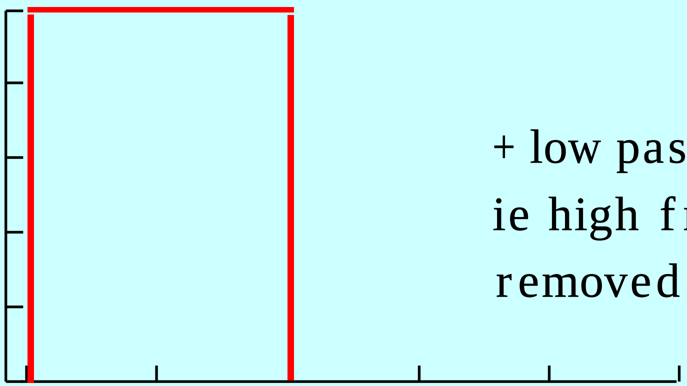


or

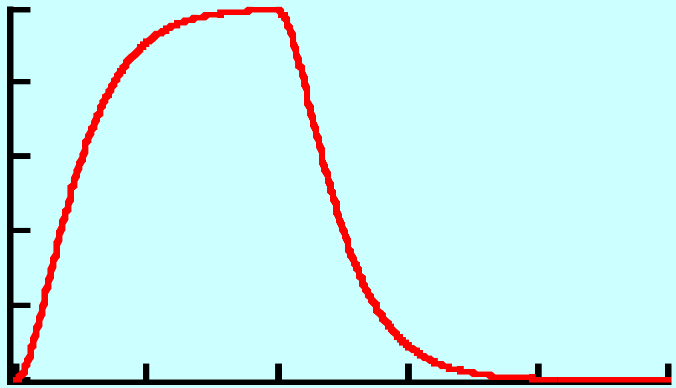


usually when doing so, amplitudes of frequencies in output are different from those input,

ie spectral content is changed or some frequencies <u>filtered</u> out



+ low pass filter
ie high frequencies
removed



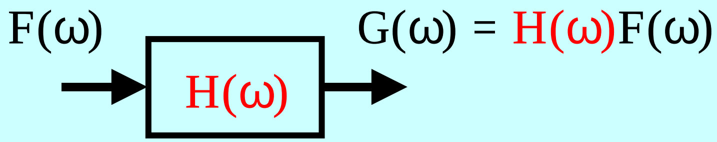
frequently want to analyse signals and systems in terms of frequency content, as well as behaviour in time

Transfer Function

• Inputs to, and outputs from, system considered as sum of components, with each component a single frequency

$F(\omega) = A(\omega)\, e^{j\omega t}$

$\omega = 2\pi f$



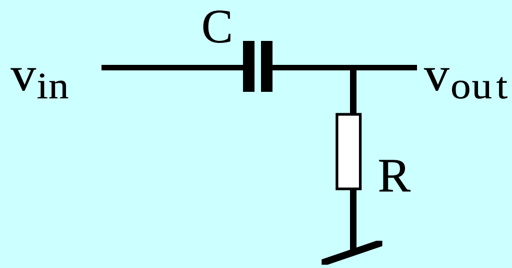
H(ω) is transfer function of system = G(ω)/F(ω)

generally complex so introduces both **phase** and **amplitude** changes

Eg high pass filter $H(\omega) = v_{out}(\omega)/v_{in}(\omega)$

characteristic frequency referred to as pole

$f = 1/2\pi\tau = 1/2\pi RC$



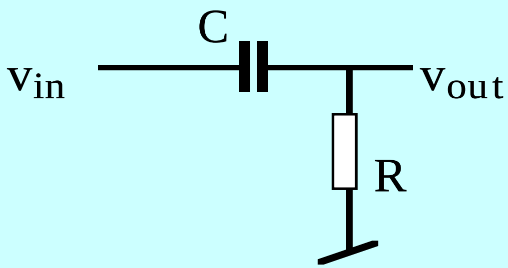
We will find the Fourier (or Laplace) transform an important tool for handling this kind of thing

High and low pass filters

- High pass filter

$H(\omega) = R/(R + 1/j\omega C)$

$= j\omega\tau/(1 + j\omega\tau)$



$\tau = RC$

response to voltage **step** ~ $e^{-t/\tau}$

- Low pass filter

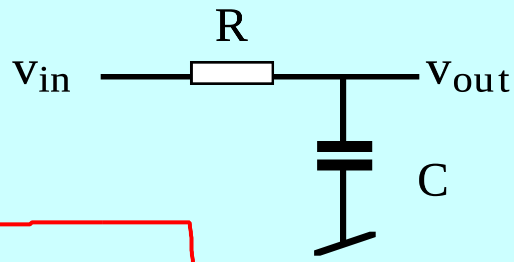
$H(\omega) = (1/j\omega C)/(R + 1/j\omega C)$

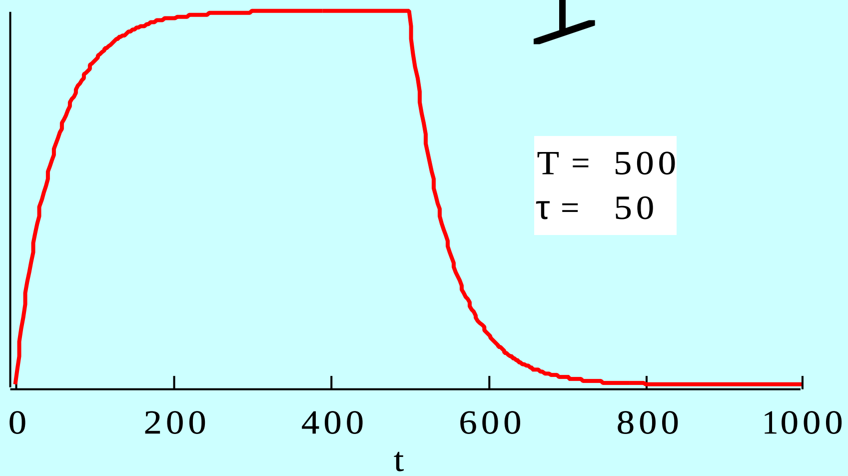
$= 1/(1 + j\omega\tau)$

"roll-off" 6dB/octave at high frequencies

response to voltage **step** ~ $1 - e^{-t/\tau}$

rise time: usually define as 10-90%





Frequency behaviour

- Bode plot

  display transfer function of a circuit as function of frequency

  $H(f) = |H(f)|e^{j\varphi}$

  $= Re[H(f)] + j\,Im[H(f)]$

  Usual to plot gain on log-log scale and phase vs log f

- 3db point

  frequency at which power is half maximum

  voltage = x $1/\sqrt{2}$

- High frequency behaviour

  low pass filter $H \sim 1/\omega\tau$

  -20dB for each f decade

  (-6dB per octave)