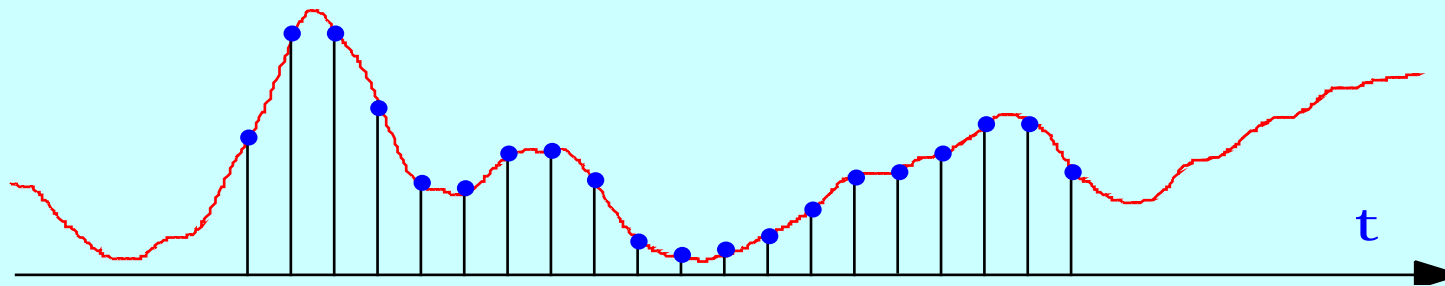


Sampling and digital processing

- Up to now all signals have been considered to be continuous in time but much modern digital electronics uses sampled signals
 - a series of levels taken in coincidence with a clock pulse
 - the amplitude can be represented by a digital value
- We can consider a sampled signal to be convolution of the continuous signal with discrete function



- Start with a puzzle..
 - sample sine wave at fixed frequency (10kHz)
 - vary frequency of sine wave from 250Hz to ~180kHz
 - program calculates frequency from samples, and displays result
 - is this what we expect?

Sampling

- One of the most important applications is processing of sequences of sampled signals derived from continuous analogue waveforms

under certain circumstances a continuous time signal can be completely represented by samples at points equally spaced in time

- Surprising?

moving images (cinema/video)

pointilliste paintings...

not intuitively obvious - infinite no of signals need infinite no of samples

why so important?

dramatic advances in digital technology, now possible to:

sample to convert continuous signal to discrete

process - with discrete time system

convert back to continuous

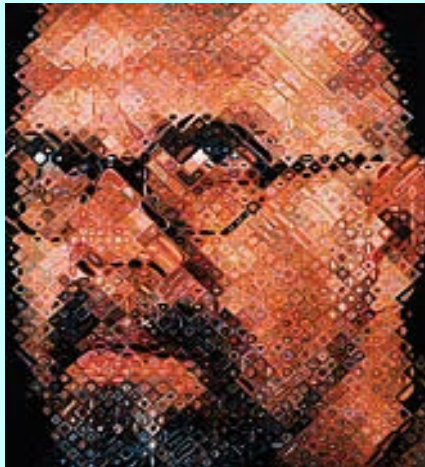
digital audio is a good example

Chuck Close

- US b 1940

"Phil"

Original size 9ft x 7ft



self portrait 1997

the images themselves (jpegs) are also examples of sampling



Fourier transform of periodic functions

- The Fourier series expands in terms of natural frequencies of the system

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk_0 t} \quad a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk_0 t} dt \quad k_0 = \frac{2\pi}{T}$$

outside interval $t = T$, function repeats

- What is the Fourier series for $x(t)$?

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk_0 t} dt \quad \text{so} \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk_0 t}$$

- Consider a signal $x(t)$ with FT $X(\omega)$ which is a unit impulse in frequency

$$X(\omega) = \delta(\omega - k_0) \quad \text{then} \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - k_0) e^{j\omega t} d\omega = \frac{1}{2\pi} e^{jk_0 t}$$

- therefore

$$X(\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta(\omega - k_0) \quad x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{jk_0 t} = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

- ie the Fourier transform of a periodic series of impulses in time
- is a series of (magnitude $2\pi/T$) impulses in frequency at harmonics k_0

