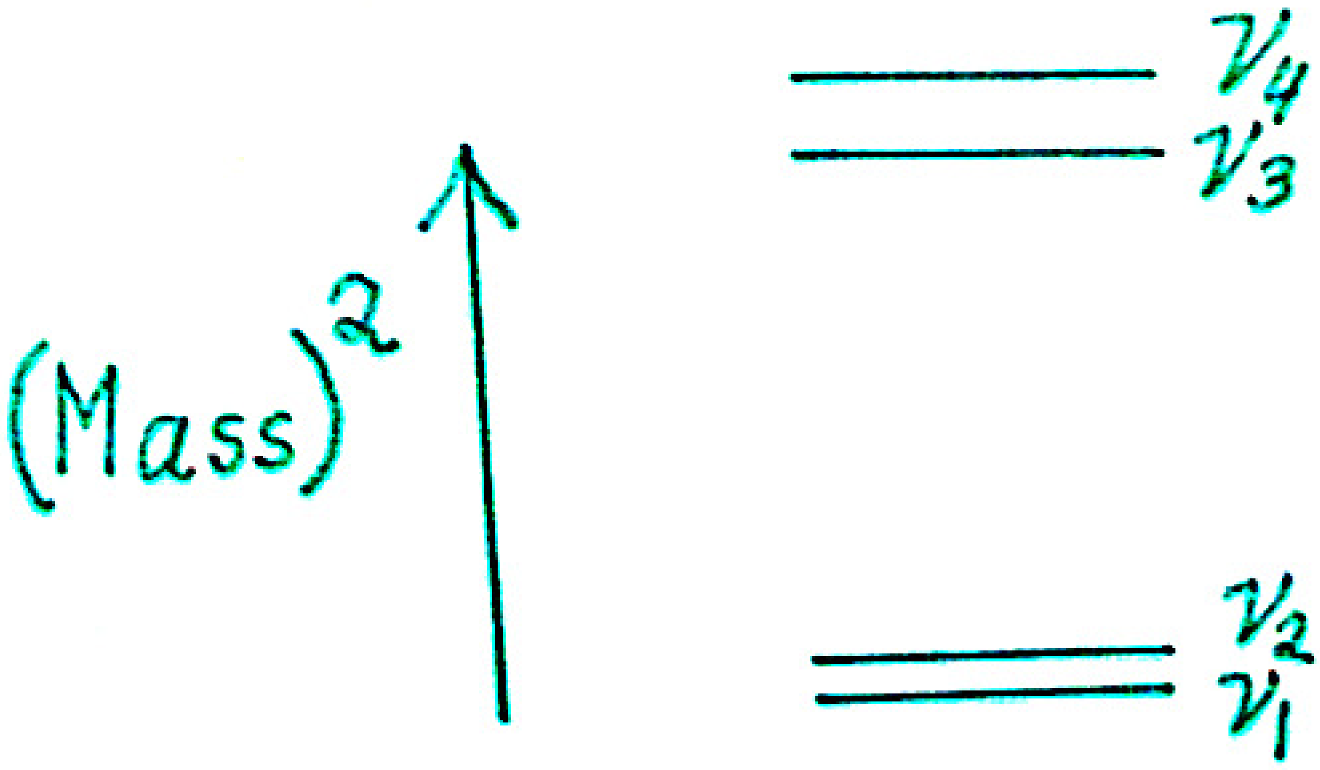
ν.1

# Neutrino Masses and Mixing

There is some spectrum of 3 or more neutrino mass eigenstates $\nu_i$:



$$\text{Mass}(\nu_i) \equiv m_i$$

$l_e \equiv e,\ l_\mu \equiv \mu,\ l_\tau \equiv \tau$

When $W^+ \longrightarrow l_\alpha^+ + \nu_\alpha$ ,

$e$, $\mu$, or $\tau$

the produced neutrino state $|\nu_\alpha\rangle$ is

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* \, |\nu_i\rangle$$

Neutrino of flavor α

Unitary Leptonic Mixing Matrix

fraction of $\nu_i = |\langle\nu_\alpha|\nu_i\rangle|^2 = |U_{\alpha i}$

If there are, say, four mass eigenstates, then one linear combination of them,

$$|\nu_{sterile}\rangle = \sum_i U^*_{si} \, |\nu_i\rangle ,$$

has no normal weak couplings.

More than 3 neutrinos $\Longrightarrow$
A new kind of neutrino.

