Theoretical implications of neutrino oscillations.

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Abstract

We review the possible origins of neutrino mass and the progress that has been made towards understanding the pattern of neutrino masses and mixings. The relation of the masses and mixings of the leptons to the quarks is explored.

1 Introduction

The observation of neutrino oscillation has raised several crucial questions concerning the origins of the theory of the fundamental interactions and the origin of mass [1]. The first question is how many light neutrinos and is this number related to the numer of quark and charged leptons? The second immediate question is why are they so light in comparison to the other states of the Standard Model and is the mass a Dirac mass, conserving lepton number or a Majorana mass, violating lepton number? Thirdly what is the spectral pattern; is there near degeneracy, an hierarchy or an inverted hierarchy? Finally what is the underlying origin of fermion mass generating the hierarchical pattern of quark, charged lepton and neutrino masses and giving rise to small mixing angles in the quark sector and large mixing angles in the lepton sector.

2 How many light neutrinos?

Given the fact that in field theory radiative corrections communicate between different sectors of the theory there are only two ways of protecting a light state from large radiative corrections to its mass. The first is if the state is composite in which case the virtual momentum of the intermediate states in radiative corrections are cut-off at the scale that the strong binding force becomes weak. However neutrinos are so much lighter than the energy scales we have already probed without finding composite structure that it looks extremely improbable that they be composite. Therefore their mass should be protected by a symmetry from getting large radiative corrections.

In the Standard Model the neutrinos, ν_{Li}, are protected by Lepton number conservation from getting Majorana mass (there are no right-handed neutrino states so there are no Dirac masses). There are three conserved lepton numbers corresponding to the three generations. What about the possibility of additional light fermion states? There cannot be a normal fouth family neutrino because it would contribute to the Z width at an unacceptable level. However there could be additional light "sterile" states not transforming under the Standard Model, provided they too are protected by a symmetry. A simple example is if the sterile state transforms under an additional Abelian symmetry for the Majorana mass term for this neutrino breaks this symmetry. Supersymmetry also provides candidates for additional light fermion states. The vector supermultiplet has gaugino fermion partners of the gauge bosons which cannot get a mass until supersymmetry or the gauge symmetry is broken. The Higgs bosons have Higgsino fermion superpartners which cannot be different in mass from the Higgs until supersymmetry is broken. Any of the neutral members of these new fermion states, "neutralinos", are candidates to mix with the Standard Model neutrino states. The masses of these new states is not very small (of the order of the supersymmetry breaking scale) but they can generate small neutrino masses via a see-saw mechanism in which they couple to ν_{Li} via small R-parity violating couplings. Another possible source of new light states motivated by string theories are modulini - the superpartners of the numerous moduli fields. The vacuum expectation values of the moduli fix the couplings and the compactification characteristics of the theory. They and their partner modulini are massless until supersymmetry is broken. In summary, the message is that there is no theoretical obstruction to having additional sterile neutrino states which may play a role in generating neutrino masses.

3 Why so light?

Given that the neutrinos are protected from getting mass the question why they are so light is equivalent to the question why the symmetry is broken so weakly. There are several possible explanations for this. One possibility is the see-saw mechanism communicating the breaking from a very high scale such as the scale of Grand Unification via large couplings or from a much lower scale, such as the supersymmetry breaking scale, via smaller R-parity violating couplings. A second possibility occurs in theories with additional space dimensions in which the coupling communicating the breaking of the symmetry can be extremely small. A third possibility is that the breaking of the symmetry occurs in radiative order only and thus is suppressed by the smallnes of such corrections.

