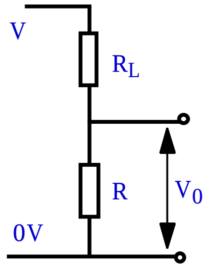
Instrumentation Problem Sheet 3 Answers



(1) $V_0 = VR(R+R_L)^{-1}$

$dV_0/dT = (dV_0/dR)(dR/dT)$ and $dR/dT = R\alpha$

so $dV_0/dT = VR_L R\alpha(R+R_L)^{-2}$

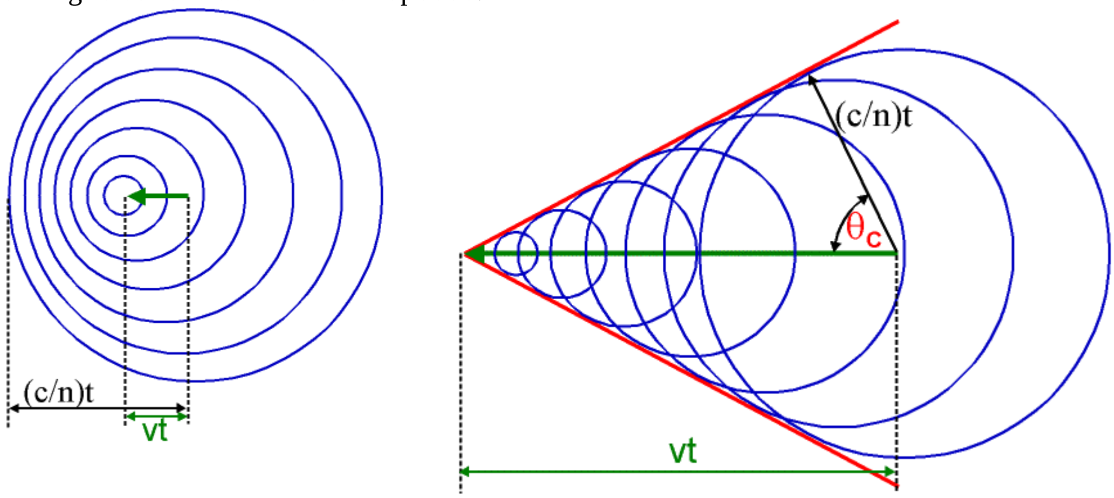
Power in thermistor $P = V_0^2/R = V^2R(R+R_L)^{-2} < 10^{-3}W$

so $R_L > 10k\Omega$ and $dV_0/dT = 40mV/K$

40mV/K would be easy to measure, even with a DVM, so there is some margin for reducing power further.

Given that the sensitivity will fall approximately as $1/R_L$, since $R \ll R_L$, it would not be a good idea to increase the margin too much. A factor 5 might be reasonable, although 10 is probably too conservative, unless one is willing to make some effort with the measuring circuit. This would be possible using op-amps if the temperature precision was required, as we'll discuss later.

(2) The conditions are shown in the following figures, illustrating a charged particle travelling from right to left at different speeds.



v < c v > c

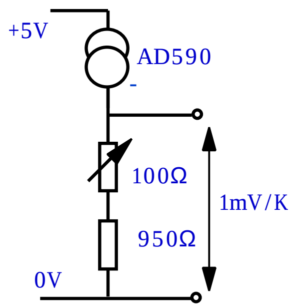
The circles (spheres in 3-d) represent the wavefronts of light emitted at different points along the trajectory. When the light travels more slowly ($c_{medium} = c/n$) than the particle in the medium it is possible for the wavefronts to be in phase. It can easily be seen that this is when $\cos\theta = 1/\beta n$.

It may be helpful to work it out practically with paper and compass: draw a line representing the path of the particle. Choose the speed of light in the material compared to the particle – eg 0.5. Take a compass and draw a circle centred at the right end of the line representing the distance travelled by the light since it was emitted. It should have a radius half the length of the line.

Take a second point, half way along the line. Draw a circle centred on this point with radius a quarter the line length. And so on... At the left end of the line the circle will have zero radius. Finally draw a line tangential to all the circles – which is the wavefront.

If you have difficulty getting your head around this one, you may be surprised to know you are already very familiar with the phenomenon. Imagine a duck on a still pond (nonrelativistic, compared to the speed of waves on the water) or a boat on a lake (usually relativistic). The bow wave produced by the boat is analogous to Cerenkov radiation.

(3) The sensible starting point would be to follow the manufacturer's recommendation as shown in the figure. The load resistance is set to be approximately 1kΩ but can be adjusted by the smaller trimming potentiometer so that the output is exactly 1mV/K. To achieve the ±0.5K spec it will be important to select the AD590M version, as the other versions are less precise (unless one is lucky enough to get an exceptionally good one at the lower price).