Neutrino flavor change depends only on the splittings

$$\Delta m^2_{ij} \equiv m_i^2 - m_j^2 ,$$

not on individual masses.

When only 2 neutrinos count,

$$\begin{array}{cccc} & \nu_1 & \nu_2 & \\ \nu & [\cos\theta & \sin\theta] & \leftarrow \text{Mixing angle} \end{array}$$

Voluminous atmospheric neutrino data are well described by –

$$\nu_\mu \rightarrow \nu_\tau$$

$$1.6 \times 10^{-3}\,eV^2 < \Delta m^2_{atm} < 3.9 \times 10^{-3}\,eV^2$$

$$\sin^2 2\theta_{atm} > 0.92$$

(90% CL)

Best Fit $\Rightarrow \begin{cases} \Delta m^2_{atm} = 2.5 \times 10^{-3}\,eV^2 \\ \sin^2 2\theta_{atm} = 1.0 \end{cases}$

From Super-K. Compatible values from MACRO, Soudan, and the K2K Long BaseLine experiment.

Bethe: "Mixing angles are small."

Nature: Only the small ones are!

" behind this

## Solar

For the $^8B$ (high-energy) solar neutrinos, the S(udbury) N(eutrino) O(bservatory) studies—

NC $\nu_\odot d \to \nu n p \Rightarrow \phi_e + \phi_{\mu\tau}$

ES $\nu_\odot e \to \nu e \Rightarrow \phi_e + 0.15\,\phi_{\mu\tau}$

CC $\nu_\odot d \to e p p \Rightarrow \phi_e$

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Including SK $\nu_\odot e \to \nu e$ data,

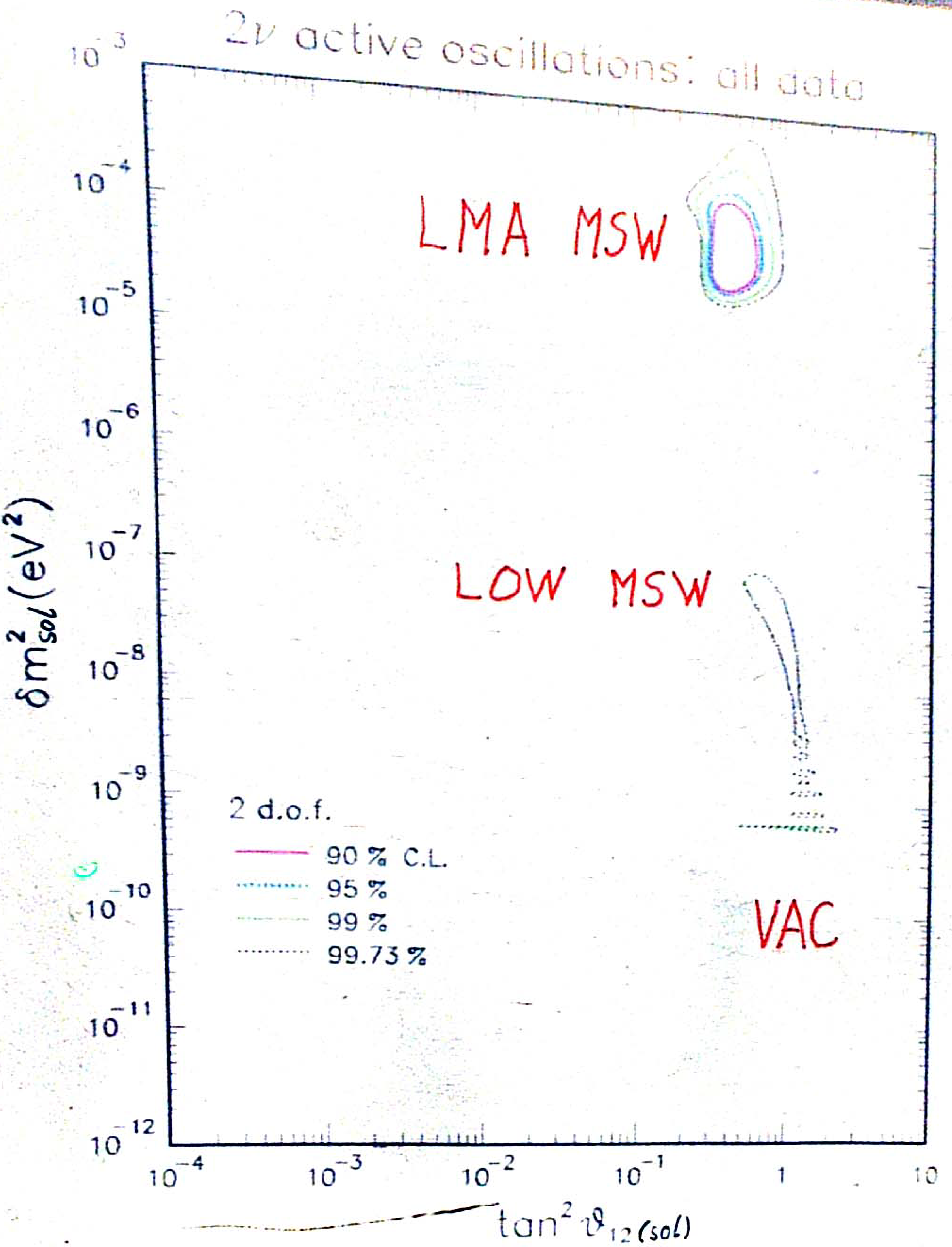
$$\phi_{\mu\tau} = (3.45^{+0.65}_{-0.62}) \times 10^6/\text{cm}^2\,\text{sec}.$$