3.1 Small neutrino masses : the see-saw mechanism

The "see-saw" mechanism [2] follows naturally in the case that right-handed neutrinos exist. Suppose that there is no weak isospin 1 Higgs field and hence there are no mass terms of the $\nu_L \nu_L$ type. In this case there are just two possible neutrino masses

$$m_{Dirac}\overline{\nu_L}\nu_R + M_{Majorana}\nu_R\nu_R \equiv \begin{pmatrix} \overline{\nu_L} & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix} \begin{pmatrix} \overline{\nu_L} \\ \nu_R \end{pmatrix} \tag{1}$$

The Dirac mass is similar to an up quark mass and one's naive expectation is that they should have similar magnitude. On the other hand, the Majorana mass term is invariant under the Standard Model gauge group and does not require a stage of electroweak breaking to generate it. For this reason, one expects the Majorana mass to be much larger than the electroweak breaking scale, perhaps as large as the scale of the new physics beyond the Standard Model; for example, the Grand Unified scale or even the Planck scale. Diagonalising the mass matrix gives the eigenvalues

$$\begin{aligned} m_{Heavy} &\simeq M_M \\ m_{Light} &\simeq \frac{m_D^2}{M_M} \end{aligned} \tag{2}$$

The see-saw mechanism generates an effective Majorana mass for the light neutrino (predominantly ν_L) by mixing with the heavy state (predominantly ν_R) of mass M_M. It is driven by an effective Higgs $\Phi^{I_W=1}$ made up of $H^{I_W=1/2}\ H^{I_W=1/2}\ /\ M_M$ (hence the two factors of m_D in eq.(2)). Eq.(2) shows that a large scale for the Majorana mass gives a very light neutrino. For example with $M_M = 10^{16}$ GeV and m_D taken to be the top quark mass gives

$$m_{Light} \simeq 3.10^{-3} \text{ eV}$$

This estimate shows that it is quite natural to have neutrinos in a mass range appropriate to give, for example, solar neutrino oscillations. However, in many cases, larger masses capable of explaining the other oscillation phenomena are possible because the Majorana mass for the right handed neutrinos is often smaller than the Grand Unified mass. A Majorana mass for the right-handed neutrino requires a Higgs carrying right-handed isospin 1 (in analogy with the left-handed case when it needed left-handed isospin 1). If this field is not present (for example in level one string theory this is always the case) one may get a double see-saw because the Majorana mass for the right-handed neutrino is also generated by an effective Higgs, made up of $H^{I_{W,R}=1/2}H^{I_{W,R}=1/2}/M'$, where M' denotes a scale of physics beyond the Grand Unification scale. Taking this to be the Planck scale (probably the largest reasonable possibility) and $< H^{I_{W,R}=1/2} >$ to be the Grand Unified scale (it breaks any Grand Unified group) one finds

$$M_M \simeq \frac{(10^{16})^2}{10^{19}} \text{ GeV}$$

giving

$$m_{Light} \simeq 1 \text{ eV}$$

Thus, one sees that the see saw mechanism naturally gives neutrino masses in the range relevant to neutrino oscillation measurements. Moreover, as the neutrino mass is proportional to the Dirac mass squared, taking the Dirac mass of each family of neutrinos to be of the order of the equivalent up quark mass, one obtains a large hierarchy between different families of neutrino. This is what is required if one is to explain several oscillation phenomena, for it allows the existence of several mass differences.

3.2 Small neutrino masses - large new dimensions

It is interesting to see how a very small coupling responsible for neutrino mass arises in a theory with additional space dimensions. In such theories the solution to the hierarchy problem is supposed to be due to the absence of any fundamental mass scale much larger than the electroweak breaking scale [4] and so there is no large mass scale to suppress the communication of the symmetry breaking as happens in the see-saw mechanism. However, in such theories, the weakness of the gravitational interactions does not require a fundamental Planck scale of $O(10^{18} GeV)$. The reason is readily seen if we compute the Newtonian potential between two point masses in the underlying $(4+\delta)$ dimensional theory. It is given by

$$V_{gravity}(r) = \frac{m_1 m_2}{M_{Planck,4+\delta}^{\delta+2}} \cdot \frac{1}{r^{\delta+1}}$$

where $M_{Planck,4+\delta}$ is the fundamental mass scale in the $(4+\delta)$ dimensional theory (the string tension in a string theory). This form applies at distances much smaller than the size, R, of the δ additional compact dimensions. At distances larger than their size we have

$$V_{gravity}(r)|_{r>>R} = \frac{m_1 m_2}{M_{Planck,4+\delta}^{\delta+2}} \cdot \frac{1}{R^\delta} \cdot \frac{1}{r}$$

This shows that the Planck scale in our 4 dimensional theory is made larger by the spreading of flux in the new dimensions. It is given by

$$M_{Planck,4}^2 = M_{Planck,4+\delta}^{\delta+2} \, R^\delta \tag{3}$$

For a large new dimension (measured in the fundamental unit $M_{Planck,4+\delta}$) we have $M_{Planck,4+\delta} \, R \gg 1$ and so we can see that the large size of the Planck mass (and the associated feebleness of the gravitational force) may be due to the existence of a new large dimension.

