Signal processing

•Now identified: Noise sources Amplifier components and basic design

How to achieve “best” signal to noise?

•Possible constraints

power consumption

ability to provide power & extract heat, material for cooling

layout of system (space, cables,...)

signal rate

eg. signal "pileup" vs E resolution

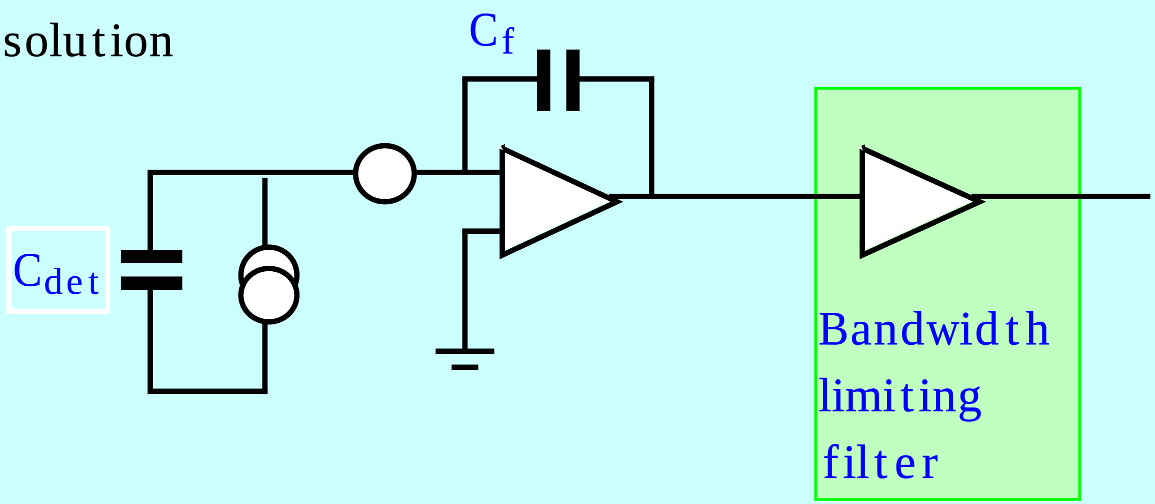
•Two methods

Pulse shaping

time invariant filter

Pulse sampling

time variant filter



Pulse shaping

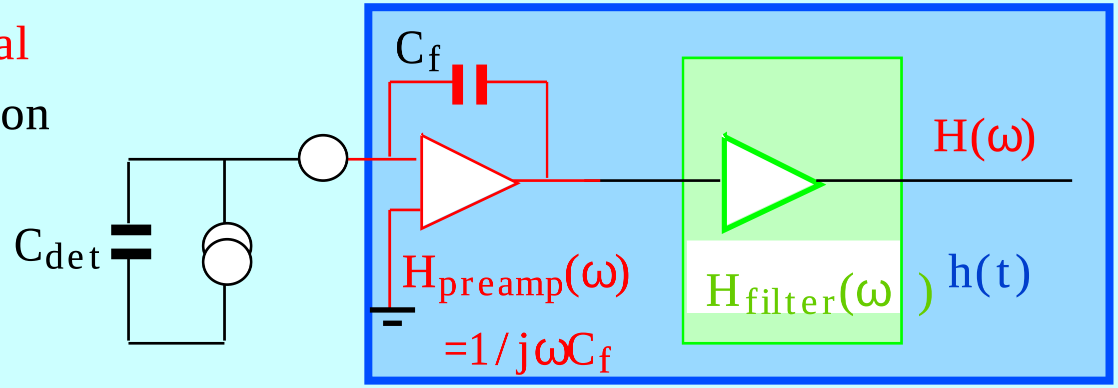
- Preamplifier pulse shape is impractical

  step (exponential) with long duration

  short, sharp peak

- Calculate signal and noise in t and f

  using results so far



- Signal

$$S_{out}(\omega) = S_{in}(\omega).H_{preamp}(\omega).H_{filter}(\omega) = S_{in}(\omega).H(\omega)$$

$$S_{out}(t) = \int S_{in}(t)*h_{preamp}(t)*h_{filter}(t) = \int S_{in}(t)*h(t)$$ **convolution**

- Noise

$$\sigma^2 = \int I_n^2 |H(\omega)|^2 \, df = \int (e_n^2 \omega^2 C^2 + i_n^2) \, |H(\omega)|^2 \, df$$