(5.5σ from zero)

The nuclear processes that power the sun make only $\nu_e$

- Neutrinos do change flavor.

Lisi



Global results

Cl, Ga, SK, SNO data
Analysis in terms

$$\text{From } \mu^+ \to e^+ \nu_e \bar{\nu}_\mu \;\rightarrow\; \bar{\nu}_\mu \to \bar{\nu}_e$$

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\%$$

(LSND)

KARMEN: No signal.

But —

Distance $L$ (LSND) $\sim 30$ m

" $L$ (KARMEN) $\sim 18$ m

Combined statistical analysis ⇒

$$0.2 \lesssim \Delta m^2_{LSND} \lesssim 1\,\mathrm{eV}^2 \;\&\; .003 \lesssim \sin^2 2\theta_{LSND} \lesssim .03$$

— or —

$$\Delta m^2_{LSND} \simeq 7\,\mathrm{eV}^2 \;\&\; \sin^2 2\theta_{LSND} \simeq .004$$

might explain both experiments.

(Church, Eitel, Mills, Steidl)

# Null Disappearance Experiments

These limit —

- $\bar{\nu}_e \longrightarrow \bar{\nu}_{\mu,\tau,s}$ with $\Delta m^2 \gtrsim 10^{-3}\,\text{eV}^2$

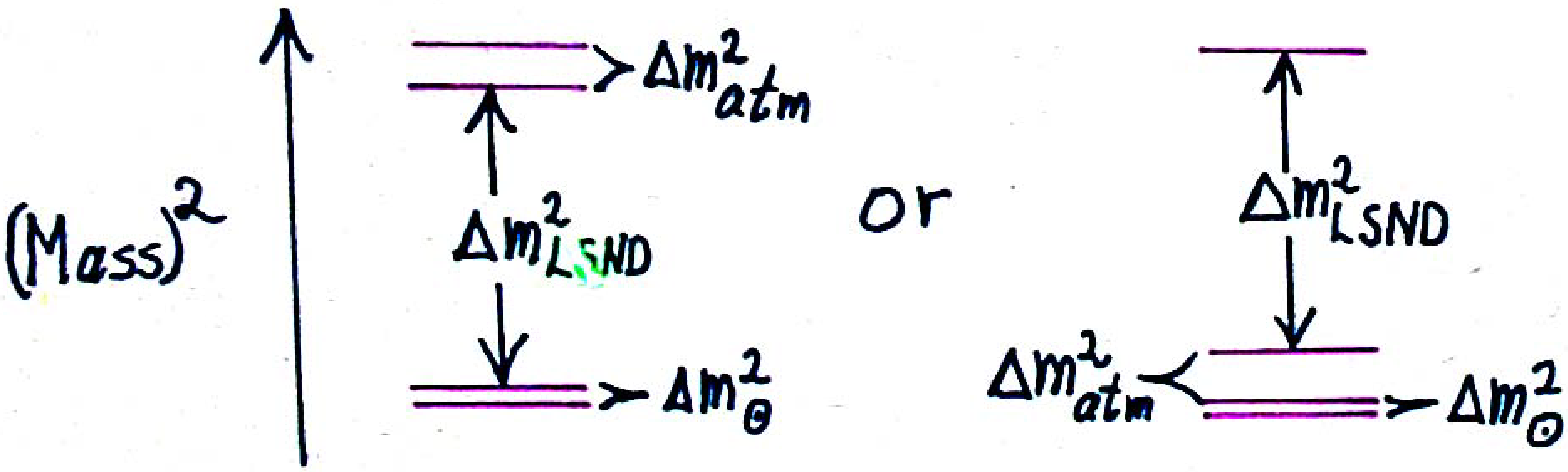
- $\nu_\mu \longrightarrow \nu_{e,\tau,s}$ with $\Delta m^2 \gtrsim 1\,\text{eV}^2$