In these theories the smallness of Newton's coupling is due to flux spreading of the graviton in the additional dimension. If the right-handed neutrino is also able to propagates in the new large dimension its flux too will spread in the extra dimensions and this will lead to a suppression of the Yukawa coupling of the neutrino to the Higgs. As a result the neutrino mass will be naturally small, having a similar see-saw form to that in eq(2) suppressed by an inverse power of the four dimensional Planck mass[5]. Indeed it is known that candidate sterile neutrino must exist in supersymmetric versions of such theories, namely the modulini associated with the moduli setting the size of the new dimension. To make this idea more explicit consider a five dimensional (5D) space with coordinates (x_i, y). The interaction of the right-handed bulk neutrino, $\nu_R(x.y)$, with the Standard Model lepton doublet and Higgs states, $l(x)$, $h(x)$, confined to a 4-D brane is given by

$$S = \lambda \int d^4x l(x) h(x) \nu_R(x, y = 0)$$

If the fifth dimension is compact (here, compactified on a circle) we may expand the neutrino wave function in terms of its Kaluza Klein tower of states

$$\nu_R(x, y) = \frac{1}{\sqrt{2\pi R M_*}} \sum_{n=-\infty}^{\infty} \nu_R^n(x) e^{iny/R}$$

where the normalisation factor has been chosen so that the states, $\nu_R^n(x)$, have canonical normalisation for their kinetic terms - this factor accounts for the flux spreading into the fifth dimension. Here R is the radius of compactification and M_* is the fundamental mass scale. The neutrino mass is given by

$$\begin{aligned} m_\nu &= \frac{\lambda < h >}{\sqrt{2\pi R M_*}} \\ &= \frac{\lambda < h > M_*}{M_{Planck,4}} \\ &\approx 10^{-4}\lambda eV.(\frac{M_*}{1TeV}) \end{aligned}$$

where we have used the fact that the 5D mass scale is related to the 4D Planck scale by $2\pi R M_* = M_{Planck,4}/M_*$. Thus we see the effect of flux spreading is to suppress the neutrino coupling in the same way as the gravitational coupling is suppressed since both propagate in the extra dimension(s). As a result one naturally obtains a see-saw mechanism suppressed by the large Planck mass even though there are no fundamental mass scales of this magnitude in the theory. For M_* of order $1TeV$, a scale that avoids the hierarchy problem, one readily obtains neutrino masses of the correct order.

3.3 Small neutrino masses - radiative generation

As discussed above, yet another possibility is that neutrino masses are zero at tree level and are generated at some order in perturbation theory. A simple example is given in (non-MSSM) supersymmetric extensions of the Standard Model in which there are $R-$parity violating couplings generate the lepton-number violating couplings needed to give a Majorana mass. For example the coupling $h\nu d\widetilde{d}^c$ generates a neutrino mass at one loop order via a d quark $\widetilde{d}^c$ squark intermediate state given by [6]

$$m_\nu = \frac{h^2}{16\pi^2}\frac{m_d^2 A_{SUSY}}{m_{\widetilde{d}}^2}$$

where A_{SUSY} is a soft trilinear supersymmetry breaking parameter and $m_{\widetilde{d}}$.is the down squark mass. For a reasonable choice of the supersymmetry breaking parameter this gives

$$\frac{m_\nu}{m_d} \approx 10^{-5}h^2$$

small without the need for a GUT mass.

4 What is the spectral pattern?

In Figure 1 four viable mass patterns are shown together with the composition of the states. The implications of each of these possibilities is different so we will discuss tham in turn.

