Ring Coolers

R B Palmer

Brookhaven National Laboratory, Upton, NY, 11973, USA

Abstract. The simulated performance of three different 6D muon cooling rings are presented. Two of these used solenoid focusing and one used quadrupoles. All three performed better than the linear cooling channel described in Study-2[1]. The best example increased the 6D phase space density by a factor of 162, compared with the linear channel's factor of only 15. However, none of the simulations used fully realistic magnetic fields, and absorber and rf cavity windows were not included. Injection/extraction is discussed.

Submitted to: *J. Phys. G: Nucl. Phys.*

PACS numbers: 1315, 9440T

1. Introduction

If muons alternately pass through a material absorber, and acceleration; and if there is sufficient focusing at the absorber, then the muon's transverse phase space is reduced, i.e. the muons are cooled in the transverse dimensions. Straggling in the material causes the momentum spread, and thus the longitudinal phase space to rise. Momentum spread can be reduced if dispersion is introduced and a wedge absorber placed such that high momentum particles pass through more material than low momentum particles, but the beam width is increased (Fig. 1). The process is primarily an exchange of emittance between the longitudinal and transverse dimensions, but when combined with transverse cooling in the material, all three dimensions can be cooled.

Several studies alternating significant transverse cooling with special sections devoted to emittance exchange were relatively unsuccessful[2]. But more recently, useful cooling in all dimensions has been achieved in several studies of rings, in which longitudinal and transverse cooling are alternated more rapidly, or are combined in a single wedge absorber.

In discussing cooler performance, it is useful to define a Merit Factor M:

$$M = \frac{\text{Initial 6D Emittance}}{\text{Final 6D Emittance}} \times \text{Transmission}$$

We will discuss particular examples of three different concepts, and then deal with some common problems.

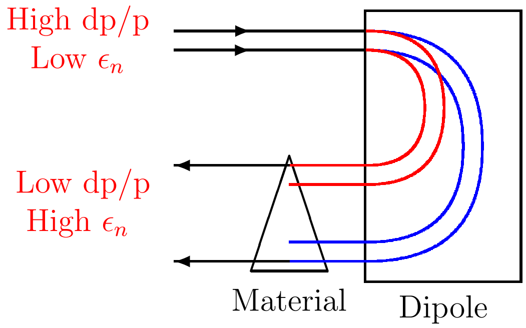

Figure 1. Emittance Exchange Concept

2. Balbekov 6D Cooling Ring[3]

The ring layout is shown in Fig. 2 with parameters given in table 1. In this example the transverse cooling with a liquid hydrogen absorber is alternated with emittance exchange in a lithium hydride wedge in separate straight sections. The hydrogen absorber is at the center of a 6.67 m straight solenoid focused channel with a field rising to 5.1 T at the absorber. 200 MHz rf is placed within the solenoid, on either side of the absorber. Dispersion in the following shorter straight is generated by a 45 degree dipole bend. The short straight solenoid channels have field reversals and lithium hydride wedges at their center.