The resistors should be chosen to have a low thermal coefficient of resistance α, which is expected to vary with T as $R = R_0(1+\alpha T)$. To meet the ±0.5°C spec, we need

$I\Delta R = IR_0\alpha\Delta T \ll 0.5mV$ with $\Delta T = 1K$ (or the measurement range)

$IR_0 = 298mV$ at the calibration temperature (assume ≈25°C)

so $\alpha\Delta T \ll 0.5/300 \sim 10^{-3}$

which should not be a problem. Typical resistor temperature coefficients are ~200-300ppm (parts per million), or less, if required.

Note: this calculation really assumes that what is being measured is a stable temperature. One could demand that the temperature error should be less than 1K (ie +0.5 -(-0.5)) over the entire temperature range, which is much more demanding. In this case, we require

$\alpha\Delta T \approx \alpha 200 \ll 0.5/T_{max} \sim 10^{-5}$

Where T_{max} is the upper temperature limit (150°C = 423K). In practice this is not a reasonable thing to do since this is a systematic error and would normally be taken care of by calibrating the system and applying a correction to the readings. Otherwise you would put too great demands on the resistor requirements for a practical (or affordable) solution.

To convert from K to °C it is necessary to subtract 273.2mV from the voltage measured across the load. This is easy to do using op-amp circuits, as we will discuss very soon.

(4) **The original problem omitted one vital piece of information, that the gain of the tube was 5×10^7. Sorry!**

The peak signal charge is is $10 \times 1.6\ 10^{-19} \times 5\ 10^7$ = 80pC, in 10ns this corresponds to a peak current of $80pC/10^{-8}$ = 8mA with ~1μs on average between pulses (probably Poisson distributed!)

The average signal current is $10 \times 1.6\ 10^{-19} \times 5\ 10^7 \times 10^6$ = 0.08mA

For such an average current it would probably be adequate to operate the tube with a bias current of 2mA, ie 25x the average signal current, provided sufficient capacitors were present in the final stages of the chain to supply the peak currents required. The resistor at each stage is then

$R = 200V/2mA = 100k\Omega$.

The charge stored in the capacitors across the final stage should be many times (eg x100) the peak signal charge to maintain good linearity. So $C \times 200V \geq 100 \times 80pC$ and $C \geq 40pF$.

Manufacturers often recommend appropriate divider chain circuits.

At high pulse rates, the linearity can also fall because of space charge in the tube (ie the high electron density present can disturb the field in the tube). This can be corrected by increasing the voltage across the last few stages.

(5) An electron gains kinetic energy from electric field and generates more electrons through their impacts on the next dynode, which converts ke into ionised electrons which then escape from the dynode with low kinetic energy. After each stage the gain in ke is T=200eV, assuming the initial energy is zero (this is a reasonable assumption, even if not exact). The electron velocity after each gain stage is $(2T/m)^{1/2}$. The average velocity per stage is half of this. Thus

$$\langle v \rangle = 0.5(2T/m)^{1/2}$$

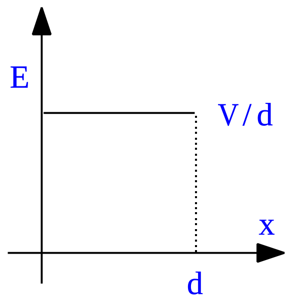
$m = 511\text{keV}/c^2$, T = 200eV so $\langle v \rangle = 0.5c(400/511000)^{1/2} = 0.014c = 4.2\ 10^6\ \text{m.s}^{-1}$

The transit time is

$$\Delta t = 12\text{mm}/4.2\ 10^6\ \text{m.s}^{-1} \approx 3\text{ns}$$

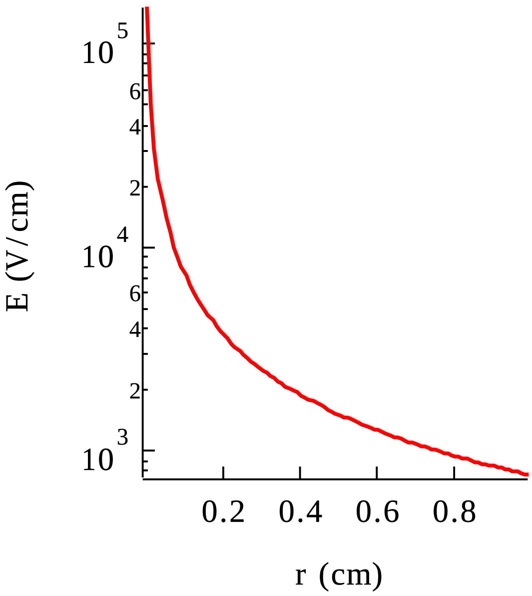
Note how short the transit time is, and one can expect the dispersion in time to be equally small, so the output pulses will be very fast, by which we usually mean short rise time and duration.