4.1 Near degeneracy

In this case the neutrinos are very nearly degenerate. Since the oscullation experiments only measure mass (squared) differences the average mass can be much larger than is the case if the masses are hierarchical. To generate the degeneracy needs a family symmetry to ensures it is stable against radiative corrections [7]. An example of such a symmetry [8] is provided by

Figure 1: Four possible spectral patterns capable of explaining the experimental indications of neutrino mass. The composition of the neutrino mass eigenstates in terms of the current neutrino eigenstates is shown,

$SO(3)$ in which the three left-handed lepton doublets of the Standard Model make up a triplet of $SO(3)$. In the case that the symmetry is unbroken the neutrinos must be degererate and the mass splitting is generated by small symmetry breaking effects. Given the near degeneracy the large mixing angles in the neutrino sector is quite natural. Making the mass as large as possible cosmologically interesting abundances are possible.

$$\Omega_\nu \leq (0.05 - 0.5)\left(\frac{0.5}{h}\right)^2$$

It is easy to avoid conflict with double beta decay bounds by arranging that the degenerate mass corresponds to a Pseudo Dirac mass in the $e\mu$ sector ensuring that $e - \mu$ number is conserved and eliminating double beta decay processes.

4.2 Normal hierarchy

The second spectral pattern of Figure 1 has a "normal" distribution in the sense that the heaviest neutrino has a mass $m_@ \simeq \sqrt{\Delta m^2_@}$ and the next has mass $m_\odot \simeq \sqrt{\Delta m^2_\odot}$ where Δm^2 denotes the mass squared difference relevant to the oscillation measurement. For the case of three generations the see-saw mechanism gives the effective neutrino masses of the form

$$M_\nu = M^\nu_D . M^{-1}_M . M^D_\nu \tag{4}$$

where the Ms are 3×3 matrices in family space. We will work in a basis where the charged leptons and Majorana mass are diagonal. Then we have

$$M^\nu_D = \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \quad M_M = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

An interesting structure emerges if the Majorana masses are hierarchical such that the first mass is so light it dominates eq.(4) [9]. In this case (Single Right Handed Dominance SRHD) we have

$$m_@ \simeq \tfrac{1}{M_1} \tag{5}$$

The condition that this should give near maximal neutrino mixing corresponds to $x_2 \sim x_3 >> x_1$. The interest in this is that, as we shall see, such a pattern occurs in the quark mass matrix. Thus, due to the see-saw mechanism, the smallness of the quark mixing angles does not imply small neutrino angles even though the neutrino Dirac mass matrix has the same form as the quark mass matrix.

What about the solar mixing angle? Again a particularly interesting structure emerges if the hierarchical form of the Majorana masses continues to dominate in the sense that the second Majorana state dominates the contribution to the second heaviest neutrino state [9]. In this case (Double Right Handed Dominance DRHD)

$$m_\odot \simeq \tfrac{1}{M_2}\sum_i y_i^2, \quad y_2 \sim y_3 \sim y_1$$

and the near maximal measurement of neutrino masses implies $y_2 \sim y_3 \sim y_1$. We shall discuss shortly whether this is consistent with the Dirac mass matrix of the same form as the quark mass matrix. In this scheme we have a bound on the remaining mixing angle [10, 11]

$$|U_{e3}| \geq \frac{1}{2} \sin 2\theta_{12} \frac{m_\odot}{m_@}$$

This implies $|U_{e3}|$ is close to the present bound. Such a value is encouraging for the prospect of measuring CP violation using very long baseline experiments.

4.3 Inverted hierarchy

In the case of the inverted hierarchy the two heaviest neutrinos are nearly degenerate with a Pseudo Dirac mass equal to $m_@$ while the third eigenstate has mass $m_\odot$ corresponding to the form

$$M_\nu \simeq m_@ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & \\ 1 & & 0 \end{pmatrix} + m_\odot \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

This structure gives a small contribution to $|U_{e3}|$[10]

$$|U_{e3}| \leq \frac{m_\odot^2}{2m_@^2}$$

but there is a contribution from the diagonalisation of the charged lepton masses which can increase it near to the experimental bound again [11].