$$= e_n^2 C^2 \int |\omega H(\omega)|^2 \, df + i_n^2 \int |H(\omega)|^2 \, df$$

$$\sigma^2 = e_n^2 C^2 \int [h'(t)]^2 \, dt + i_n^2 \int [h(t)]^2 \, dt$$

<u>provided e_n & i_n are white</u>

Equivalent Noise Charge

- Noise must be compared with signal - normalisation

$S_{out}(t) = \int Q\delta(u)*h(t-u)du = h(t)$ inject unit impulse ie. Q = 1

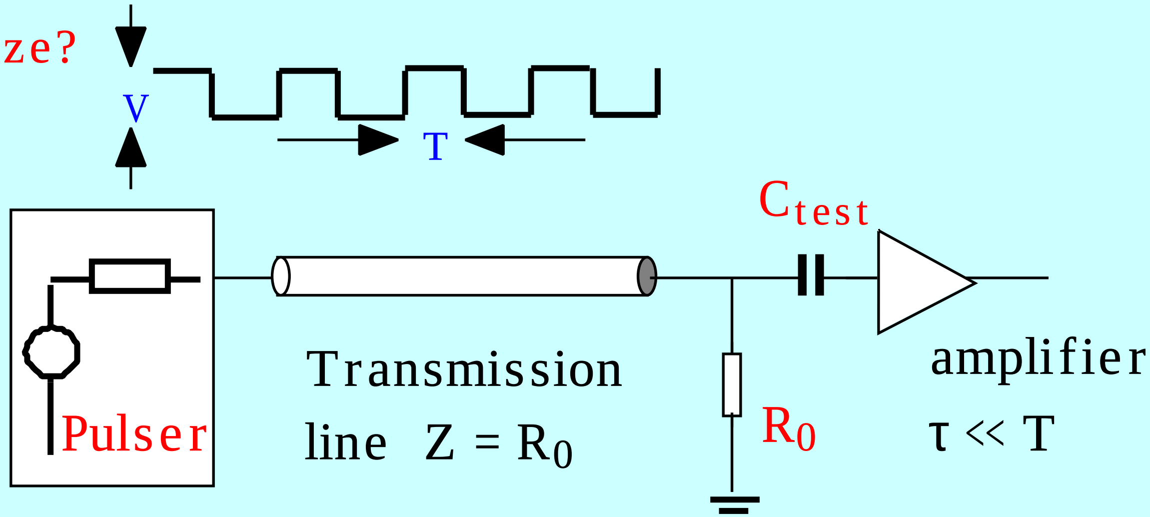
h(t) contains gain

$ENC^2 = e_n^2C^2\int [h'(t)]^2\, dt + i_n^2\int [h(t)]^2\, dt$ normalised to signal of unit amplitude

- ENC = signal which produces output amplitude equal to r.m.s. noise

desirable to measure in absolute units - e, coul, keV(Si),..

- How to inject an impulse of known size?



$Q_{test} = C_{test}V_{test} = Ne$

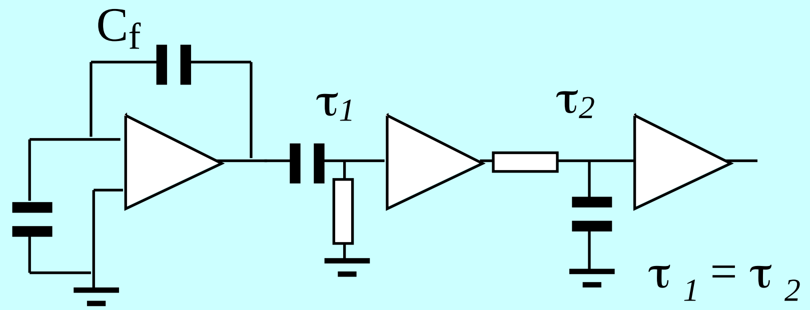
measure V_{out} for known Q_{test}

Example noise calculation

- Preamplifier + CR-RC bandpass filter = <u>equal τ</u> high & low pass filters

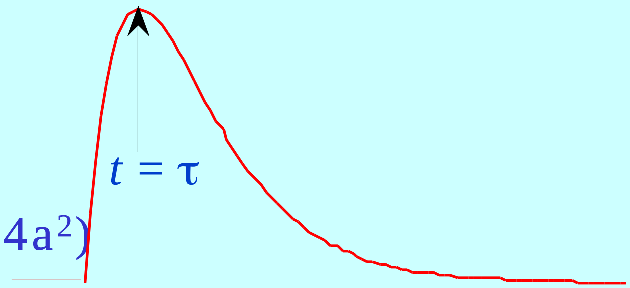
$h(t) = e^{1}(t/\tau)e^{-t/\tau} = e^{1}(at)e^{-at} = 1 \text{ at } t = \tau$

$h'(t) = e^{1}a(e^{-at} - at\,e^{-at}) = e^{1}ae^{-at}(1-at)$



$[h(t)]^2 = e^2a^2t^2e^{-2at}$

$[h'(t)]^2 = e^2a^2e^{-2at}(1-2at+a^2t^2)$



$I_0 = \int_0^\infty e^{-2at}dt = [e^{-2at}/(-2a)]_0^\infty = 1/(2a)$

$I_1 = \int_0^\infty te^{-2at}dt = [te^{-2at}/(-2a)]_0^\infty + (1/2a)\int_0^\infty e^{-2at}dt = 1/(4a^2)$

