

Jargon, names & concepts

- **Linear systems** will be a frequent assumption

input signal = $f(t)$ output = $g(t)$

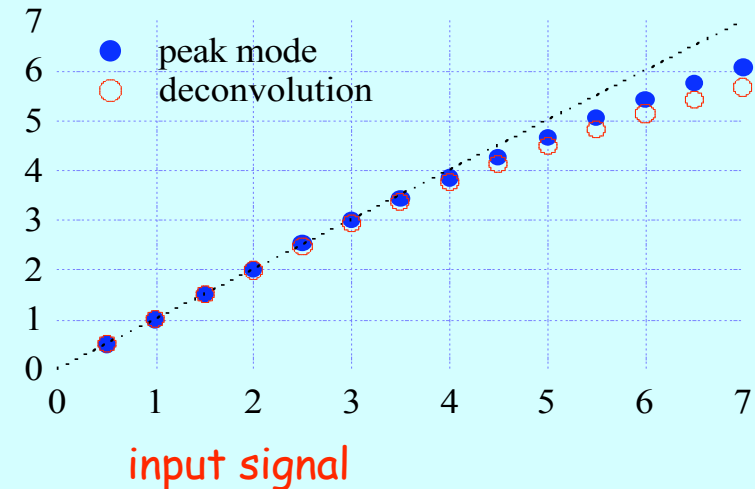
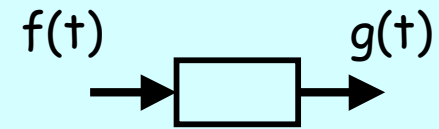
expect output to vary with input as $Af(t) \rightarrow Ag(t)$

not always the case,

eg amplifiers frequently exhibit saturation

can arise for several reasons

output
signal



- **Superposition**

important principle in many areas of physics & mathematical physics

If $f_1(t) \rightarrow g_1(t)$ and $f_2(t) \rightarrow g_2(t)$

then $af_1(t) + bf_2(t) \rightarrow ag_1(t) + bg_2(t)$

Decibels

- decibels (dB)

signal magnitudes cover wide range so frequently prefer logarithmic scale

$$\text{Number of dB} = 10\log_{10}(P_2/P_1)$$

often measuring voltages in system: $\text{dB} = 20 \log_{10}(V_2/V_1)$

- Not an absolute unit and sometimes encounter variants

dBm: dB with $P_{in} = 1\text{mW}$

Dynamic range

- In most systems there will be a smallest measurable signal

if there is noise present, it is most likely to be related to the smallest signal distinguishable from noise

3 x rms noise? 5 x rms noise?

or quantisation unit in measurement

- and a largest measurable signal

most likely set by apparatus or instrument, eg saturation

- Dynamic range = ratio of largest to smallest signal

often expressed in dB or bits

eg 8 bits = dynamic range is 256_{10}

= 48dB (if signal is voltage)

Precision

- many measurements involve detection of particle or radiation quantum (photon)

simple presence or absence sometimes sufficient = binary (0 or 1)

other measurements are of energy

- why do we need such observations?

primary measurement may be energy

eg medical imaging using gammas or high energy x-rays, astro-particle physics

extra information to improve data quality

removes experimental background, eg Compton scattered photons mistaken for real signal

optical communications - pressure to increase "bandwidth" - eg number of telephone calls carried per optical fibre

wavelength division multiplexing - several "colours" or wavelengths in same fibre simultaneously

- what is ultimate limit to precision?

Statistical limit to energy measurement

- Assume no limit from anything other than sensor
often not realistic assumption, but best possible case