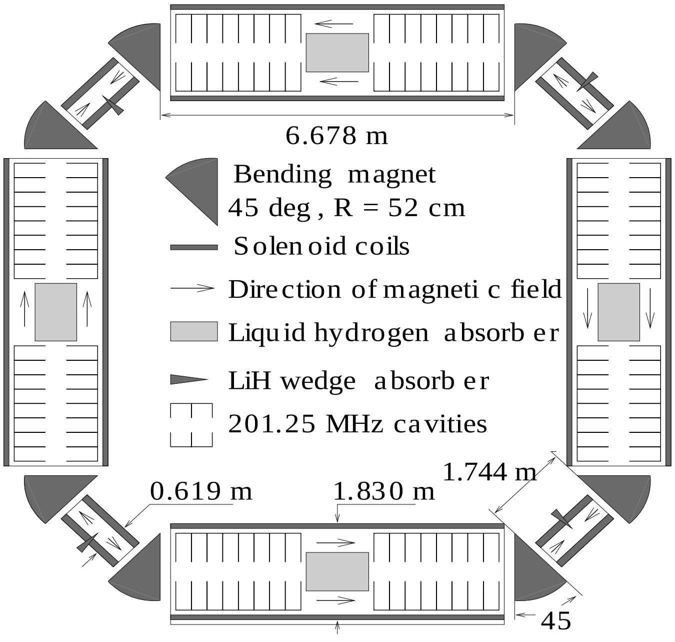

Figure 2. Layout of Balbekov Ring

Table 1. Balbekov Ring Parameters

Circumference	36.963 m
Repeat length	9.25 m
Mean Muon Momentum	227 MeV/c
Max B_z	5.155 T
rf Frequency	205.69 MHz
rf Gradient	15 MV/m

Table 2. Balbekov Ring Performance

	Before	After	Ratio
$\epsilon_\perp$ (πmm)	12	2.1	5.7
$\epsilon_\parallel$ (πmm)	15	6.3	2.4
ϵ_6 (π^3cm^3)	2.2	0.028	79
N/N_0, no decay	1	0.71	0.71
N/N_0, inc. decay	1	0.48	0.48
Merit	1	38	**38**

Performance is given in table 2. Good cooling is observed in all dimensions, with relatively good transmission (48%). The merit factor is 38. A later independent simulation of the ring by R. Fernow [4] using ICOOL[5] with some possible differences of parameters, observed a merit factor of 94.

The separation of transverse and longitudinal cooling results in a relatively long repeat (9.25 m) and results in the presence of integer resonances within the momentum acceptance. It is not known if this is a significant problem.

Neither simulation was fully realistic. Both used unphysical hard ended fields, no space was provided for injection/extraction, and no hydrogen or rf windows were included. Efforts to use realistic fields are under way. When space for injection and extraction was inserted into one side of the ring, the merit factor dropped to 3.9, but no further optimization was attempted.

These problems may be soluble, but they are clearly difficult.

3. Quadrupole Ring[6]

The use of quadrupoles, instead of solenoids, is motivated by the greater experience in their use. Lattices were studied with both separated transverse cooling and emittance exchange, and with them combined. The best results were obtained with them combined. In this case there is a relatively short repeat (3.8 m), a small phase advance per repeat, and no integer resonances within the momentum acceptance.

Table 3 gives the parameters. Figure 3 shows the lattice parameters for one of the 8 identical cells in the ring. The hydrogen wedge is at the center of the cell, focus quadrupoles are on each side, followed by combined function bend/defocus magnets, beyond which are two more focus quadrupoles. The rf is inserted between cells, where the dispersion is zero.

After design using SYNCH, simulation was carried out with ICOOL, without windows, and with hard edged fields. Neither higher order multipoles, nor end effects were simulated. No space for injection/extraction was included.

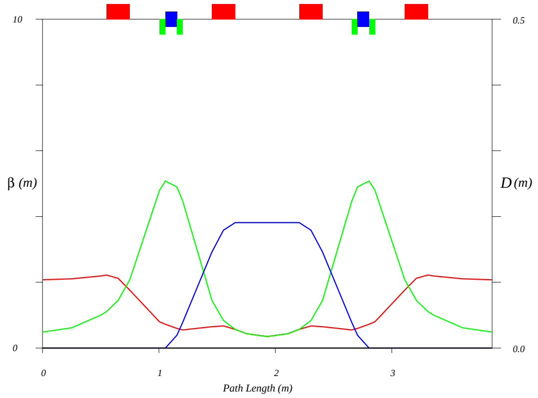

Figure 3. Layout of Quadrupole Ring

Table 3. Quadrupole Ring Parameters

Circumference	31 m
Cell Length	3.8 m
Momentum	250 MeV/c
Magnet aperture (full)	40 cm
Max pole tip field	2 T
rf Frequency	200 MHz
rf Gradient	16 MV/m