$I_2 = \int_0^\infty t^2e^{-2at}dt = [t^2e^{-2at}/(-2a)]_0^\infty + (1/2a)\int_0^\infty 2te^{-2at}dt = 1/(4a^3)$

so $\int_0^\infty [h(t)]^2dt = e^2a^2/(4a^3) = e^{-2}\tau/4$

$\int_0^\infty [h'(t)]^2dt = e^2a^2[1/(2a) - 2a/(4a^2) + a^2/(4a^3)] = e^2a/4 = e^2/(4\tau)$

$$ENC^2 = \frac{e^2}{8}\left(\frac{4kTR_sC_{tot}^2}{\tau} + 2eI\tau\right)$$

1/f noise ignored

Factor (1/2) from requiring frequencies from 0 to ∞

Improved time invariant filters

- Optimal filter = infinite exponential cusp

  $h(t) = \exp(-|t|/\tau_{opt})$

  gives equal contributions from series and parallel sources, if...

  $\tau_{opt} = C_{tot}^{2}.R_s.R_p$ R_s and R_p are equivalent noise resistances

  impractical in real systems

- Practical filters - wide range of possibilities !

  CR-RC^n type n ~1-2

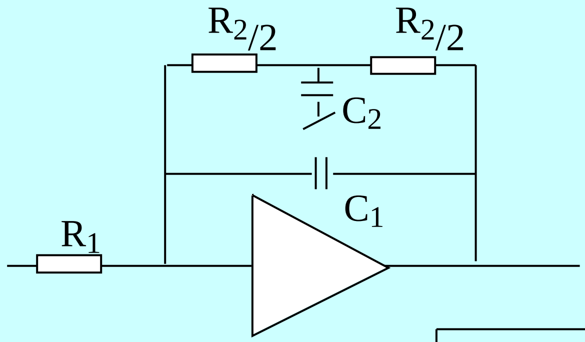
  RC = low pass filter CR= high pass

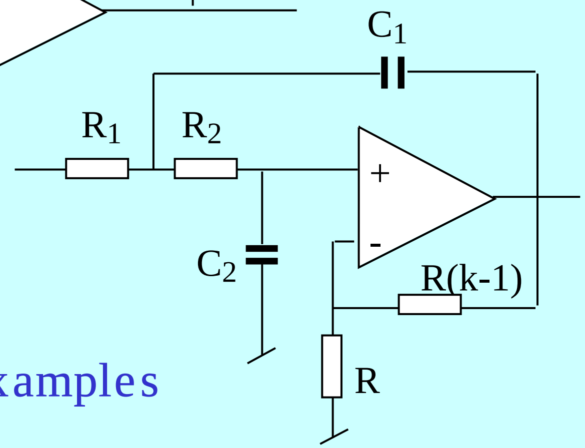
  easy to implement

  CR-RC^n n ~5-7

  semi-gaussian ≈ symmetric pulses

  active filters based on op-amp configurations





examples

Noise after pulse shaping

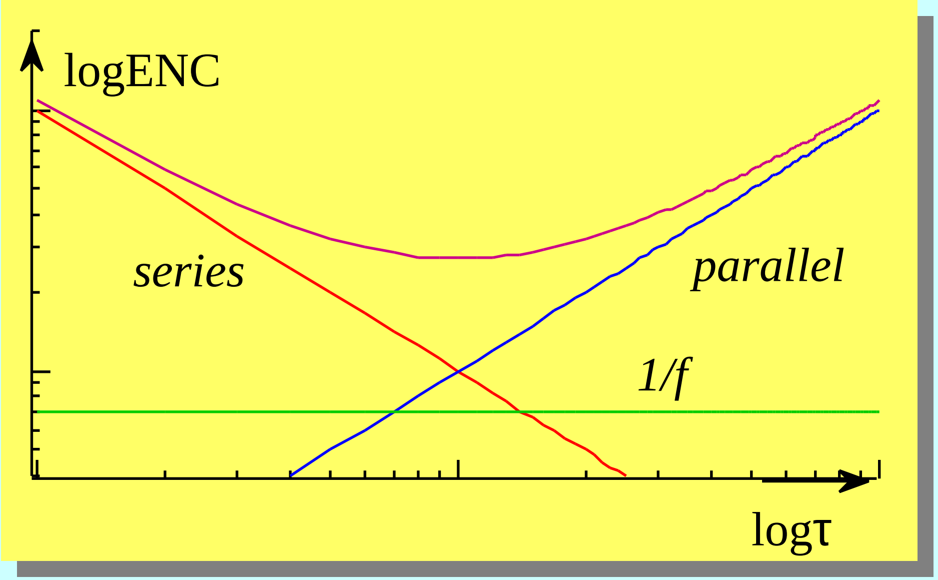
- General result is

$$ENC^2 = \alpha e_n^2 C^2/\tau + \beta i_n^2 \tau + \gamma C^2$$

α,β depend on pulse shape

calculate in t or f

γ : 1/f - can be computed in f only



- a minimum noise can be achieved with a given shaping time constant chosen depending on magnitudes of noise sources

