Lepton flavour violating μ-e conversion rate for various nuclei

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Abstract. The coherent μ-e conversion rate for general LFV interactions for various nuclei is calculated. The branching ratio increases for the light nuclei such as $Z \lesssim 30$, are largest for $Z = 30 - 60$, and decreases for heavy nuclei with $Z \gtrsim 60$. Although this tendency of Z dependence is the same for different types of coupling constants, there are significant differences in Z dependence of branching ratios. The atomic number dependence of the conversion ratio calculated here is useful to distinguish theoretical models with lepton flavor violation.

1. Introduction

The search for the lepton flavour violation (LFV) is one of the most interesting subjects in particle physics. The μ-e conversion in muonic atoms, or neutrinoless muon capture, is an expected signal of the LFV to be observed. The μ-e conversion rate has been calculated by several authors [1, 2], and its experimental upper bound is given to be 6.1×10^{-13} [3]. Further experiments are ongoing and planned to improve the bound down to $10^{-17} - 10^{-18}$ [4, 5]. The PRISM project [5] in the JAERI-KEK joint project is of particular interest among them since it is planned to make use of a muon storage ring, the muon beam from which is free from contamination of electrons and pions. It will then be possible to carry out the experiments with various target atom, and thus to optimize the target atom for the search. We thereby evaluate the μ-e conversion rate for nuclei of a wide range of atomic numbers. The atomic number dependence of the conversion ratio calculated here turns out to be useful to distinguish theoretical models with LFV. More details are found in the reference [6].

2. Formula of μ-e conversion rate

It is necessary to specify the interaction Lagrangian and the initial and final states in order to calculate the μ-e conversion amplitudes and rates. The LFV interaction Lagrangian is taken to be most general form which contributes to the μ-e conversion in nuclei [7]. It consists of photonic interaction terms and four-Fermi interaction terms, and the magnitude of coupling constant in each term depends on the interaction of the new physics in which the lepton flavour conservation is violated. The initial state in the μ-e conversion process is the 1s state of the muonic atom, and the final electron

§ Present address: Institute for Advanced Study, Princeton, NJ 08540, USA state is the eigenstate with an energy of $m_\mu - \varepsilon_b$, where ε_b is the binding energy of the 1s state of muonic atom. The final state of the nucleus is taken to be same as the initial state, which is referred to as the coherent process; the fraction of this process is generally larger than incoherent one approximately by a factor of the mass number of the target nuclei.

Then the formula of the μ-e conversion rate can be written in terms of the five overlap integrals $D, S^{(\mathrm{p,n})}$ and $V^{(\mathrm{p,n})}$ as

$$\omega_{\mathrm{conv}} = 2G_\mathrm{F}^2 \left| A_\mathrm{R}^* D + \tilde{g}_{\mathrm{LS}}^{(\mathrm{p})} S^{(\mathrm{p})} + \tilde{g}_{\mathrm{LS}}^{(\mathrm{n})} S^{(\mathrm{n})} + \tilde{g}_{\mathrm{LV}}^{(\mathrm{p})} V^{(\mathrm{p})} + \tilde{g}_{\mathrm{LV}}^{(\mathrm{n})} V^{(\mathrm{n})} \right|^2 + 2G_\mathrm{F}^2 \left| A_\mathrm{L}^* D + \tilde{g}_{\mathrm{RS}}^{(\mathrm{p})} S^{(\mathrm{p})} + \tilde{g}_{\mathrm{RS}}^{(\mathrm{n})} S^{(\mathrm{n})} + \tilde{g}_{\mathrm{RV}}^{(\mathrm{p})} V^{(\mathrm{p})} + \tilde{g}_{\mathrm{RV}}^{(\mathrm{n})} V^{(\mathrm{n})} \right|^2 . \quad (1)$$

Here G_F is the Fermi constant, and $A_{\mathrm{R,L}}^*$ and $\tilde{g}_{\mathrm{LS,LV,RS,RV}}^{(\mathrm{p,n})}$ are the coupling constants relevant to the LFV interaction. The overlap integrals are defined by

