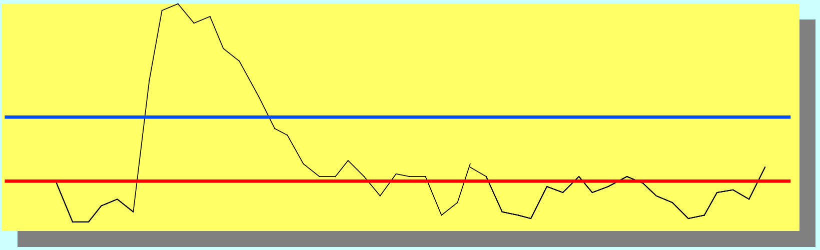
Noise

• What is NOISE? *A definition:*

Any unwanted signal obscuring signal to be observed

two main origins



• EXTRINSIC NOISE examples...

pickup from external sources unwanted feedback

RF interference from system or elsewhere, power supply fluctuations

ground currents

small voltage differences => currents can couple into system

may be hard to distinguish from genuine signals *but* AVOIDABLE

Assembly & connections, especially to ground, are important

• INTRINSIC NOISE

Fundamental property of detector or amplifying electronics

Can't be eliminated but can be MINIMISED

Origins of noise in amplifying systems

•1. Thermal noise

Quantum-statistical phenomenon

Charge carriers in constant thermal motion

macroscopic fluctuations in electrical state of system

•2. Shot noise

Random fluctuations in DC current flow

originates in quantisation of charge
non-continuous current

•3. 1/f noise

Characteristic of many physical systems

least well understood noise source

commonly associated with interface states in MOS electronics

Thermal noise (i)

- Einstein (1906) , Johnson, Nyquist (1928)

  e.g. resistor: *~10^{23} possible states*

  macroscopic statistical average over micro-states

- Experimental observation

  Mean voltage $\langle v \rangle = 0$

  Variance $\langle v^2 \rangle = 4kT.R.\Delta f$ Δf = observing bandwidth

  $\sigma(v) = \sqrt{\langle v^2 \rangle} = 1.3\ 10^{-10}\ (R.\Delta f)^{1/2}$ volts at 300K

  e.g. R = 1MΩ Δf = 1Hz $\sigma(v) = 0.13\mu V$

  gaussian distribution of fluctuations in v

  Noise power = $4kT.\Delta f$

  independent of R & q independent of f - WHITE

- Quantum effects

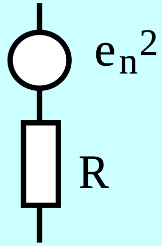
  Normal mode energies: $kT \approx hf/(e^{hf/kT} - 1)$ $kT \gg hf$

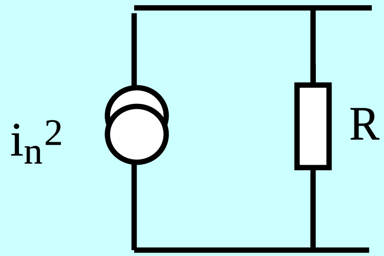
  at 300°K kT = 0.026 eV $hf = kT$ at $f = 6.10^{12}$ Hz

Thermal noise (ii)

- Circuit representations

  Noise generator + noiseless resistance R





- Spectral densities

  mean square noise voltage or current per unit frequency interval

  $w_V(f) = 4kTR$ (voltage) $w_I(f) = 4kT/R$ (current)

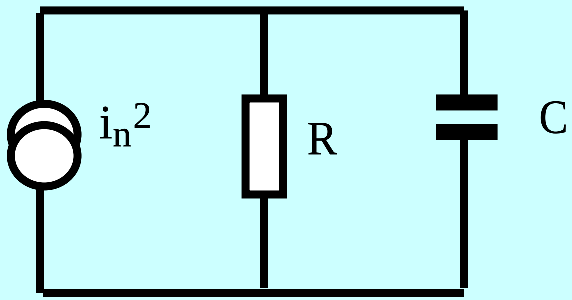
- Why not infinite fluctuation in infinite bandwidth?

  A-1: QM formula -> 0 at high f

  A-2: real components have

  capacitive behaviour (high f)

  or inductive (low f).