These limits are important constraints on the neutrino mass spectrum.

# If LSND is Confirmed

At least 4 mass eigenstates are required.

The spectrum looks like –



2+2 3+1

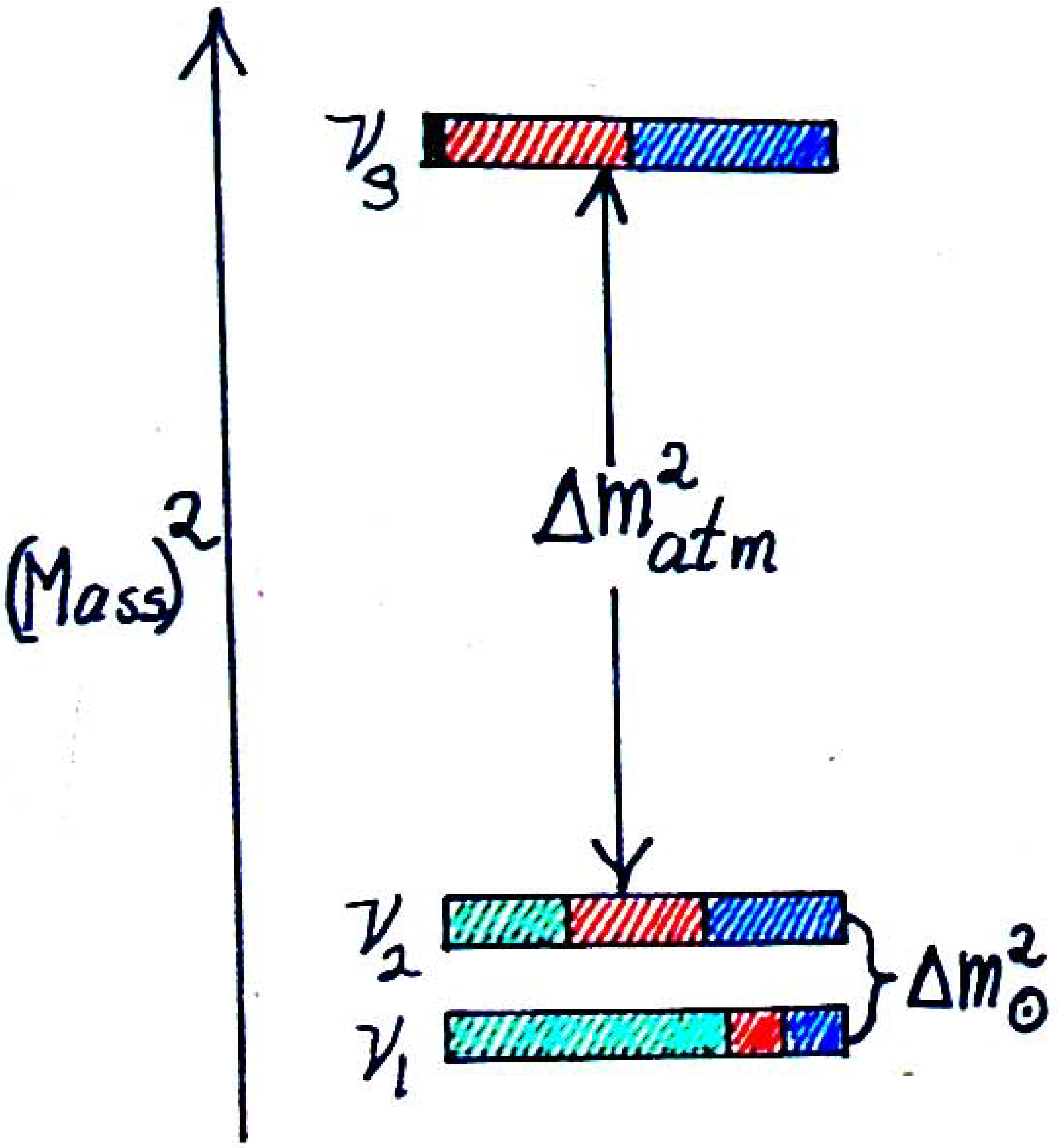
or "upside-down" version.