5 What is the origin of fermion mass structure?

Of course the fundamental question raised by the pattern of gauge couplings, fermion masses and mixing in the Standard Model is what is the underlying origin of the structure? There are several promising indications that it may be due to an extension of the symmetries of the Standard Model. The most significant is the unification of gauge couplings which suggests an underlying Grand Unified structure such as $SU(5)$ or $SO(10)$. In them both the quarks and leptons of a single family are related. If this is the case it seems difficult to understand why quark mixing angles are small while neutrino mixing angles are large [12]. In SU(5) the left handed lepton doublet together with the (charge conjugate) right handed quarks make up a 5 dimensional representation. Thus the mixing in the neutrino sector is related to the mixing of the right handed down quarks which is not measured by charged weak processes and so might be large. In $SO(10)$ this seems unlikely because all the members of a single family, plus the right handed neutrino belong to a single 16 dimensional representation and so, if $SO(10)$ breaking effects are small, the mixing in the neutrino sector are related to the mixing in both the left- and right- handed sectors.

However, as we have seen, due to the see-saw mechanism, our naive expectations about the relation between mixing angles may not be correct and equality of mixing in the Dirac mass matrices of quarks and leptons may be quite consistent with neutrino oscillations! This is easy to see in the limit of a strong hierarchy of neutrino Majorana masses, $M_2 << M_{1,3}$ which gives (c.f eq.(5) - the difference is because we have chosen M_2 rather than M_1 to be the lightest state)

$$m_@ \simeq \frac{1}{M_2} \sum_i y_i^2,$$

In the linit $y_2 \sim y_3 >> y_1$ the mixing angle $(U_{MNS})_{23}$ relevant for atmospheric neutrino mixing is large. This pattern for the elements of the neutrino Dirac mass matrix can be compared with the measured values of the down quark mass matrix (I use the down quark matrix since the up quark is poorly determined)) [13]

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.5\varepsilon^3 & 0.4e^{i20}\varepsilon^3 \\ 1.5\varepsilon^3 & \varepsilon^2 & 1.3\,\varepsilon^3 \\ 0.4e^{i20}\varepsilon^3 & 1.3\,\varepsilon^3 & 1 \end{pmatrix} \qquad \varepsilon = 0.15 \tag{6}$$

One sees that the $(2,2)$ element is quite close to the $(2,3)$ element, just as is wanted in the neutrino sector!

This illustrates the point that there may be a close connection between the mass patterns of the quark and lepton mass matrices consistent with large neutrino mixing angles. To illustrate how far this connection can be taken and the implications for an underlying Grand Unified and family symmetry I will summarise the form that results from a $SU(3)$ family symmetry [14, 15] in which the three generations of left-handed quarks and leptons are in the triplet representation as are the charge conjugate of their right handed partners. In addition there is a Grand Unified symmetry ($SO(10)$) which relates quarks and leptons. The symmetry constrains the Dirac mass matices of down quarks and charged leptons to have the form

$$\frac{M^d}{m_b} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon \\ -\varepsilon^3(z+y\varepsilon) & \varepsilon^2(w+u\varepsilon) & \varepsilon^2(w-u\varepsilon) \\ -\varepsilon^3(z+y\varepsilon) & \varepsilon^2(w-u\varepsilon) & 1 \end{pmatrix} \tag{7}$$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} \varepsilon^8 & \varepsilon^3(z+(x+y)\varepsilon) & \varepsilon^3(z+(x-y)\varepsilon \\ -\varepsilon^3(z+y\varepsilon) & \varepsilon^2(a_l w+u\varepsilon) & \varepsilon^2(a_l w-u\varepsilon) \\ -\varepsilon^3(z+y\varepsilon) & \varepsilon^2(a_l w-u\varepsilon) & 1 \end{pmatrix} \tag{8}$$