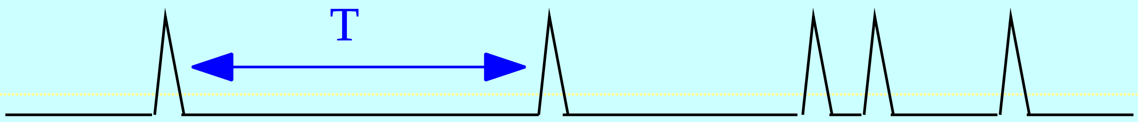


with R and C in parallel $\langle v^2 \rangle = kT/C$

Shot noise

- Poisson fluctuations of charge carrier number

  eg arrival of charges at electrode in system - induce charges on electrode



quantised in amplitude and time

- Examples

  electrons/holes crossing potential barrier in diode or transistor

  electron flow in vacuum tube

$\langle i_n^2 \rangle = 2qI.\Delta f$ WHITE (NB notation e = q)

I = DC current

gaussian distribution of fluctuations in i

1/f noise

- White noise sources frequently dominate in many real systems

however frequency dependent noise is also common

- 1/f noise is a generic term for a wide range of phenomena, possibly not always related

Power spectral density

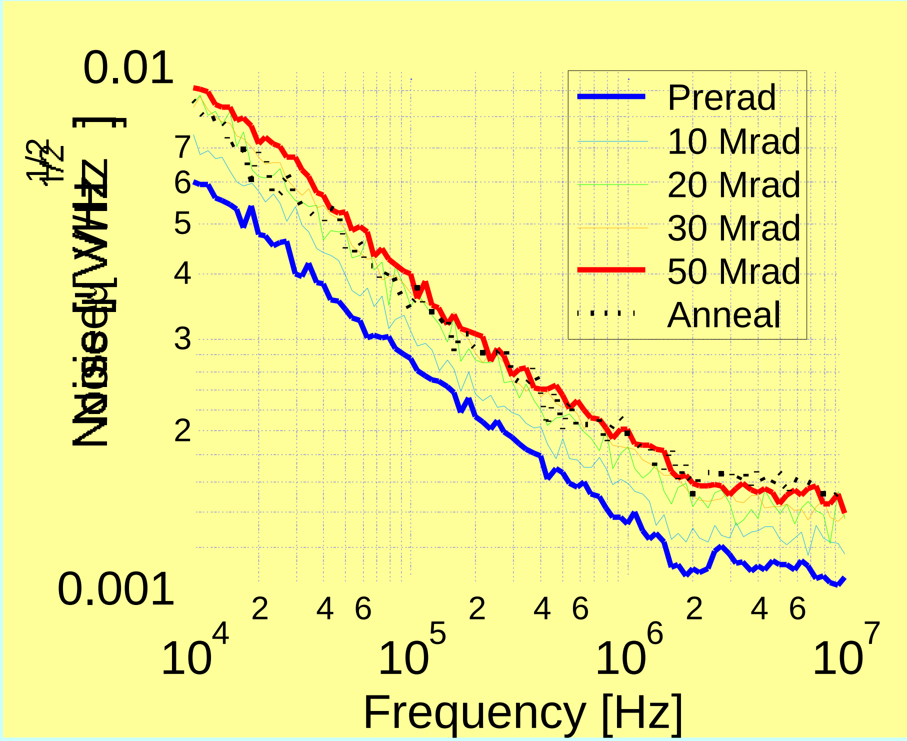
$w(f) = A_f / f^n$ n ~ 0.8-1.5 typical

- Most important for MOS FET devices,

often dominates

but can also arise in other circumstances

e.g. dielectrics,...



pMOS transistor noise spectrum

An explanation for 1/f noise

- Silicon MOS transistors are very sensitive to oxide interface

    typically populated by band-gap energy levels (traps)

    traps exchange charge with channel - ie. emit and capture electrons or holes

- Traps have lifetime to retain charge $h(t) \sim e^{-t/\tau}$

    Expect a range of traps with different time constants, distributed with $p(\tau)$

    in frequency domain $H(\omega) \sim 1/(1+j\omega\tau)$

    Deduce frequency spectrum by integrating over all values of τ

    $w(f) \sim \int_0^\infty p(\tau)|H(\omega)|^2 d\tau$

    If $p(\tau)$ = constant, ie all time constants equally probable

    $w(f) \sim \int_0^\infty d\tau/(1 + \omega^2\tau^2)$ [standard integral, put $\tan\theta = \omega\tau$]