- Useful point of comparison: CR-RC bandpass filter

$$ENC^2 = \frac{e^2}{8}\left(\frac{4kTR_s C_{tot}^2}{\tau} + 2eI\tau + 4A_f C_{tot}^2\right)$$

only 36% worse than theoretical optimal filter

Some numerical values

- An approximate numerical value

$$ENC^2[e^2] \approx \left(\frac{24^2 R_s[k\Omega] C_{tot}^2[pF]}{\tau[\mu s]} + 100^2 \tau[\mu s] \right)$$

- using CR-RC filter, ignoring 1/f noise

ie

$I = 1nA$ $\tau = 1\mu s$ $ENC_p \approx 100e$

$R_s = 10\Omega$ $C = 10pF$ $\tau = 1\mu s$ $ENC_s \approx 24e$

Time variant filters

- Alternative to pulse shaping

  filters based on summation

- Sample & hold method

  initially switches S_0 B_1 B_2 open, S_1 S_2 closed

  switch S_0 is Reset

  V_{out} = output from charge sensitive preamplifier

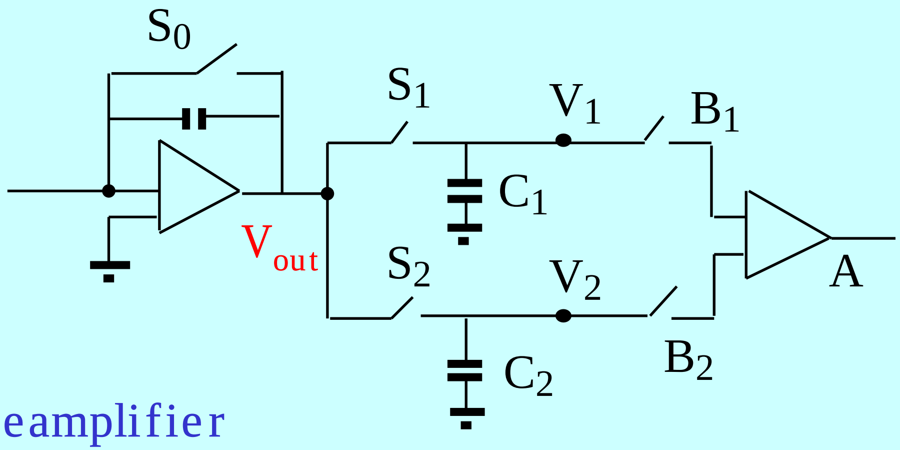
  open S_1 : preserves V_{out} on C_1

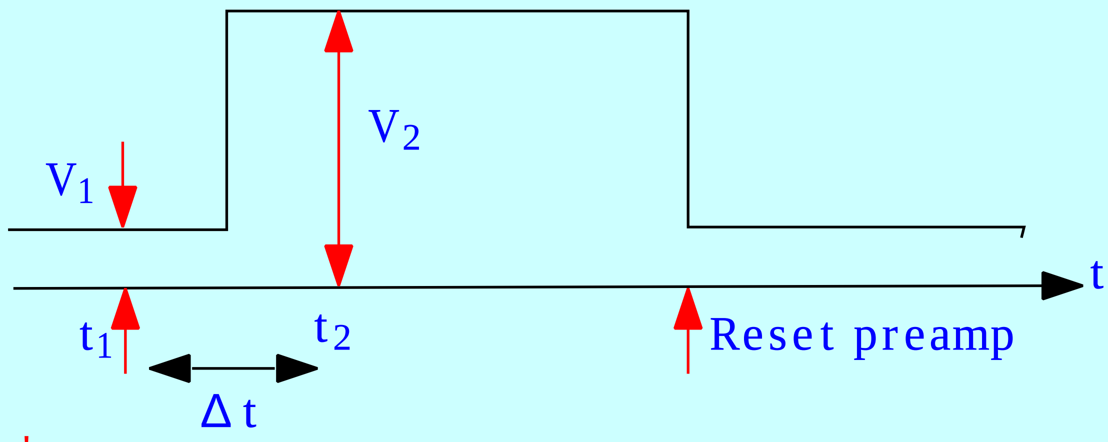
  after time Δt open S_2 : preserves V_{out} on C_2

  then, close B_1 and B_2:

  output $A = V_1 - V_2$

  reset preamp later



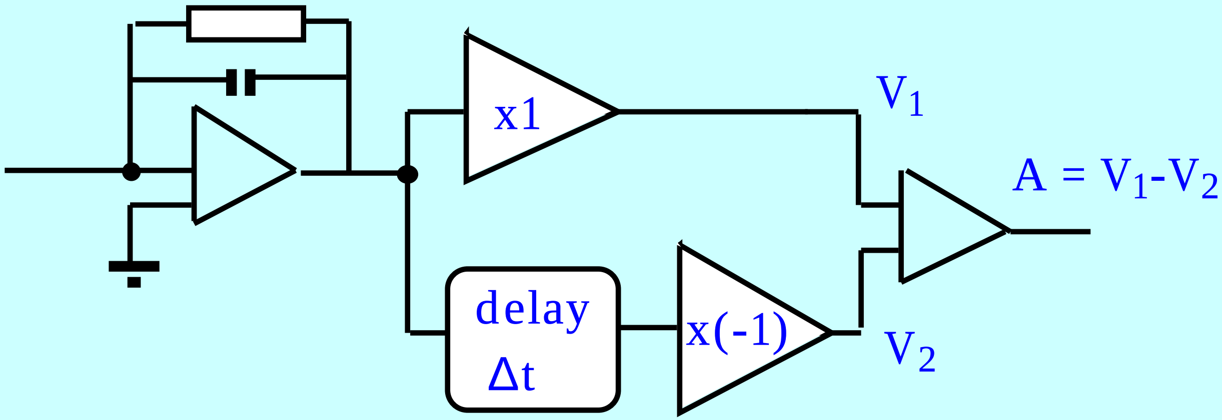


- need to know when signal will arrive!

  Switched capacitor easy to implement convenient for MOS technology

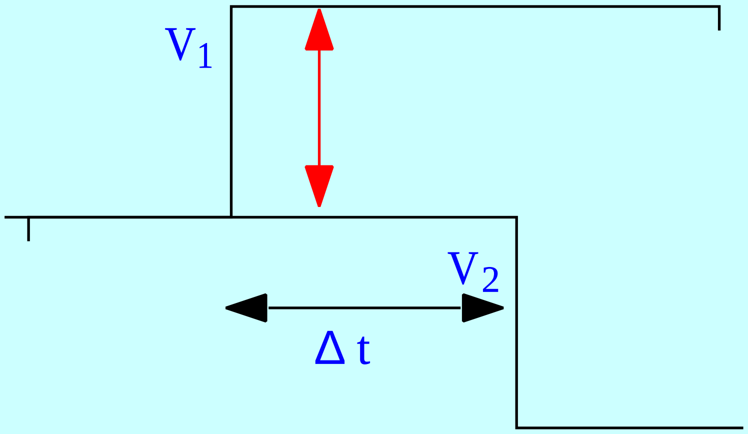
Time variant filters

•Can perform same process with delay line



delay line Double Correlated Sampling

•delay Δt << RC of preamplifier

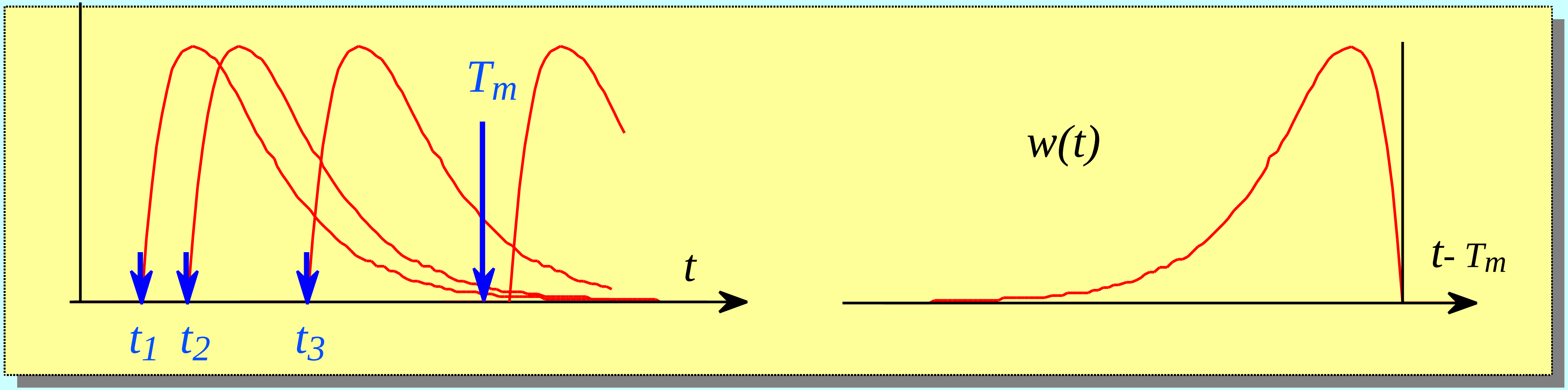


•How to analyse noise performance of time variant systems?

Weighting function

- What output is produced at T_m by impulse at time t?

  consider all t - defines weighting function



- Time invariant filter — w(t) is mirror image of h(t)

- Time variant filter — usually need to think carefully

- Noise calculation — $ENC^2 = e_n^2 C^2 \int [w'(t)]^2 dt + i_n^2 \int [w(t)]^2 dt$

Integrating Analogue to Digital Converter (ADC)