# WHAT DO THE OBSERVATIONS IMPLY?

If LSND is confirmed . . . [4ν]

If LSND is not confirmed, nature may contain only 3 neutrinos.

Assuming LMA-MSW,
the spectrum looks like —



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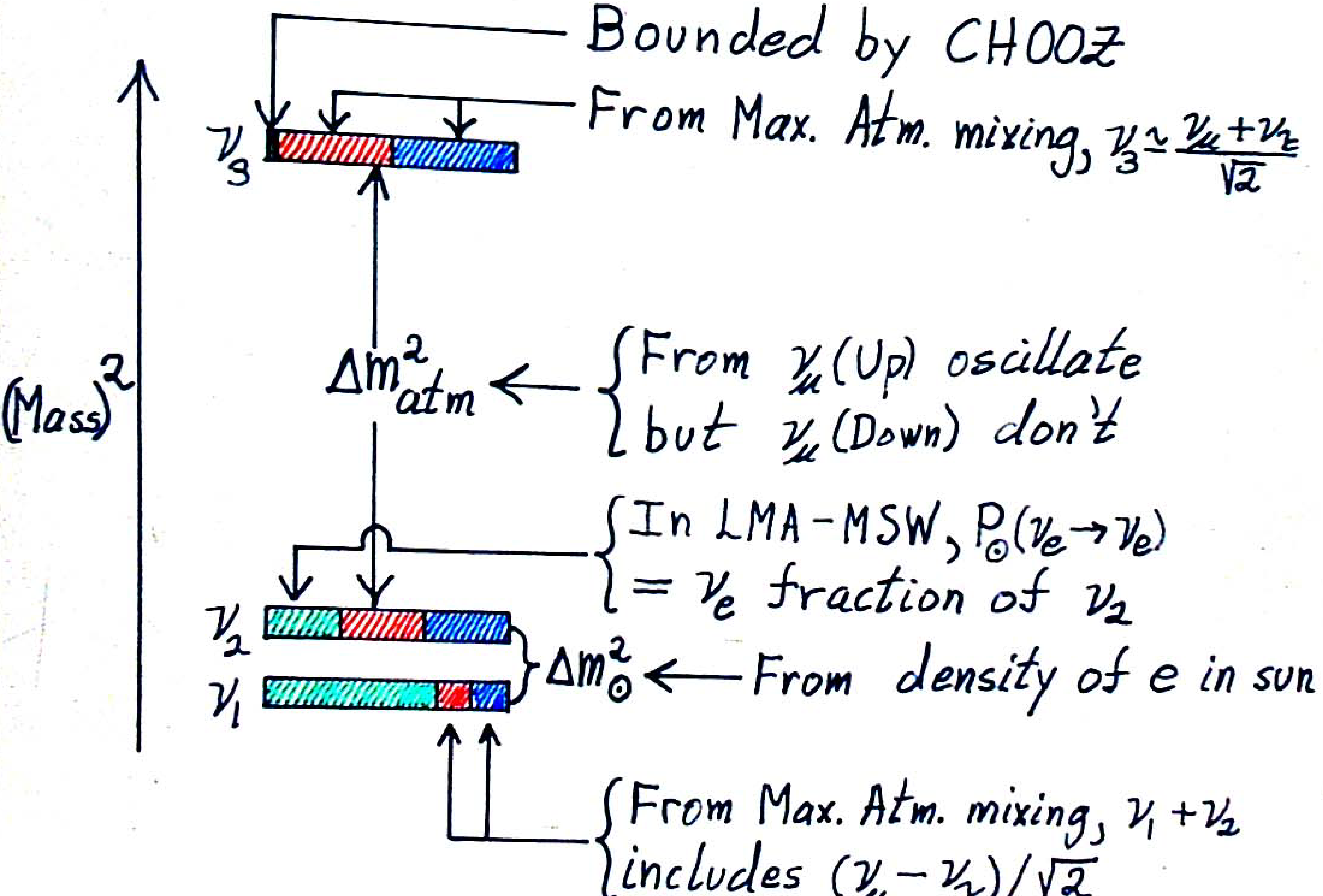


🟩 $\nu_e\,[|U_{ei}|^2]$ 🟥 $\nu_\mu\,[|U_{\mu i}|^2]$ 🟦 $\nu_\tau\,[|U_{\tau i}|^2]$

The spectrum could be  instead of .