    $= A/f$

    Many other real-life processes have $e^{-t/\tau}$ time distributions -

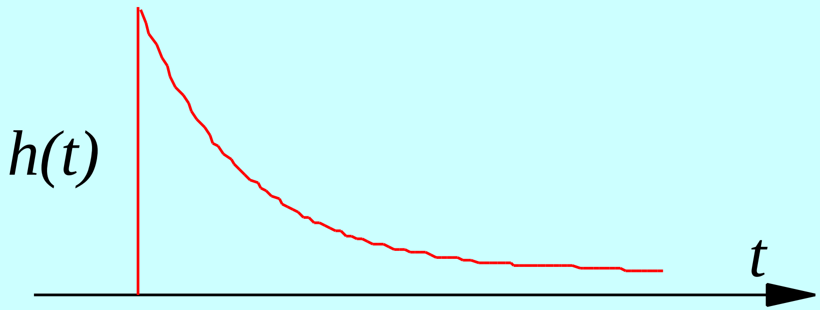
    typical of random, Poisson-type processes

Campbell's theorem - time domain

- Most amplifying systems designed to be linear

  $S(t) = S_1(t) + S_2(t) + S_3(t) + \ldots$

- Impulse response h(t) = response to δ

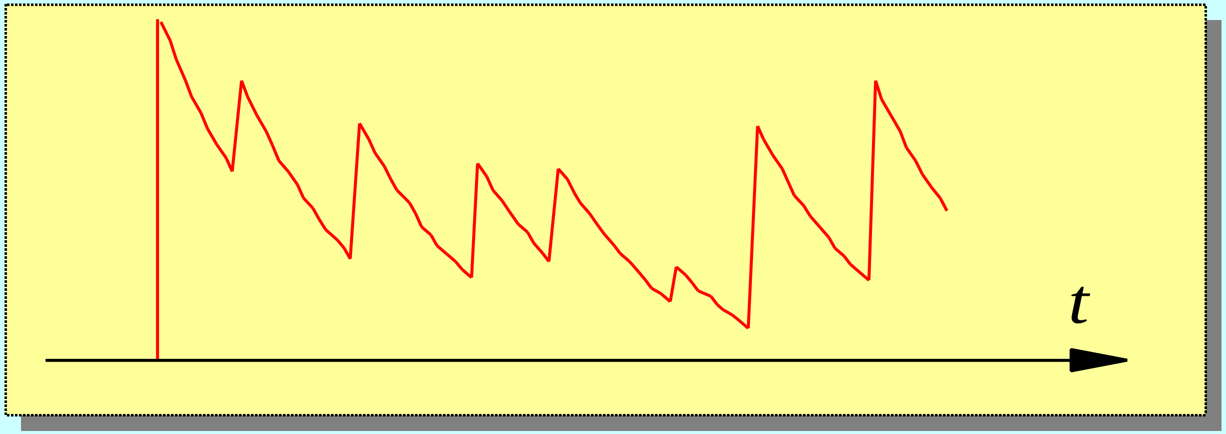


- In a linear system, if random impulses occur at rate n

average response $\langle v \rangle = n \int_{-\infty}^{t_{obs}} h(t)dt$ (A)

variance $\sigma^2 = n \int_{-\infty}^{t_{obs}} [h^2(t)]dt$ (B)

i.e. sum all pulses preceding time, t_{obs}, of observation



Campbell's theorem - frequency domain

- Recall relationship between impulse response h(t) and transfer function H(ω)

$H(\omega) = \int_{-\infty}^{\infty} h(t).e^{-j\omega t}\, dt \qquad h(t) = \int_{-\infty}^{\infty} H(\omega)e^{j\omega t} df \qquad H(\omega) = v_{out}(\omega) / v_{in}(\omega)$

- Rewrite (B) using Parseval's Theorem

$$\int_{-\infty}^{\infty} h^2(t)\, dt = \int_{-\infty}^{\infty} |H(\omega)|^2\, df = 2\int_0^{\infty} |H(\omega)|^2\, df$$

h(t) is real and thus
$H(-\omega) = H^*(\omega)$

so $\sigma^2 = n\int_{-\infty}^{\infty} h^2(t)\, dt = n\int_{-\infty}^{\infty} |H(\omega)|^2\, df$

This relates noise spectral densities at input and output:

$w_{out}(f) = w_{in}(f)\, |H(\omega)|^2$ can use theorem to calculate system response to noise

- eg. shot noise Consider impulse response to be impulse (ie unchanged!)