• Integrate signal during application of gate - another time variant filter

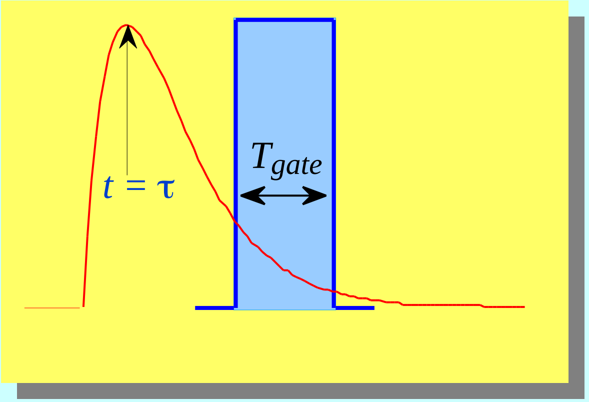
convert charge to digital number

• = **convolution** of pulse shape with gate

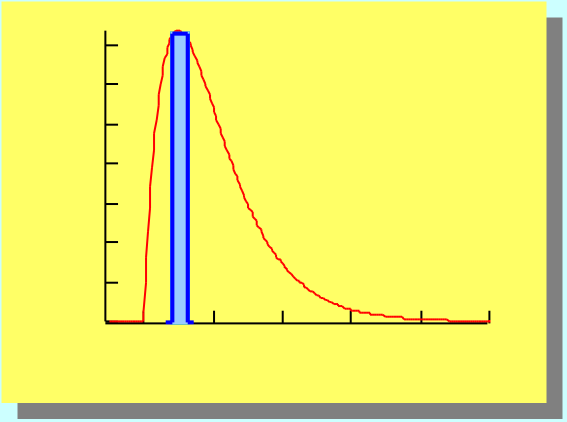
so

$w(t) = h(t) * g_{gate}(t)$

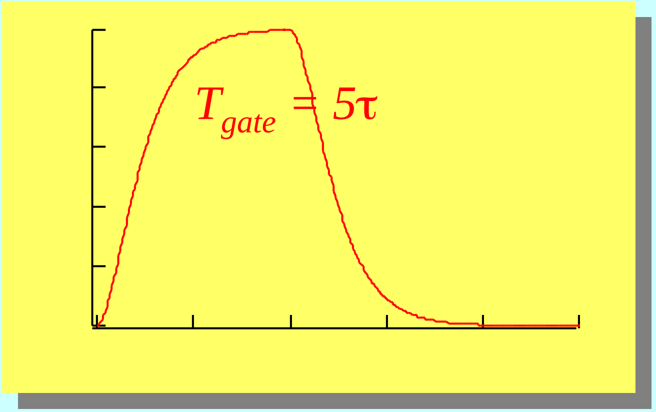
(ignoring t reflection)



• $T_{gate} << \tau$



$T_{gate} >> \tau$



$w(t) = h(t)$

new, wider weighting function

can change filtering and **increase** or **decrease** noise

Digitisation noise

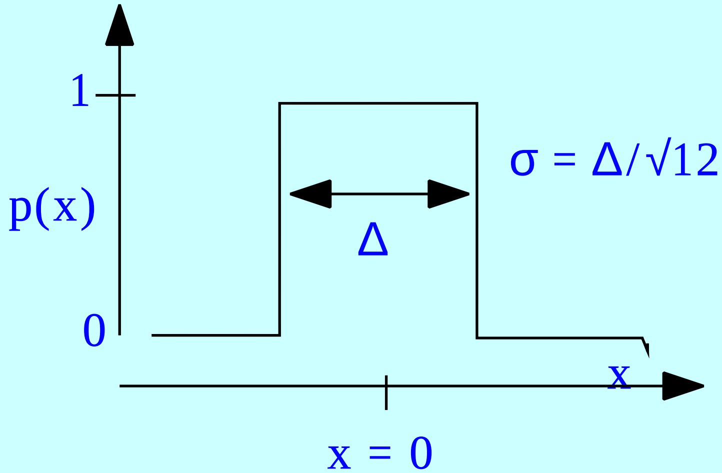
- Eventually need to convert signal to a number

  quantisation (rounding) of number = noise source

  the more precise the digitisation, the smaller the noise

- After digitisation all that is known is that

  signal was between $-\Delta/2$ and $\Delta/2$



$\langle x\rangle = \int x.p(x).dx / \int p(x).dx$

$\sigma^2 = \langle x^2\rangle = \int x^2.p(x).dx \;/ \int p(x).dx$

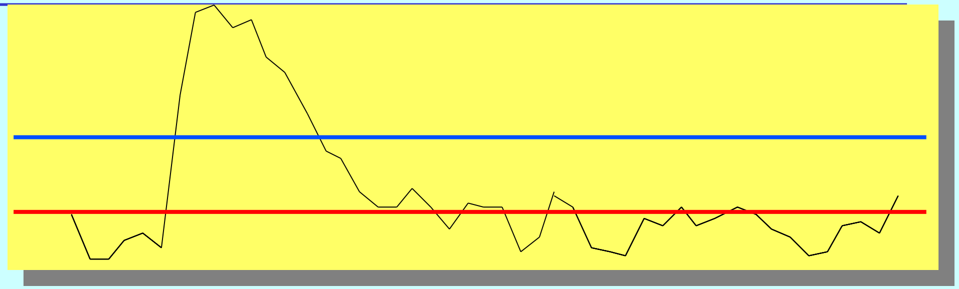
$\int p(x).dx \qquad = \int_{-\Delta/2}^{\Delta/2} dx \;= [x]_{-\Delta/2}^{\Delta/2} = \Delta$