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty \mathrm{d}r r^2 [-E(r)](g_\mathrm{e}^- f_\mu^- + f_\mathrm{e}^- g_\mu^-), \quad (2)$$

$$S^{(\mathrm{p})}, V^{(\mathrm{p})} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r r^2 Z \rho^{(\mathrm{p})} (g_\mathrm{e}^- g_\mu^- \mp f_\mathrm{e}^- f_\mu^-), \quad (3)$$

$$S^{(\mathrm{n})}, V^{(\mathrm{n})} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r r^2 (A-Z) \rho^{(\mathrm{n})} (g_\mathrm{e}^- g_\mu^- \mp f_\mathrm{e}^- f_\mu^-), \quad (4)$$

where m_μ is the muon mass, $E(r)$ is the electric field in nuclei, $\rho^{(\mathrm{p,n})}(r)$ is the normalized density distribution of the proton and neutron in the nuclei, and $g_{\mu,\mathrm{e}}^-(r)$ and $f_{\mu,\mathrm{e}}^-(r)$ are the radial part of the wave function of the muon and the electron. The minus and plus sign in (3) and (4) corresponds to $S^{(\mathrm{p,n})}$ and $V^{(\mathrm{p,n})}$, respectively.

3. Numerical results

Figure 1(a) shows the five overlap integrals as functions of the atomic number Z. Here the proton density is taken from the reference [8], and the neutron density is assumed to be equal to the proton density. This is not necessarily a good approximation to calculate $S^{(\mathrm{n})}$ and $V^{(\mathrm{n})}$ for heavy nuclei, but the result turns out to be consistent within 10% to the result obtained from the experimental neutron distribution data [6]. We see that the overlap integrals increase as functions of Z for light nuclei up to $Z \sim 30$, and saturate or decrease for heavy nuclei. Although the tendency is the same, each overlap integral has different Z dependence, especially for heavy nuclei.

The conversion rate ω_{conv} and the branching ratio $B_{\mu\mathrm{N}\to\mathrm{eN}} \equiv \omega_{\mathrm{conv}}/\omega_{\mathrm{capt}}$, where ω_{capt} is the rate of the ordinary muon capture, can be straightforwardly calculated through equation (1) once a theoretical model with LFV is specified. We consider the following three cases: (i) Only the photonic dipole LFV operator is relevant; (ii) Only the scalar four-Fermi LFV operator is relevant; (iii) Only the vector four-Fermi LFV coupling constants $\tilde{g}_{\mathrm{LV}}^{(\mathrm{p})}$ and/or $\tilde{g}_{\mathrm{RV}}^{(\mathrm{p})}$ are relevant.Figure 1(b) presents the μ-e conversion branching ratio normalized by the value for aluminum ($Z = 13$). We can see that, for all three types, the branching ratio increases with Z for $Z \lesssim 30$, is largest for $30 \lesssim Z \lesssim 60$, and decreases for $Z \gtrsim 60$. It is also seen that the conversion ratios are very different in heavy nuclei depending on the type of interaction. From this property we may be able to distinguish models beyond the Standard Model through several experiments with different targets.

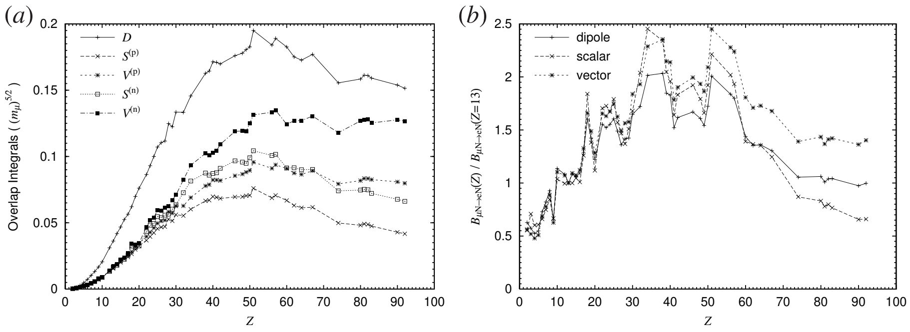

Figure 1. (a): The overlap integrals are plotted as functions of the atomic number Z. Neutron density distributions are assumed to be equal to charge density distributions. (b): The μ-e conversion ratios for typical theoretical models are plotted as functions of the atomic number Z. The solid, the long dashed, and the dashed lines represent the cases where the photonic dipole, scalar, and vector operator dominates, respectively. The proton and neutron distribution are taken to be the same, and the conversion ratios are normalized by the conversion ratio in aluminum nuclei ($Z = 13$).

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