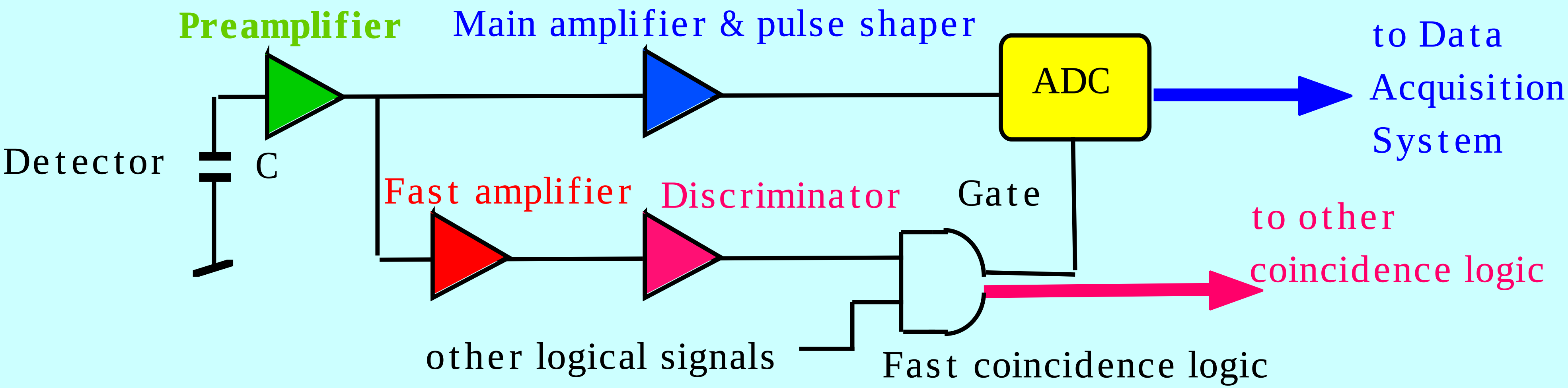
$h(t) = e\delta(t) \Rightarrow H(\omega) = 1.e$

$\sigma^2 = n\, e^2 \int_{-\infty}^{\infty} \delta^2(t)\, dt = ne^2 \int_{-\infty}^{\infty} |H(\omega)|^2\, df = 2ne^2 \Delta f$

but $n = I/e \Rightarrow \sigma^2 = \underline{2eI\, \Delta f}$

Amplifier systems for spectroscopy

• typical application - precise measurements of x-ray or gamma-ray energies



• pre-amplifier *first stage of amplification*

• main amplifier - *adds gain and provides bandwidth limiting*

ADC - analogue to digital conversion - *signal amplitude to binary number*

• fast amplifier and logic -

start ADC ("gate") and flag interesting "events" to DAQ system

- most signals arrive randomly in time.

Other logic required to maximise chance of "good" event, eg second detector

"Rules" of low noise amplifier systems

- Combine uncorrelated noise sources in quadrature

  $e^2_{tot} = e_1^2 + e_2^2 + e_3^2 + \ldots + i_n^2 R^2 + \ldots$

  follows from Campbell's theorem

  consider as combinations of gaussian distributions

- First stage of amplifier dominates

  noise originates at input

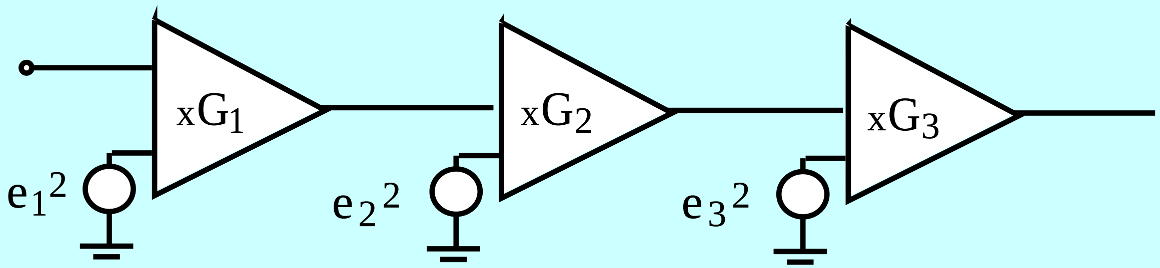
- Noise is independent of amplifier gain or input impedance

  so noise can be referred to input

- In real systems both are approximations - but normally good ones

  so often sufficient to focus on input device

Amplifiers - dominance of input stage



- Amplifier systems (and amplifiers!) usually consist of several stages

  impractical to put all gain at one location - power, heating, material, size,...