Corresponding to the flavor content shown,

$$U \approx \begin{array}{c|ccc|} & \multicolumn{2}{c}{\text{Close pair}} & \text{Isolated} \\ & \nu_1 & \nu_2 & \nu_3 \\ \hline \nu_e & c\, e^{i\frac{\alpha_1}{2}} & s\, e^{i\frac{\alpha_2}{2}} & s_{13} e^{-i\delta} \\ \nu_\mu & -\frac{s}{\sqrt{2}} e^{i\frac{\alpha_1}{2}} & \frac{c}{\sqrt{2}} e^{i\frac{\alpha_2}{2}} & \frac{1}{\sqrt{2}} \\ \nu_\tau & \frac{s}{\sqrt{2}} e^{i\frac{\alpha_1}{2}} & -\frac{c}{\sqrt{2}} e^{i\frac{\alpha_2}{2}} & \frac{1}{\sqrt{2}} \\ \hline \end{array}$$

$c \equiv \cos\theta_\odot$ , $s \equiv \sin\theta_\odot$ , $s_{13} \equiv \sin\theta_{13}$

With LMA − MSW,

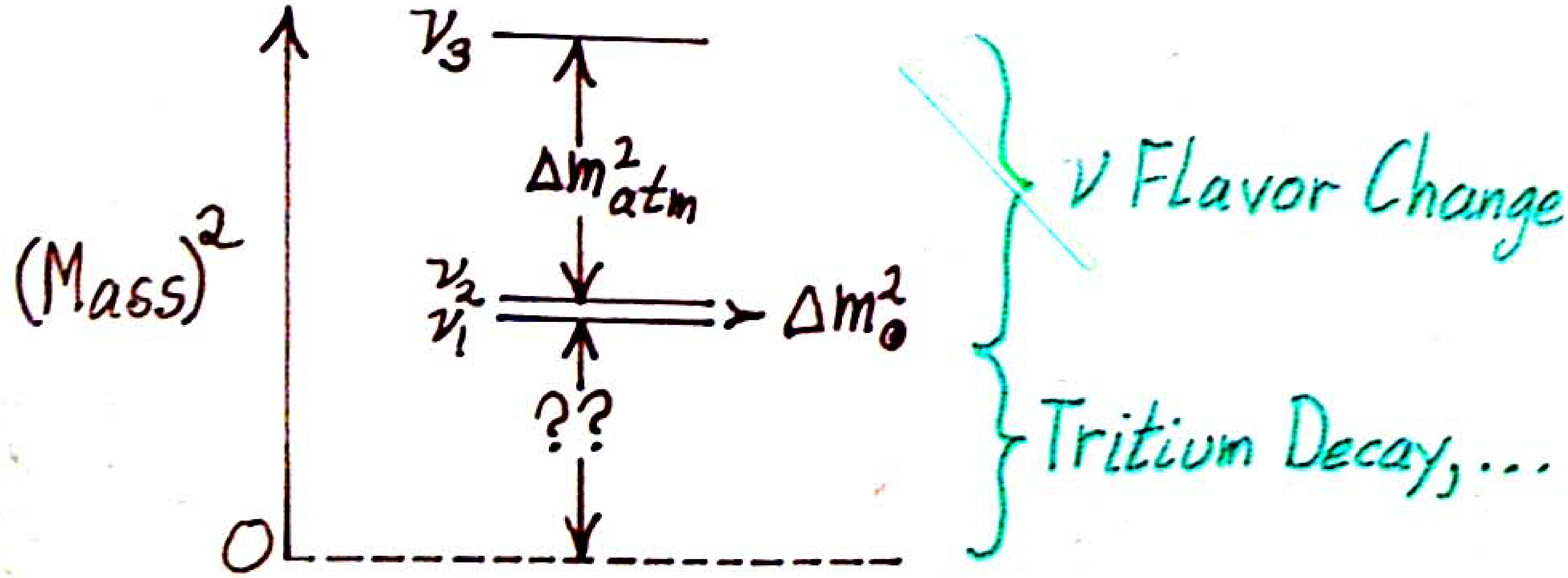
$0.24 \lesssim \sin^2\theta_\odot \lesssim 0.38$ (90%CL) (Lisi)