$\int x^2.p(x).dx \quad = \int_{-\Delta/2}^{\Delta/2} x^2.dx \;= [x^3/3]_{-\Delta/2}^{\Delta/2} = 2\Delta^3/24$

so $\quad \sigma^2 = \Delta^2/12$

- ie statistical noise which is proportional to digitisation unit

Level crossing statistics



•Binary counting systems have noise !

fluctuations cause threshold crossing

•Rate of zero (level) crossing f_Z

proportional to spectral density of noise w(f)

$$f_Z = 2\left[\frac{\int_0^\infty f^2 w(f)\,df}{\int_0^\infty w(f)\,df}\right]^{1/2}$$

•General proof is complicated:

imagine noise at several distinct frequencies: f_0, f_1,...

$w(f) = \delta(f-f_0) + \delta(f-f_1) + \ldots$

$$f_Z = 2\left[\frac{\int_0^\infty f^2 \delta(f-f_0)\,df}{\int_0^\infty \delta(f-f_0)\,df}\right]^{1/2} + 2\left[\frac{\int_0^\infty f^2 \delta(f-f_1)\,df}{\int_0^\infty \delta(f-f_1)\,df}\right]^{1/2} \ldots = 2f_0 + 2f_1 + $$

factor 2 because crossings can occur in both directions

Positive level crossing rate

- If we know level crossing rate f_V

  by measurement or calculation

- Rate of crossing, in positive direction, of level V

  is

$$f_v = \frac{f_Z}{2} \exp\left(-v^2 \Big/ 2\sigma^2 \right)$$

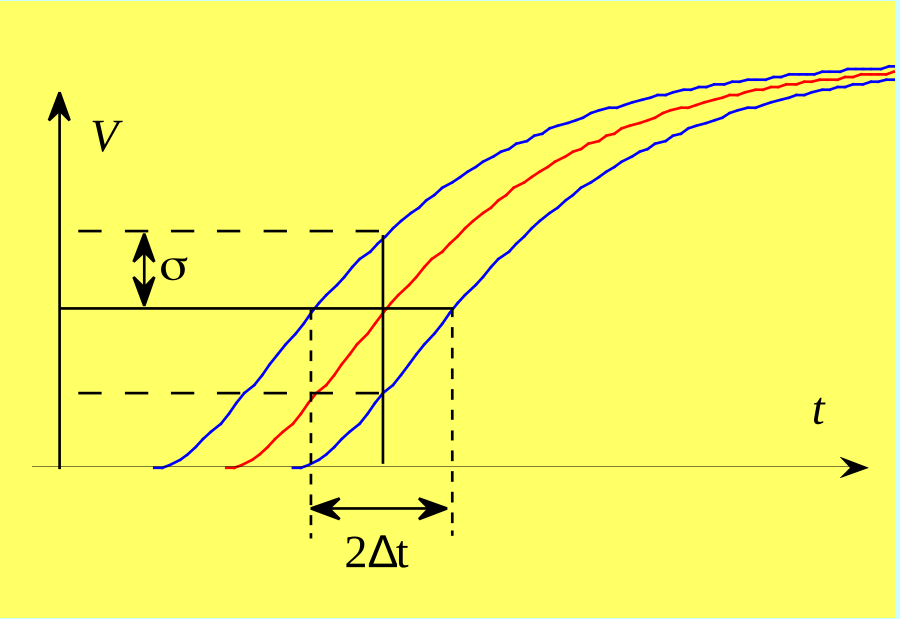
- because noise has gaussian distribution of amplitudes

  factor 1/2 because one direction

- so improvement for any other threshold X, compared to threshold Y

$$\frac{f_X}{f_Y} = \exp\left(-(X^2 - Y^2) \Big/ 2\sigma^2 \right)$$

Time measurements and noise



• When did signal cross threshold ?

noise causes "jitter"