- Calculate signal and noise at output

  $S_{out} = G_1.G_2.G_3 S_{in}$ for 3 stage system, but can easily extend to N

  $(e_{out})^2 = G_1^2.G_2^2.G_3^2 e_1^2 + G_2^2.G_3^2 e_2^2 + G_3^2 e_3^2$

  $(e_{out} / S_{out})^2 = (e_1^2 + e_2^2/G_1^2 + e_3^2 /G_1^2.G_2^2)/ S_{in}^2$

- Desirable to maximise gain at input stage

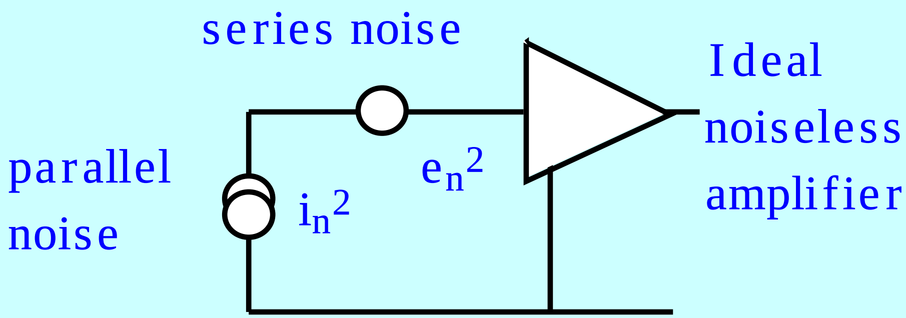
  eg stage 1 boosts signal enough for transmission down cable and should be large enough that environmental noise is not significant

Amplifiers - location of noise sources

- Normal to partition noise sources

  not fundamental to calculations

  but can simplify!

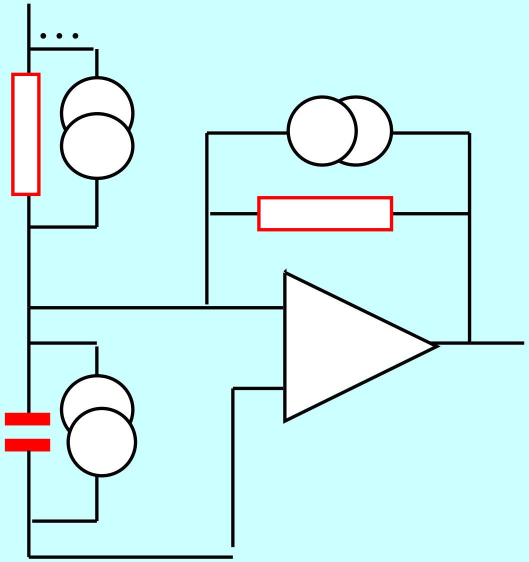


- Parallel noise sources appear as currents at input

  detector leakage current

  bias resistors

  feedback resistor



Series noise sources appear as voltage at input

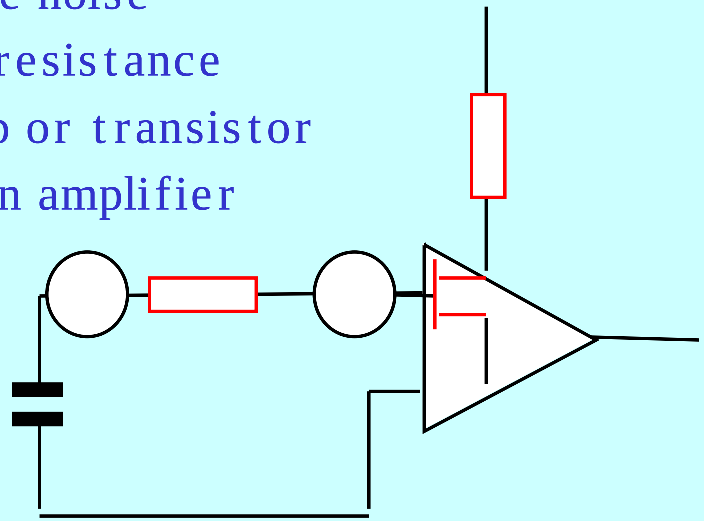
transistor gate noise

device series resistance

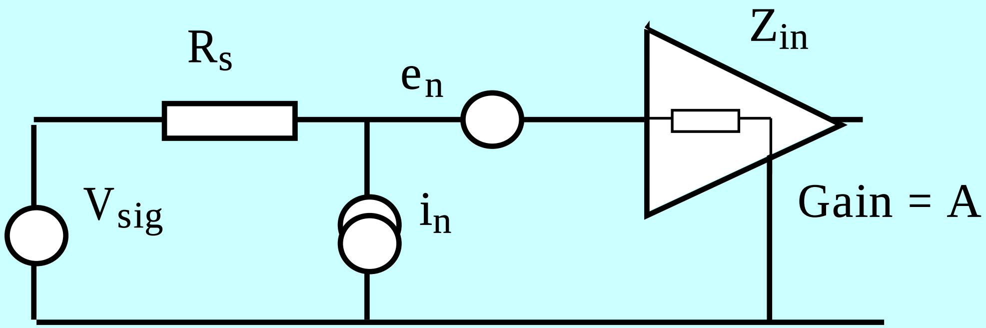
microstrip or transistor

load resistor in amplifier

...