Here z, w and u are real coefficients and x and y complex coefficients of order 1. The parameter $\varepsilon \simeq 0,15$ is the $SU(3)$ symmetry breaking parameter and an additional Abelian family symmetry orders the breaking in the different matrix elements in a way that reproduces the form of eq.(6). We have also included subdominant contributions developing the first terms in a power series in ε. The important thing to note is that the underlying $SU(3)$ symmetry requires the equality of the leading terms of order ε^3 in the $(1,2)$ and $(1,3)$ elements and the leading terms of order ε^2 in the $(2,2)$ and $(2,3)$ elements. The same applies to the nonleading terms. This will be important in generating near bi-maximal neutrino mixing. The form of the lepton mass matrix, eq(8), is the same as that of the quark mass matrix, eq.(7) and the coefficients are related due to the $SO(10)$ Grand Unified symmetry. In particular the $(1,2)$ and the $(1,3)$ elements are equal leading to the relations $Det(M_d) = Det(M_l)$ and $m_b = m_\tau$ at the Grand Unified scale. After including radiative corrections to continue to the laboratory scale, these relations are in excellent agreement with experiment. The coefficient a_l also comes from $SO(10)$ but in this case the fields generating the masses have a Clebsch Gordon coefficient giving $|a_l| = 3$. This gives an excellent value for m_μ, $m_\mu = 3m_s$ at the GUT scale leading, after radiative corrections, to $m_\mu \simeq m_s$ at low energies [16] which, together with the two previous relations, generates the charged lepton masses from the down quark masses.

The up quark and netrino mass matrices are also related by the GUT giving

$$M^d \leftrightarrow M^u \quad M^l \leftrightarrow M^\nu \quad \varepsilon \leftrightarrow \varepsilon' \quad a_l \leftrightarrow a_\nu$$

The appearance of the different expansion parameter $\varepsilon = 0.05$ is due to the $SO(10)$ breaking effects which are necessarily present. Finally the Majorana mass matrix is very nearly diagonal and strongly hierarchical. This leads to the DRHD discussed in Section 4.2 with the two heaviest neutrinos having the form

$$\begin{aligned} \nu_a \propto &\quad z(\nu_\mu + \nu_\tau) + y\varepsilon'(\nu_\mu - \nu_\tau) \\ \nu_b \propto &\quad z\nu_e + r(\nu_\mu - \nu_\tau - y\varepsilon'(\nu_\mu + \nu_\tau)/z) \end{aligned}$$

where

$$r = \sqrt{2}(u - a_\nu wy/z) \quad = O(1)$$

This naturally gives near bi-maximal mixing in the neutrino sector. There are two choices for a_ν consistent with $|a_l| = 3$, namely $|a_\nu| = 3$ and $a_\nu = 0$. The former gives the LOW solar solution while the latter gives the LMA solution [15]. This example shows that, contrary to our first impressions, near bimaximal mixing is consistent with a GUT relation between neutrino masses and quark masses.

6 Summary and Conclusions.

The new data on quark and lepton masses and mixing is testing our ideas for the origin of fermion masses and mixings and raises interesting questions. The multiplicity of light neutrino-like states sheds light on the symmetries of the low energy theory and additional sterile neutrinos would indicate new symmetries. There are several ideas for the origin of the hierarchy of masses of the neutrinos. These include approximate symmetries such as lepton numbers, new family symmetries supersymmetry, or new space dimensions giving rise to anomalously small Yukawa couplings responsible for fermion mass or the see-saw mechanism in which the large mass of the messenger state makes the neutrinos anomalously light. The pattern of quark and lepton masses and mixings suggests there may be a relation between quarks and leptons. This may be due to an underlying Grand unified symmetry such as $SU(5)$ or $SO(10)$ or directly due to the underlying string structure which may not have a GUT symmetry below the compactification scale. The connection between the mixing angles in the quark and lepton sectors and the underlying mass matrix structure is masked by the see-saw mechanism, In one particular limit with a very strong hierarchy between the Majorana masses the connection becomes simple and the large mixing angles measured in the lepton sector correspond to small mixing angles. In this scheme the quark and lepton Dirac matrices may have quite similar forms, with differences due to simple Clebsch Gordon coefficients coming from an underlying GUT. An encouraging feature of the models of neutrino mass for all the spectral patterns is that the remaining mixing angle U_{e3} is likely to be large, close to the present bounds and so CP violation in the lepton sector may be measurable in very long baseline neutrino experiments.

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