From bounds on reactor $\bar{\nu}_e$ oscillation,

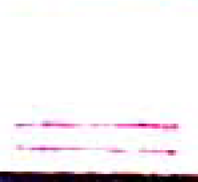
$\sin^2\theta_{13} \lesssim 0.03$ (90% CL) (CHOOZ, Palo Verde)

# What Would We Like To Find Out?

- * How many neutrino species are there? Are there sterile neutrinos?
- * What are the $(\text{Mass})^2$ splittings between the mass eigenstates $\nu_i$? How far above zero does the whole pattern lie?



Is LMA-MSW truly the mechanism of solar neutrino flavor change?

Is the spectral pattern  or  ?

* What is the flavor content of the mass eigenstates?

- Do $\nu_1$, $\nu_2$, and $\nu_3$ contain $\nu_\mu$ and $\nu_\tau$ in exactly equal proportion (Maximal Mixing)? What is the deviation from maximality?
- What is $\theta_\odot$? How is the $\nu_e$ content split between $\nu_1$ and $\nu_2$?
- Is the small $\nu_e$ piece of $\nu_3$ nonzero? How large is it?
- $\not{CP}$ in oscillation is proportional to

* Is each mass eigenstate —

- A Majorana particle $(\bar{\nu}_i = \nu_i)$

or

- A Dirac particle $(\bar{\nu}_i \neq \nu_i)$

$(\beta\beta_{0\nu})$

* Do neutrino interactions violate CP?

- In oscillation?

  (From phase $\delta$ in $U$)

- In $\beta\beta_{0\nu}$?

  (From phases $\alpha_{1,2}$ & $\delta$ in $U$)

Observing ~~$\cancel{CP}$ in~~ neutrino physics would establish that $\cancel{CP}$ is not

* What is the origin of neutrino flavor physics?

- Is it new physics at a high mass scale? Where? What's there?
- Does the see-saw mechanism generate $\nu$ masses?
- Do symmetries play a role in $\nu$ masses and mixing?
- What is the connection between $\nu$ flavor physics and quark flavor physics?

# The Impact of a Neutrino Factory and $\nu_\mu, \nu_e$ Superbeams

Coming Long Base Line (LBL) experiments will try to confirm the atmospheric

Distance

$$P(\nu_\mu \to \nu_\tau) = \underbrace{\sin^2 2\theta_{atm}}_{4|U_{\mu3}|^2|U_{\tau3}|^2} \sin^2\left(\Delta m^2_{atm} \frac{L}{4E}\right)$$

Energy

by observing —

$\nu_\mu$ disappearance

$\nu_\tau$ appearance

undulation $\sin^2(\Delta m^2_{atm} \frac{L}{4E})$ with $1/E$

and determining —

$\Delta m^2_{atm}$

(K2K MINOS CNGS)

With new or upgraded beams, determine —

$$1 - \sin^2 2\theta_{atm}$$

$$\sim \left( \frac{\text{Symmetry Breaking in } \nu \text{ Mass Matrix}}{\nu_\mu - \nu_\tau \text{ Mixing} \quad \text{''} \quad \text{''} \quad \text{''}} \right)^2 .$$

With a ν Factory and the $\nu_\mu$ and possible $\nu_e$ Superbeams of the future —

Go after $\theta_{13}$ $[|U_{e3}|]$ !

- Show it is nonzero

# How big do we expect $\theta_{13}$ to be??

## A prejudice

In gauge theory,

$$U = X_\ell X_\nu = \begin{bmatrix} B & B & \theta_{13} \\ B & B & B \\ B & B & B \end{bmatrix} ; \; B \equiv \text{Big}$$

$X_\ell$: Diagonalizes $\ell$ mass matrix

$X_\nu$: Diagonalizes $\nu$ mass matrix

Except for $U_{e3} \sim \theta_{13}$,

$$\text{all } U_{\alpha i} = \sum_j (X_\ell)_{\alpha j} (X_\nu)_{j i} \text{ are Big}$$

It would take a special cancellation (caused by a symmetry??) for $U_{e3}$ to

very much smaller than all other $U$

$$P(\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu) = P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \cong \frac{1}{2}\sin^2 2\theta_{13}\,\sin^2\left(\Delta m^2_{atm}\frac{L}{4E}\right)$$

With $\theta_{13}$ in hand –

Determine if the spectral pattern is

or

using matter effects.