Amplifiers - reference to input



assume:
signal source and associated impedance
noise sources
amplifier with gain & input impedance

- noise at output $E_{no}^2 = A^2 \{e^2_n Z^2_{in}/(Z_{in} + R_s)^2 + i^2_n R^2_s Z^2_{in}/(Z_{in} + R_s)^2\}$
- transfer function $K = V_{out}/V_{in} = A.V_{sig}.Z_{in}/(Z_{in} + R_s)V_{sig} = AZ_{in}/(Z_{in} + R_s)$
- noise at input $E_{ni}^2 = E_{no}^2/K^2 \Rightarrow \mathbf{E^2_{ni} = e^2_n + i^2_n R^2_s}$ no Z_{in} or A dependence

easy to show analogous result $\mathbf{I^2_{ni} = i^2_n + e^2_n/R^2_s}$ choice is for convenience

in most detector systems, there is a current signal source and a parallel capacitance

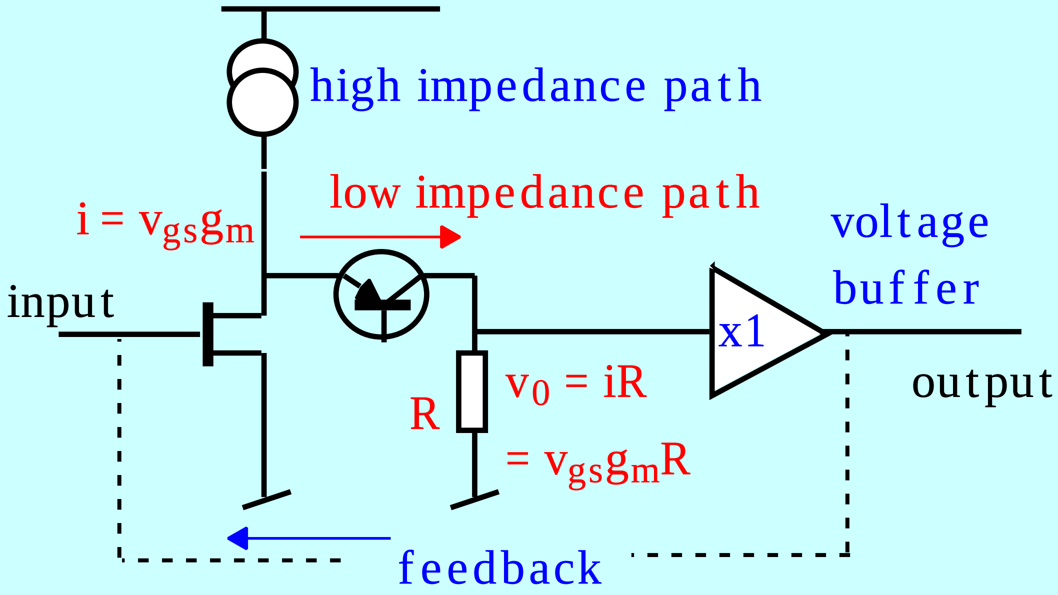
- then the spectral distribution of noise at the input is affected

$\mathbf{I^2_{ni} = i^2_n + e^2_n \omega^2 C^2}$ no longer white

Real amplifiers

- The first amplifier stage dominates - reason for distinction of pre-amplifier

  most low-noise amplifiers are based on very simple concepts



voltage amplifier

$G = g_mR$

add feedback to define type

eg. charge or current sensitive

within the preamplifier we expect the input device to be the most important

- To understand noise performance we need to study input device

  bipolar or FET

Bipolar transistor noise

- Two shot noise sources

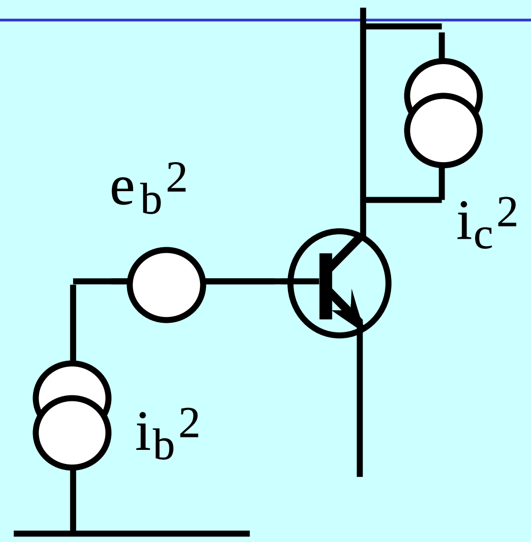
  Fluctuations in I_C $\quad i_c^2 = 2eI_C\Delta f$