6 (i) in a parallel plate gas chamber the electric field is constant between the two surfaces, which are here separated by a distance d with an applied voltage V.

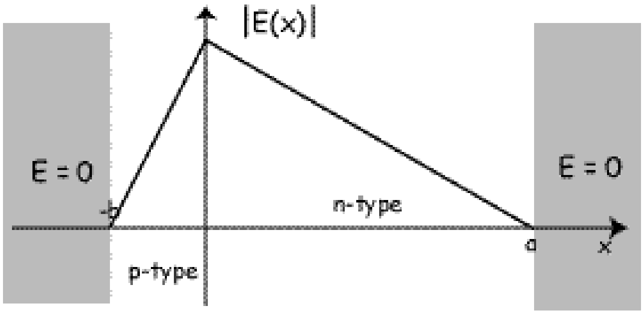


(ii) for the cylindrical proportional wire chamber, I used the example conditions given in the lectures, so V_0 = 4000V, a = 50µm, b = 1cm.

Then E = 755V/r

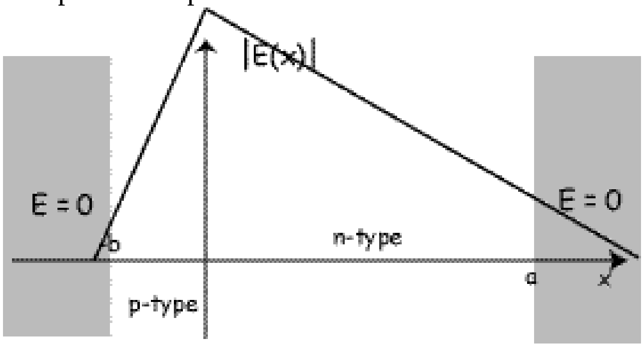


(iii) (a) The electric field in a silicon diode should have been discussed in lectures on semiconductors. When it is partly depleted ($V < V_D$) the field extends to a depth a in the lesser doped region and to a depth b in the heavily doped region. It has a maximum at the junction, which is approximated by a very sharp boundary.

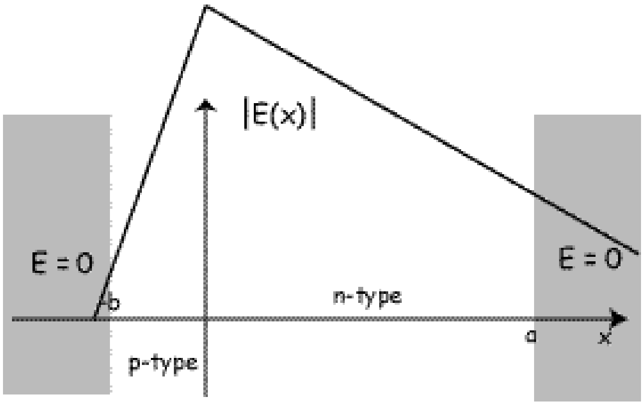


Note the regions which are not depleted (shaded in the diagram) where E = 0, and charge carriers move by diffusion alone. Typically the lesser doped region is n-type where the doping densities are ~10^{12} cm^{-3} and several orders of magnitude larger in the p-type region, which is therefore ~1µm or much less in depth. The maximum value of the field can be shown to be 2V/d, where V is the applied voltage and d the depletion depth.

(b) When the diode is depleted to its full thickness, the field distribution looks similar, except the magnitude is larger and E = 0 only at the surface of the lightly doped region. However, the depth of the field does not extend too much further into the p-type region because of the heavy doping.



(c) the over-depleted case is shown here. The field continues to increase, with the same dependence on depth, but therefore has a finite value at the n-type surface, while there is still a thin field-free region under the p-type surface. This is a region where charge recombination is likely in photo-detectors.



(Note that in all these figures, the depth of the p-type junction has been drawn grossly out of scale. Typically it is a fraction of a µm, compared to 300-500µm in a n-type silicon particle detector, or 50-100µm in a photodiode).