Table 4. Quadrupole Ring Performance

	Before	After	ratio
ϵ_x (πmm)	8.5	3.4	2.5
ϵ_y (πmm)	5.2	1.2	4.2
$\epsilon_{\|}$ (πmm)	14	3.8	3.7
ϵ_6 (π^3cm^3)	0.62	0.015	39
N/N_0, inc. decay	1	0.41	.41
Merit	1	16	**16**

The performance is given in table 4. The final transverse emittance is seen to be similar to that of Balbekov's ring, but the momentum acceptance, and thus initial emittance is lower. The smaller momentum acceptance appears to be a result of the weaker focusing of quadrupoles (that always defocus in one of two directions) compared with that of solenoids (that focus in both directions). The merit factor is 16, less than Balbekov's, but still comparable with the Study-2 linear solenoid channel.

Work is continuing on lattices similar to this one, including one lattice with no quadrupoles, dependent only on 'weak' focusing in the bends.

4. RFOFO Ring Cooler

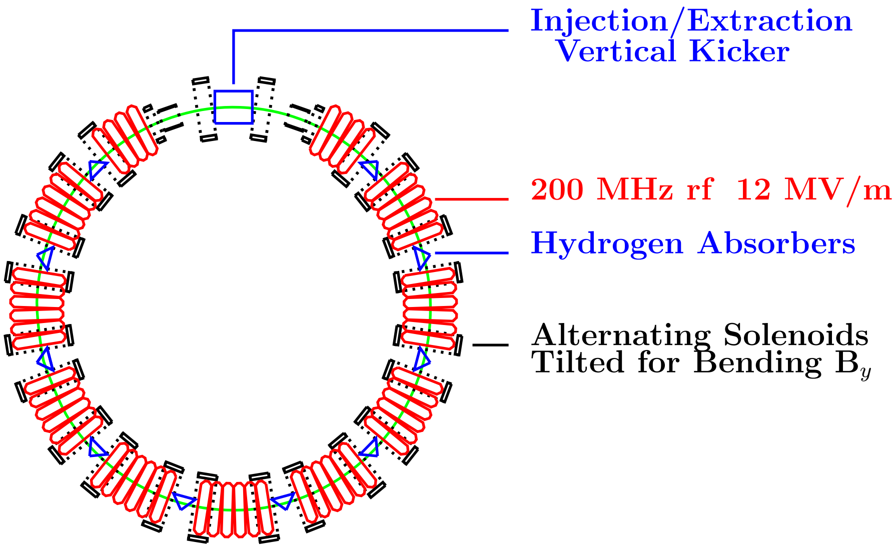

Figure 4. Layout of RFOFO Ring[7]

Figure 4 shows the layout of the "RFOFO" Cooling Ring. Focusing is provided by pairs of solenoids with alternating field direction ("FOFO" since both focus, "R" is for 'reverse' to distinguish it from the SFOFO[8] used in Study-2). As in the quadrupole ring, all cells are identical and both transverse and longitudinal cooling are obtained in the same hydrogen wedge absorbers. The parameters are given in table 5.

The solenoids are not evenly spaced: those on either side of the absorbers are closer. This increases the focusing . A bending field is generated by tilting the focus coils by alternate 2.6 degree angles. 200 MHz rf is placed inside the coils, as in Balbekov's ring. But unlike in the other rings, there is dispersion at the rf cavities.

The simulation of this ring was done with physically possible, i.e. Maxwellian, fields, but these fields were not exactly those derived from known coil positions.

Figure 4 shows an injection/extraction insertion at the top, but initial simulations were done without it. Figure 5 shows the three emittances and transmission plotted on a log scale versus distance traveled. Table 6 summarizes the performance, including the merit factor of 162.