Impulse train sampling

- We can think of a sampled waveform as a sum of different amplitude impulses

$$s(t) = x(t)p(t) \quad \text{where } p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

- This is a multiplication in time, so a convolution in frequency

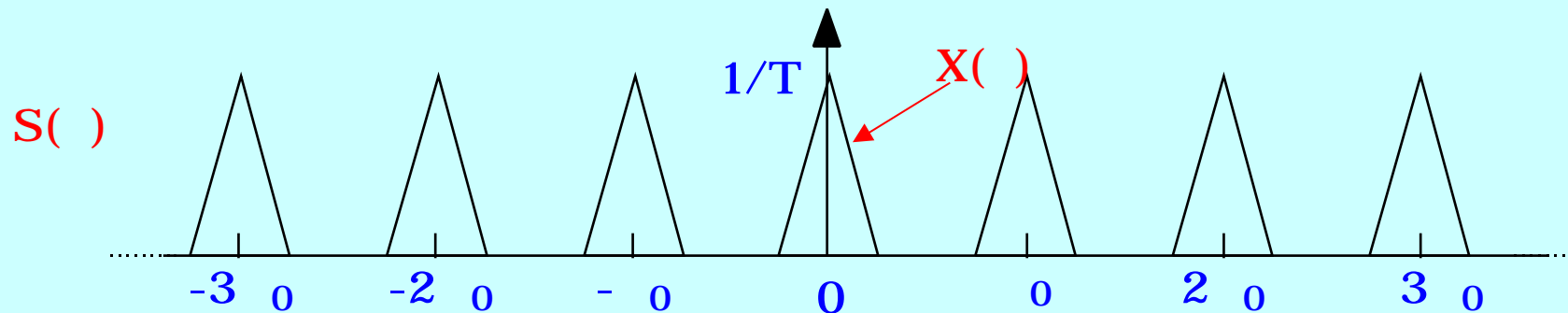
$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) p(t) e^{-j\omega t} dt \quad \text{and } P(\omega) = \int_{-\infty}^{\infty} p(t) e^{-j\omega t} dt$$

$$S(\omega) = \int_{-\infty}^{\infty} x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT) e^{-j\omega t} dt = \sum_{k=-\infty}^{\infty} X(\omega) e^{-j\omega kT}$$

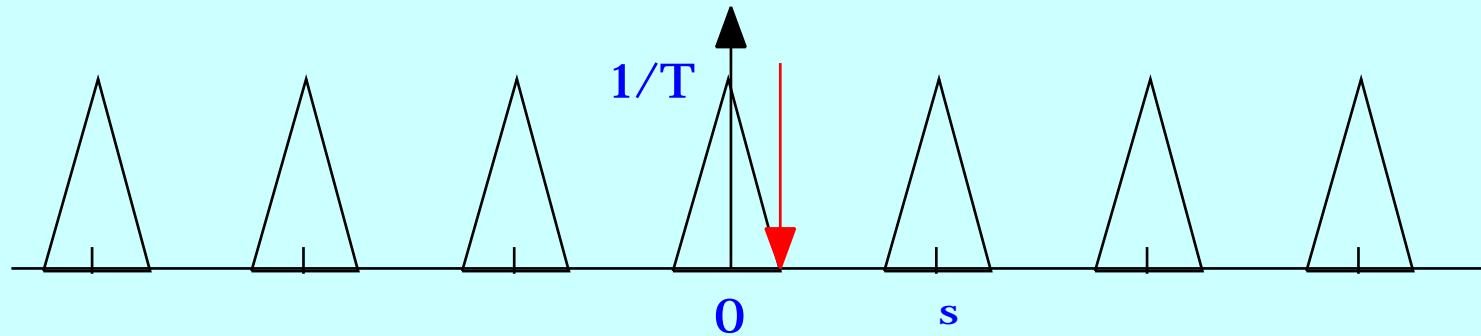
$$= \sum_{k=-\infty}^{\infty} X(\omega - k\omega_0)$$

- The FT of the sampled waveform is a series of equally spaced replicas of $X(\omega)$