  Fluctuations in I_B $\quad i_b^2 = 2eI_B\Delta f$

  but these are correlated since $I_B \approx I_C/\beta$

- Thermal noise in base & contacts $\quad e_b^2 = 4kTr_b\Delta f$

  r_b = base spreading resistance

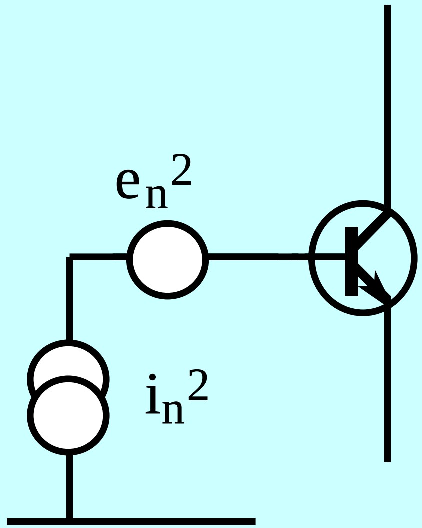


- Transfer noise sources to input

  use $\quad v_{be} = i_c r_e = (kT/qI_c)i_c$

  $e_n^2 = (4kTr_b + 2qI_c r_e^2)\Delta f \quad = [4kTr_b + 2(kT)^2/qI_c]\Delta f$

  $i_n^2 = 2eI_B\Delta f$



- The constraint that i^2_n, e^2_n are correlated gives a limited range of noise with best performance for high speed applications

Some useful numerical values

$kT/e \approx 0.025\ V = 25mV$ at 293K (Room temperature)

$e = 1.6\ 10^{-19}$ coulomb

- Parallel noise - what resistor is equivalent to given current?

$4kT/R_p = 2eI$

$I = 2(kT/e)(1/R_p) = 50mV/R$

eg, $1M\Omega \approx 50nA$

- Parallel noise spectral density - units and magnitudes

$i_n = [2eI_B]^{1/2} = [32e^{-18} I_B]^{1/2} = 0.6 I_B^{1/2}$ **nA/√Hz**

eg, 3A ≈ 1**n**A/√Hz 3**µ**A ≈ 1**p**A/√Hz

- Series noise spectral density - units and magnitudes

$e_n = [4kTR_s]^{1/2} = [4(kT/e)eR]^{1/2} = [1.6\ 10^{-20} R]^{1/2} = 0.13R^{1/2}$ **nV/√Hz**

eg, $60\Omega \approx 1nV/\sqrt{Hz}$

Bipolar noise example

- r_b is often small so neglect

$\beta = 100$	$I_C = 1mA$ $I_B = 10\mu A$	$I_C = 100\mu A$ $I_B = 1\mu A$
$e_n = [2(kT)^2/qI_C]^{1/2}$ $= [2(kT/q)^2(q/I_C)]^{1/2}$	0.45 nV/√Hz	1.4 nV/√Hz
$i_n = [2qI_B]^{1/2}$	1.8 pA/√Hz	0.6 pA/Hz
i_nR_S for $R_S = 1k\Omega$	1.8 nV/√Hz	0.6 nV/Hz
$[4kTR_S]^{1/2}$ for $R_S = 1k\Omega$	4.0 nV/√Hz	4.0 nV/√Hz
NF for $R_S = 1k\Omega$	0.84 dB	0.59 dB

- Noise figure - often used to characterise voltage amplifier performance

NF [dB] = $10\log_{10}$(Total noise power at input/Source noise power)

$$= 10\log_{10}\left(\frac{e_n^2 + i_n^2R_s^2 + 4kTR_s}{4kTR_s}\right) \qquad = 10\log_{10}\left(1 + \frac{e_n^2 + i_n^2R_s^2}{4kTR_s}\right)$$