This performance was obtained when the hydrogen wedge, without windows, did

Table 5. RFOFO Ring Parameters

Circumference	m	33
Cells		12
Max Bz	T	2.7
Coil Tilts	deg.	2.6
Ave Momentum	MeV/c	220
Min Trans. Beta	cm	35-40
Dispersion	cm	8
Wedge Absorber Material		H_2
Central thickness	cm	28.6
Wedge angle	deg	100
RF Cavities/cell		6
Frequency	MHz	201.25
Gradient	MV/m	12

not fully cross the aperture. Such a wedge may be hard to build. A merit factor of the order of 100 was obtained with more conventional wedges.

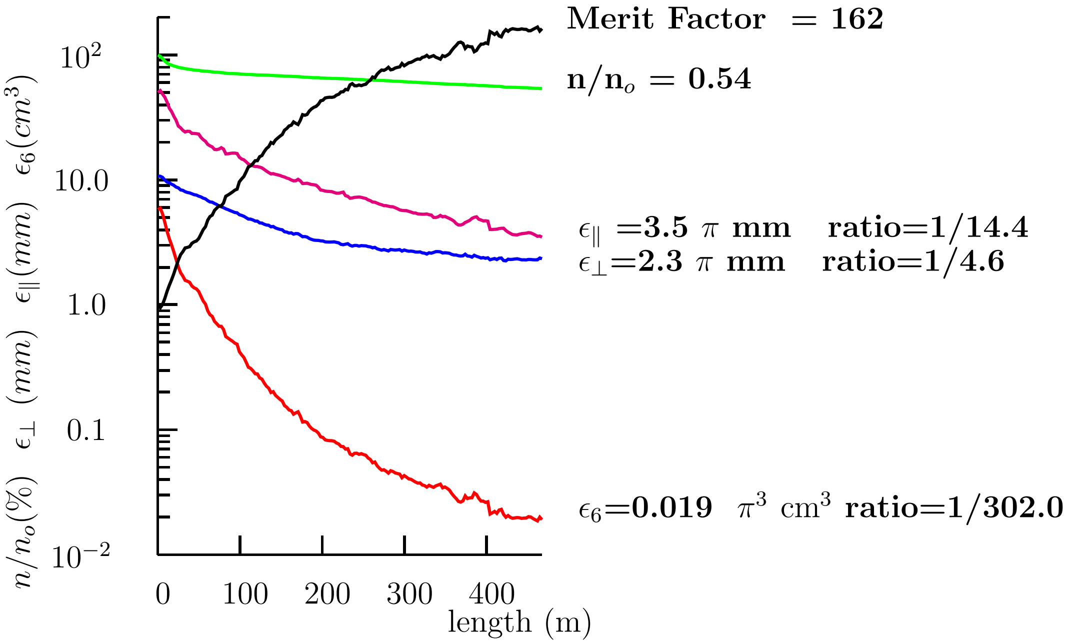

Figure 5. Performance of RFOFO Ring

When simulated with the injection/extraction insertion, but without modification, the merit factor dropped to 10. The severe drop was due to the excitation of a synchrotron resonance. With a small modification of the energy, and with some re-tuning, the merit was raised to 110.

Table 6. RFOFO Ring Performance

	Before	After	ratio
$\epsilon_\perp$ (πmm)	10.7	2.3	4.6
$\epsilon_\parallel$ (πmm)	50.1	3.5	14.4
ϵ_6 (π^3cm^3)	5.8	0.019	302
N/N_0, inc. decay	1	0.54	0.54
Merit	1	162	**162**

5. Problems Common to all Solutions

None of the simulations discussed here included either absorber or rf windows, and the inclusion of such windows with thicknesses similar to those in Study-2, seriously degrades the ring performances. Thinner absorber windows using stronger materials, and thinner rf windows at liquid nitrogen temperature need to be developed.

Another common problem is the kicker needed for injection. The minimum pulsed energy required is proportional to the square of the transverse emittance being kicked. Since the initial muon emittances discussed here are more than an order of magnitude greater than those in current applications, the kicker pulsed energy is three orders of magnitude greater: $\approx$10,000 J compared with 10-20 J with, for instance, antiproton kickers.