$$N_{\text{quanta observed}} = E/\epsilon$$

= energy deposited by radiation

energy required to generate quantum of measurement

examples

semiconductor: energy for electron-hole pair ~ few eV

gaseous ionisation detector: energy for electron-ion ~ few x 10 eV

scintillation sensor: energy per photon of scintillation light ~ 100 eV

- Basic Poisson statistics

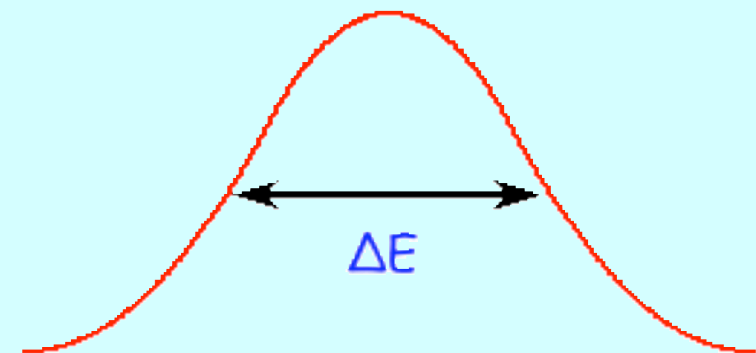
$$E_{\text{meas}} \sim N_q$$

$$\sigma^2(N_q) = N_q$$

$$\sigma(E)/E = \sigma(N_q)/N_q = 1/\sqrt{N_q}$$

expect gaussian distribution of N_q for large N_q

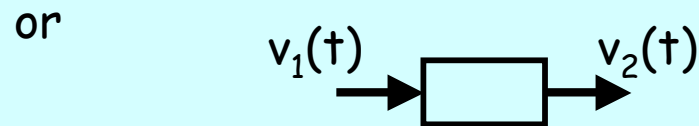
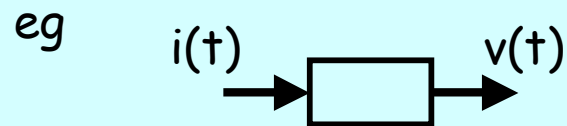
advantage in sensor with small ϵ



$$E_{\text{FWHM}} = 2.35\sigma(E)$$

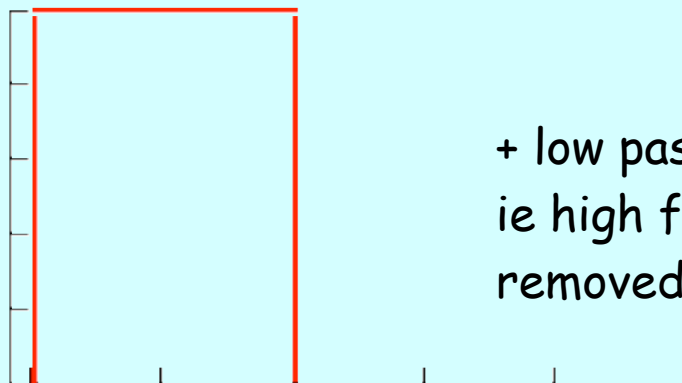
Filters

- Device or components to transform electrical signals from one form to another

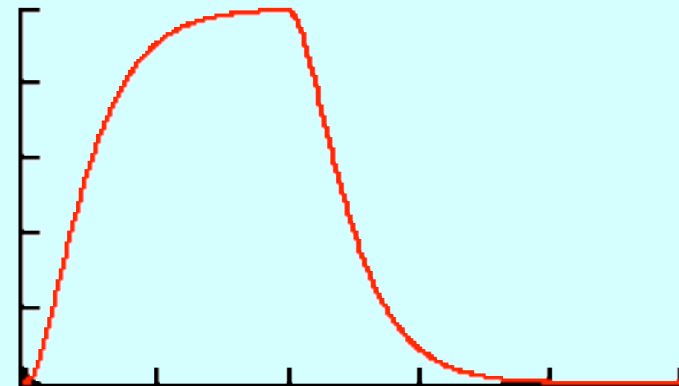


usually when doing so, amplitudes of frequencies in output are different from those input,

ie spectral content is changed or some frequencies filtered out



+ low pass filter
ie high frequencies
removed



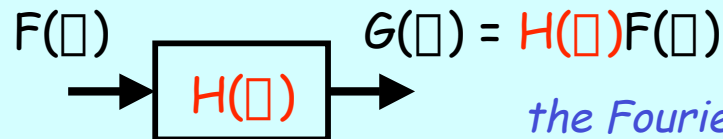
frequently want to analyse signals and systems in terms of frequency content, as well as behaviour in time

Transfer Function

- Inputs to, and outputs from, system considered as sum of components, with each component a single frequency

$$F(\omega) = A(\omega) e^{j\omega t}$$

$$\omega = 2\pi f$$



the Fourier or Laplace transform is an important tool

$H(\omega)$ is transfer function of system = $G(\omega)/F(\omega)$

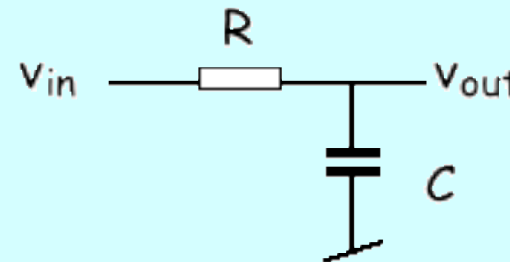
generally complex so introduces both **phase** and **amplitude** changes

- High pass filter

$$H(\omega) = R / (R + 1/j\omega C) = j\omega RC / (1 + j\omega RC)$$

$$\tau = RC$$

response to voltage **step** $\sim e^{-t/\tau}$



- Low pass filter

$$H(\omega) = (1/j\omega C) / (R + 1/j\omega C) = 1 / (1 + j\omega RC)$$

response to voltage **step** $\sim 1 - e^{-t/\tau}$

rise time: usually define as 10-90%