$\Delta t = \sigma_{noise}/(dV/dt)$

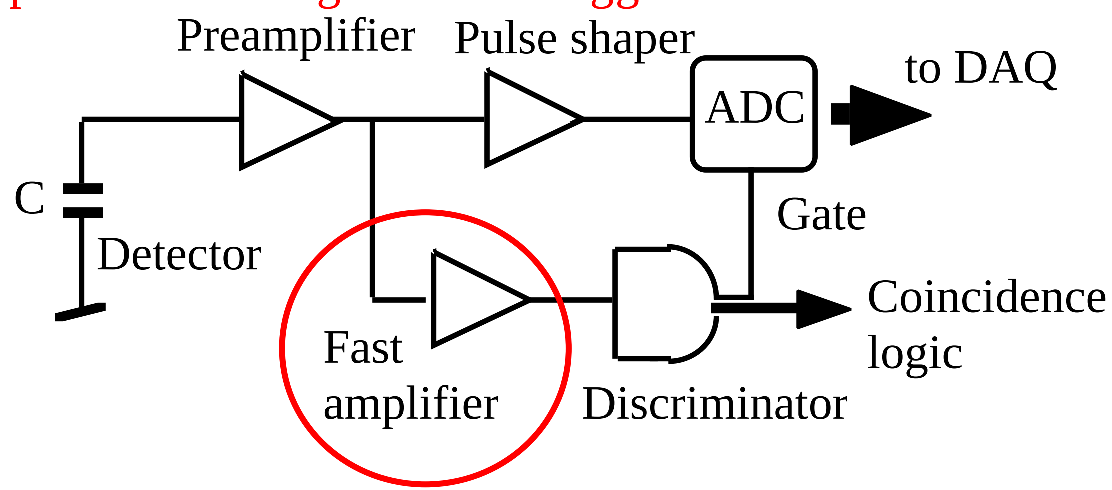
• compromise between

bandwidth (increased dV/dt)

noise (decreased bandwidth)

• limits systems where preamplifier pulse used to generate trigger

eg x-ray detection



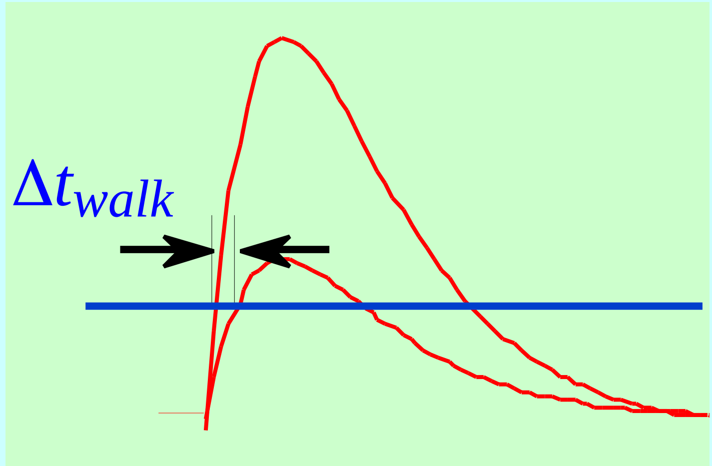
• typical preamp response

$V = V_{max}(1-e^{-t/\tau_{rise}})$

so $\Delta t \approx \sigma_{noise}\tau_{rise}/V_{max}$ $t \ll \tau$

Time and amplitude

•Time "walk" due to amplitude variation



•solutions:

constant fraction discriminator

not simple circuit but easily possible

or

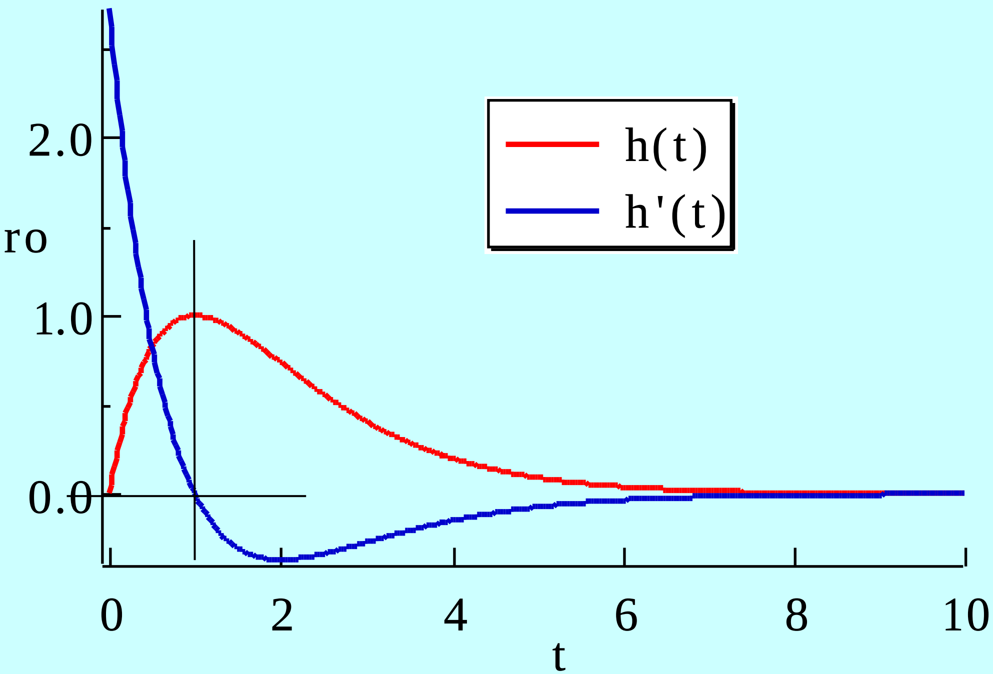
zero-crossing discriminator

differentiate pulse

measure time when output crosses zero

invariant pulse shape

always peaks at same time



•penalty

differentiation also adds noise !

more high frequencies passed