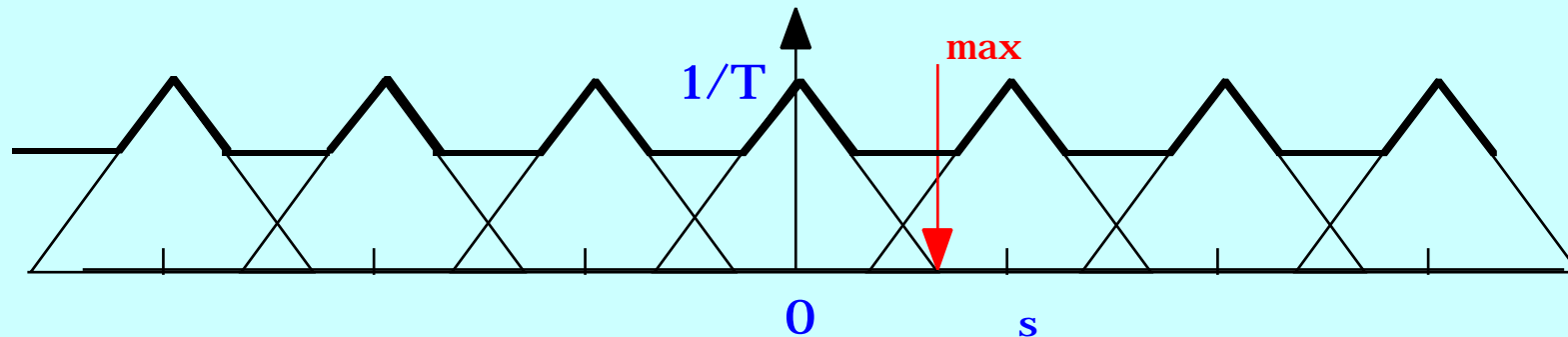
ie the FT of the signal $x(t)$ [scaled by $1/T$]



Sampling Theorem (Nyquist)



Signal can be recovered with a low- or band-pass filter



Higher harmonics will be sampled

•From this we can conclude

A continuous analogue function $x(t)$ which has a limited Fourier spectrum

ie $X(\omega) = 0$ for $\omega > \omega_{max}$

is uniquely described from its values at uniformly spaced time instants t

$t = 2\pi / s$ and $s > 2\omega_{max}$

Nyquist rate

Explanation of ...

- ..earlier puzzle

should now be clearer

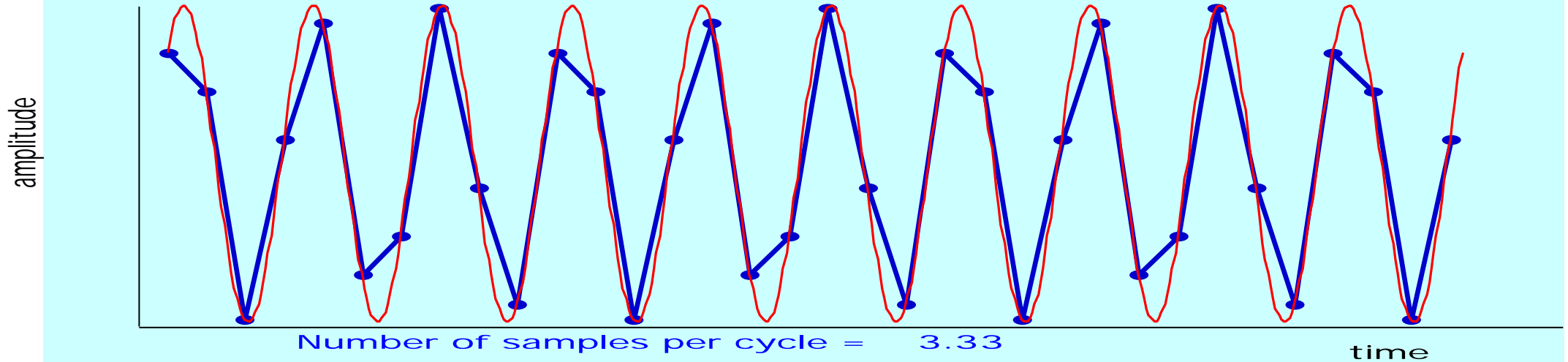
- What happens if we undersample?

ie take fewer samples than required by Nyquist for maximum frequency component in signal

- Let's take a look...

Examples

•Oversampling



•Undersampling