However, magnetic amplifiers, used in induction accelerators, can provide this kind of pulse energy at the required pulse lengths, and a kicker design based on this concept has been proposed[9].

6. Summary

Table 7 summarizes the performances of the three rings discussed, and, for comparison, the performance of the linear cooling channel in study-2[1]. It is seen that the final transverse emittances are similar in all cases, as is the transmission. The main differences in the performances are in the ratios of initial to final longitudinal emittances. There is no longitudinal cooling in the study-2 channel, only scraping. There is some longitudinal cooling in all three rings, but it is significantly larger for the RFOFO ring.

7. Conclusion

There has been much recent progress with emittance exchange in ring coolers, and simulations of three very different designs achieved significant cooling in all three dimensions. They all perform as well or better than the study 2 linear channel, yet any one of them, with circumferences less than 40 m appears likely to cost less than the 108 m Study-2 channel. But much work remains: none of the rings has been simulated

Table 7. Ring Performances

		len m	trans %	$\epsilon_\perp$ π mm	$\epsilon_\parallel$ π mm	ϵ_6 π^3cm^3	merit	End Fields
Study 2	initial	108		10.7	50.1	5.787		
	final		**50**	**2.3**	**38**	0.2	**15**	**yes**
	ratio			4.7	1.4	30		
Balbekov	initial			12	15	2.16		
	final	555	**48**	**2.1**	**6.3**	.028	**38-94**	**no**
	ratio			5.7	2.38	79		
Quadrupole	initial			8.5×5.2	14	0.62		
	final	495	**41**	**3.4×1.2**	**3.8**	.015	**16**	**no**
	ratio			2.5×4.2	3.7	39		
RFOFO	initial			10.7	50.1	5.787		
	final	468	**54**	**2.3**	**3.5**	0.019	**162**	**yes**
	ratio			4.6	14.4	302.0		

with fully realistic fields, windows have not been included, injection, extraction channels have not been fully designed, and the required kickers need development.

Bibliography

[1] S Ozaki, R B Palmer, M S Zisman, J C Gallardo, Editors,'Feasibility Study II of a Muon Based Neutrino Source', BNL-52623, June, 2001.

[2] In bent solenoids: R B Palmer, private communication ;In Helical Channels: MUC-NOTE-COOL_THEORY-146, 147, 187, & 193 ((http://www-mucool.fnal.gov/mcnotes/public/pdf/muc0146/muc0146.pdf)

[3] V Balbekov; MUC-NOTE-COOL_THEORY-232 & 246 (http://www-mucool.fnal.gov/mcnotes/public/pdf/muc0246/muc0246.pdf) (http://www-mucool.fnal.gov/notes/)

[4] R C Fernow; private communication.

[5] R C Fernow, 'ICOOL: a simulation code for ionization cooling of muon beams', Proc. 1999 Particle Accelerator Conference, p. 3020-3022.

[6] H Kirk, D Cline, Y Fukui, A Garren; Proc. Snowmass 2001, H101; SLAC-R-599; econf-C-010630, July 2002.

[7] J S Berg, R Fernow, J Gallardo, W Lau, R B Palmer, L Reginato, D Summers, and Y Zhao; MUC-NOTE-COOL_THEORY-239 (http://www-mucool.fnal.gov/mcnotes/public/pdf/muc0239/muc0239.pdf)

[8] Eun-San Kim, M Yoon, "Super FOFO cooling channel for a Neutrino Factory", MUC-NOTE-COOL_THEORY-191, Feb. 2001 (http://www-mucool.fnal.gov/mcnotes/public/pdf/muc0191/muc0191.pdf)

[9] R B Palmer, L Reginato, D Summers, 'An Induction Kicker for Muon Cooling Rings',MUC-NOTE-COOL_THEORY-256 (http://www-mucool.fnal.gov/mcnotes/public/pdf/muc0256/muc0256.pdf)