MOSFET Noise

- Gate current shot noise

  $i_g^2 = 2eI_G\Delta f$ negligibly small for most applications

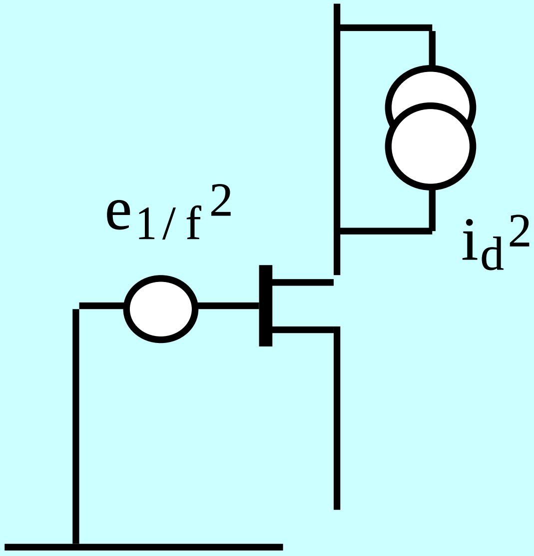
  high impedance of gate oxide (insulator) to substrate

- Thermal noise inside the transistor

  thermal current fluctuations in channel (- but **not** shot)

  $$i_d^2 = (4kT/R_n)\,\Delta f = 4kT(2/3)g_m\Delta f$$

  1/f noise $e_n^2 \approx A/f$

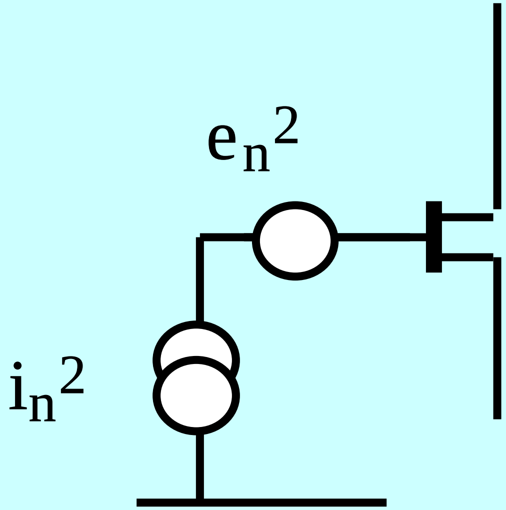


- Transfer noise sources to input

  $i_d = g_m v_{gs}$ => $e_n^2 = i_d^2/g_m^2$

  $e_n^2 = [4kT(2/3g_m) + A/f]\Delta f$

  $i_n^2 \approx 0$



MOSFET thermal noise

- Parameters are (almost) controlled by geometry and current alone

  $g_m = [2\mu C_{ox}(W/L)I_{DS}]^{1/2}$ $C_{ox} = \varepsilon_{ox}/t_{ox}$

  $e_n^2 = 4kT(2/3g_m)\Delta f \sim 1/g_m$

- How to get large g_m?

  Increase I_{DS} - but $e_n \sim I^{-1/4}$ power is a concern

  Increase C_{ox} - it scales with technology feature size so modern processes help

  1980 ~10μm 2000 ~ 0.25μm for $L_{min} \approx 0.25\mu m$ $t_{ox} \sim 5nm$

  Make W/L large

  W can be made very large, eg 2-3mm

  L can be ≈ minimum feature size

  Cooling - can gain small amount - but device heats

- Caveat carrier mobility **μ** - unfortunately not a constant

  carriers typically approach saturation velocity at high electric fields

  $1V/0.25\mu m = 4x10^4$ V/cm   so $\mu = v/E$ falls as transistors shrink

MOSFET 1/f noise

- $e_{1/f}^2 \approx A_f/f$

$A_f \approx K_f/[WL(C_{ox})^2]$

K_f is technology dependent $K_f \sim 10^{-30} - 10^{-32}$ C.cm^{-2}

PMOS transistors are significantly better than NMOS

- Corner frequency

f_{corner} where $e_{thermal} = e_f$ typically ~ 10-1000kHz

dependent on technology details, device dimensions

JFET Noise

- Almost identical to MOSFET
- \- differences are

  Negligible 1/f noise - channel is buried below surface

  Small gate current - gate is p-n diode

  both can be reduced by cooling

- JFET is interesting for high resolution spectroscopy

  allows to employ long shaping time constants (low f)

  because of the very low 1/f noise

- Noise sources referred to input

  $e_n^2 = 4kT(2/3g_m)\,\Delta f$

  $i_n^2 = 2eI_g\,\Delta f$