For $E \gtrsim 10$ GeV and $L \gtrsim 2000$ km, for example,

$$P(\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu) = P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \cong \frac{1}{2}\sin^2 2\overset{(-)}{\theta}_M\,\sin^2\left(\overset{(-)}{\Delta m^2_M}\frac{L}{4E}\right)$$

mixing angle and splitting in matter are—

$$\sin^2 2\overset{(-)}{\theta}_M = \frac{\sin^2 2\theta_{13}}{\sin^2 2\theta_{13} + (\cos 2\theta_{13} - \overset{(-)}{x}_M)^2}$$

and

$$\overset{(-)}{\Delta m^2_M} = \Delta m^2_{atm} \sqrt{\sin^2 2\theta_{13} + (\cos 2\theta_{13} - \overset{(-)}{x}_M)^2}$$

where

$$\overset{(-)}{x}_M = \overset{+}{(-)} \frac{2\sqrt{2}\, \overset{\text{Fermi constant}}{\overset{\downarrow}{G_F}}\, \overset{\text{Electron density}}{\overset{\downarrow}{N_e}}\, E}{\underbrace{\Delta m^2_{atm}}_{\equiv m^2(-) - m^2(=)}} .$$

$$\frac{\sin^2 2\theta_M}{\sin^2 2\bar{\theta}_M} \begin{cases} > 1 \; ; & \begin{smallmatrix} - \\ = \end{smallmatrix} \\ < 1 \; ; & \begin{smallmatrix} = \\ - \end{smallmatrix} \end{cases}$$

Let $P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \equiv \Delta_{\not{CP}}(\alpha\beta)$.

If there are only 3 neutrinos,

$$\Delta_{\not{CP}}(e\mu) = \Delta_{\not{CP}}(\mu\tau) = \Delta_{\not{CP}}(\tau e)$$

$$= 16\, J\, k_{12}\, k_{23}\, k_{31},$$

where

$$J \equiv \mathrm{Im}(U_{e1}^* U_{e3} U_{\mu 1} U_{\mu 3}^*) \cong \frac{1}{4} \sin 2\theta_\odot \sin\theta_{13} \sin\delta,$$

and

$$k_{ij} \equiv \sin\left(\Delta m_{ij}^2 \frac{L}{4E}\right).$$

If $\sin^2\theta_\odot = 0.3$, $\sin\theta_{13} = 0.1$, $\sin\delta = 1$,

$\Delta m^2_\odot = 5 \times 10^{-3}\,\mathrm{eV}^2$, $\Delta m^2_{atm} = 2.5 \times 10^{-3}\,\mathrm{eV}^2$,

$\frac{L}{E} = \frac{3000\ \mathrm{km}}{6\ \mathrm{GeV}}$ [1st peak of $\sin(\Delta m^2_{atm}\frac{L}{4E})$],

$$\Delta_{\cancel{CP}}(\alpha\beta) \simeq 1\% \quad .$$

In practice, $\nu$ and $\bar{\nu}$ rates will depend on —

- Genuine $\cancel{CP}$ from phase $\delta$
- Matter-induced $\nu/\bar{\nu}$ asymmetries
- CP- conserving $\nu$ parameters .

It will be necessary to disentangle things.

( Huber, Lindner, Winter

# Was Baryogenesis Made Possible by Leptonic $\not{CP}$?

Baryogenesis requires $\not{CP}$.

Standard-Model quark $\not{CP}$ is very insufficient.

Supersymmetry $\not{CP}$ requires a special corner of parameter space.

Perhaps there was —

$$\Gamma[N(\text{Heavy Majorana}) \rightarrow \ell^- + \text{Higgs}^+]$$
$$\neq \Gamma[N(\text{Heavy Majorana}) \rightarrow \ell^+ + \text{Higgs}^-].$$

This $\not{CP}$ would have produced a lepton asymmetry that would then have resulted in a baryon asymmetry.

Leptonic $\not{CP}$ may be the reason we exist.

It is important to search for leptonic $\not{CP}$.

The compelling evidence for $\nu$ mass and mixing opens a whole $\nu$ world to explore.

We have much to learn about this world.

It is important that we build on the dramatic progress that has been made.

There are technical and financial challenges to be met, but the payoff will be well worth it.

The coming years will be an exciting time.