(7) The average number of ionisations produced in thickness t cm is n = Nt. This is a random process, therefore governed by Poisson statistics. [In case you don't recall it, the Poisson formula for the probability of finding n, with mean $\langle n \rangle = \mu$ is $p(n,\mu) = e^{-\mu}\mu^n/n!$]. Thus the probability of producing zero ionisations in distance t is

$$p(0) = e^{-n} = 0.01 \qquad \text{for 99\% detection efficiency}$$

Thus $t = -\ln(0.01)/N = 4.6/N$

(8) The circuit is an inverting amplifier with a gain of $G = -R_2/R_1$ and an input impedance of R_1. The input impedance is most easily calculated by noting that the inverting input is a "virtual ground" held at 0V by the non-inverting input and the condition that $v_- = v_+$.

(9) This is a non-inverting amplifier with a gain of $G = 1 + R_2/R_1$, following a differentiator (high-pass filter) with a time constant of $\tau = RC$. The output of the system, for a 1V step input will be $Ge^{-t/\tau}$ volts. For the values given G = 22.4 and τ = 10ms.

In the frequency domain the filter has a response (transfer function) of $H(\omega) = j\omega\tau/(1+j\omega\tau)$ so

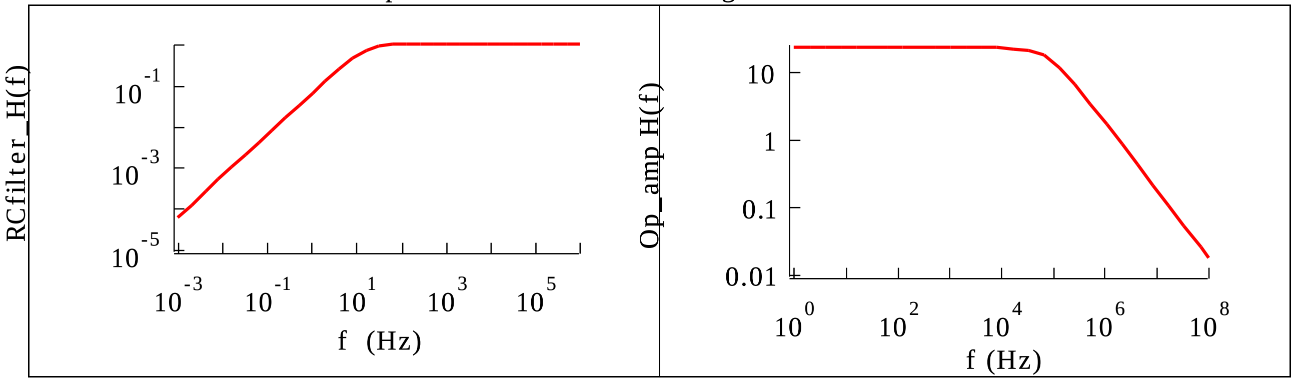
$$|H(\omega)| = \omega\tau/(1+\omega^2\tau^2)^{1/2}$$

At very high frequencies the gain of the system = G, while at 10Hz the total gain = 11.9

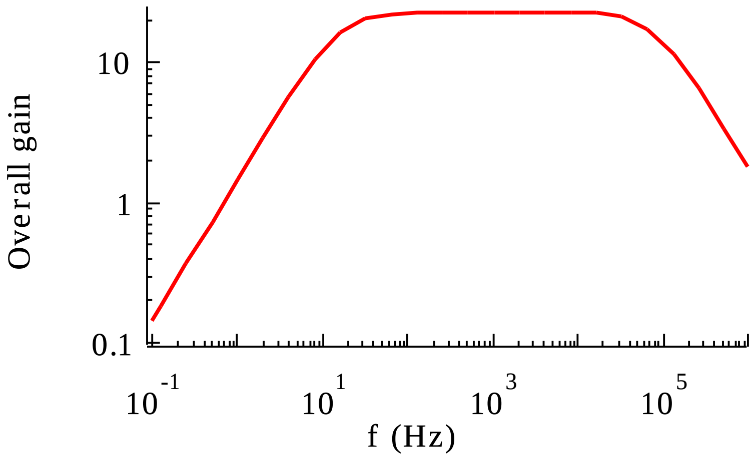
If the 80kHz pole is taken into account, the amplifier has the closed loop frequency response shown in the plot below, namely

$$|H_{AMP}(\omega)| = G/(1+\omega^2\tau_1^2)^{1/2} \text{ where } \tau_1 = 1/(2\pi x 80kHz)$$

and the overall response is the product of the high-pass filter and this. The Bode plot of the differentiator and the amplifier are shown in the figures below



So the overall frequency response is the product of these two and is shown below.



Quite a reasonable band-pass filter with a good frequency range.

(10) The output from the circuit is

$$v_{out} = -R_f(v_1/R_1 + v_2/R_2 + v_3/R_3)$$

so the result is a weighted sum of the three input voltages. To convert it to an averaging amplifier, make $R_1 = R_2 = R_3$ and choose $R_f = R_1/3$.