Neutrino Bi-Large Mixing in MSSM with Right-Handed Neutrino and Lepton Flavor Violation

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Abstract. We investigate the lepton flavor violation in the framework of the MSSM with right-handed Majorana neutrinos taking the large mixing angle MSW solution in the hierarchical, degenerate and the inverted neutrino masses. We predict the branching ratio of $\mu \to e + \gamma$ process assuming the degenerate right-handed Majorana neutrino masses. We find that the branching ratio in the degenerate neutrino mass spectrum is about 100 times smaller than the one in the inverted and the hierarchical neutrino spectra. We emphasize that the Majorana mass scale and the magnitude of U_{e3} are one of important ingredients to predict $\mathrm{BR}(\mu \to e + \gamma)$.

Super-Kamiokande has almost confirmed the neutrino oscillation in the atmospheric neutrinos, which favors the $\nu_\mu \to \nu_\tau$ process. For the solar neutrinos, the recent data of the Super-Kamiokande and the SNO also favor the neutrino oscillation $\nu_\mu \to \nu_e$ with the large mixing angle MSW solution. These results mean that neutrinos are massive, moreover, the flavor mixings are bi-large in the lepton sector.

If neutrinos are massive and flavor-mixed in the SM, there exists a source of the lepton flavor violation (LFV) through the off-diagonal elements of the neutrino Yukawa coupling matrix. However, due to the smallness of the neutrino masses, the predicted branching ratios for these processes are too small to be observable.

On the other hand, in the supersymmetric framework the situation is quite different. Many authors have already studied the LFV in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos assuming the relevant neutrino mass matrix [1]. In the MSSM with soft breaking terms, there exist lepton flavor violating terms such as off-diagonal elements of slepton mass matrices $(\mathbf{m}^2_{\tilde{\mathrm{L}}})_{ij}$, $(\mathbf{m}^2_{\tilde{\mathrm{e}}_\mathrm{R}})_{ij}$ and trilinear couplings $\mathbf{A}^{\mathrm{e}}_{ij}$. Then the decay rate can be given as

$$\Gamma(\mu \to e + \gamma) \propto \frac{\alpha_2^2}{16\pi^2} \frac{|(\triangle \mathbf{m}^2_{\tilde{L}})_{21}|^2}{m_S^8} \tan^2\beta \ , \tag{1}$$

approximately. Where α_2 is the couplng constant of $SU(2)_W$, m_S is a common SUSY particle mass. Strong bounds on these matrix elements come from requiring branching ratios for LFV processes to be below observed ratios. For the present, the most

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stringent bound comes from the $\mu \to e + \gamma$ decay ($\mathrm{BR}(\mu \to \mathrm{e} + \gamma) < 1.2 \times 10^{-11}$). However, if the LFV occurs at tree level in the soft breaking terms, the branching ratio of this process exceeds the experimental bound too much. Therefore one assumes that the LFV does not occur at tree level in the soft parameters. This is realized by taking the assumption that soft parameters such as $\left(\mathbf{m}_{\mathrm{\tilde{L}}}^2\right)_{ij}$, $(\mathbf{m}^2_{\tilde{\mathrm{e}}_\mathrm{R}})_{ij}$, $\mathbf{A}^{\mathrm{e}}_{ij}$, are proportional to the unit matrix. This assumption follows from the m-SUGRA. However, even though there are no flavor violation at tree level, it is generated by the effect of the renormalization group equations (RGE's) via neutrino Yukawa couplings. Suppose that neutrino masses are produced by the see-saw mechanism, there are the right-handed neutrinos above a scale M_R (10^{12}GeV). Then neutrinos have the Yukawa coupling matrix $\mathbf{Y}_\nu$ with off-diagonal entries in the basis of the diagonal charged-lepton Yukawa couplings. The off-diagonal elements of $\mathbf{Y}_\nu$ drive off-diagonal ones in the $\left(\mathbf{m}_{\mathrm{\tilde{L}}}^2\right)_{ij}$ and $\mathbf{A}^{\mathrm{e}}_{ij}$ matrices through the RGE's running. These terms at the low energy are approximated as

$$(\triangle \mathbf{m}^2_{\tilde{L}})_{21} \simeq -\frac{(6+2a_0^2)m_0^2}{16\pi^2}(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} \ln \frac{M_\mathrm{X}}{M_\mathrm{R}} \ , \tag{2}$$

where M_X is the GUT scale. Therefore disscussing $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ is crucial for estimating $\mathrm{BR}(\mu \to \mathrm{e} + \gamma)$. One can construct $\mathbf{Y}_\nu$ by the recent data of neutrino oscillations. Assuming that oscillations need only accounting for the solar and the atmospheric neutrino data, we take the LMA-MSW solution. Since the data of neutrino oscillations only indicate the differences of the mass square Δm^2_{ij}, the neutrinos have three possible mass spectra. The quasi-degenerate type ($m_{\nu 1} \sim m_{\nu 2} \sim m_{\nu 3}$) gives the neutrino mass spectrum as

$$m_{\nu 1} \equiv m_\nu \ , \quad m_{\nu 2} = m_\nu + \frac{\Delta m^2_\odot}{2m_\nu}, \quad m_{\nu 3} = m_\nu + \frac{\Delta m^2_{\mathrm{atm}}}{2m_\nu} \ , \tag{3}$$

and the inverted ($m_{\nu 1} \sim m_{\nu 2} \gg m_{\nu 3}$) type gives

$$m_{\nu 2} \equiv \sqrt{\Delta m^2_{\mathrm{atm}}} \ , \quad m_{\nu 1} = m_{\nu 2} - \frac{\Delta m^2_\odot}{2m_{\nu 2}} \ , \quad m_{\nu 3} \simeq 0 \ , \tag{4}$$

the hierarchical type ($m_{\nu 1} \ll m_{\nu 2} \ll m_{\nu 3}$) gives

$$m_{\nu 1} \sim 0 \ , \qquad m_{\nu 2} = \sqrt{\Delta m^2_\odot} \ , \qquad m_{\nu 3} = \sqrt{\Delta m^2_{\mathrm{atm}}} \ . \tag{5}$$

Taking the three cases of the neutrino mass spectra, one obtains the following forms, repectively,

$$\begin{aligned}
(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21} &\simeq \frac{M_\mathrm{R}}{v_u^2}\left[U_{\mu 2}U^*_{e2}(m_{\nu 2} - m_{\nu 1}) + U_{\mu 3}U^*_{e3}(m_{\nu 3} - m_{\nu 1})\right] && (6)\\
&\simeq \frac{M_R}{\sqrt{2}v_u^2}\frac{\Delta m^2_{\mathrm{atm}}}{2m_\nu}\left[\frac{1}{\sqrt{2}}U^*_{e2}\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{atm}}} + U^*_{e3}\right] \ , && \text{(Degenerate)}\\
&\simeq \frac{M_R}{\sqrt{2}v_u^2}\sqrt{\Delta m^2_{\mathrm{atm}}}\left[\frac{1}{2\sqrt{2}}U^*_{e2}\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{atm}}} - U^*_{e3}\right] \ , && \text{(Inverted)}\\
&\simeq \frac{M_R}{\sqrt{2}v_u^2}\sqrt{\Delta m^2_{\mathrm{atm}}}\left[\frac{1}{\sqrt{2}}U^*_{e2}\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{atm}}}} + U^*_{e3}\right] \ , && \text{(Hierar.)}
\end{aligned}$$

where we take the maximal mixing for the atmospheric neutrinos. Since $U_{e2} \simeq 1/\sqrt{2}$ for the bi-large mixing matrix, the first terms in the square brackets of the right hand

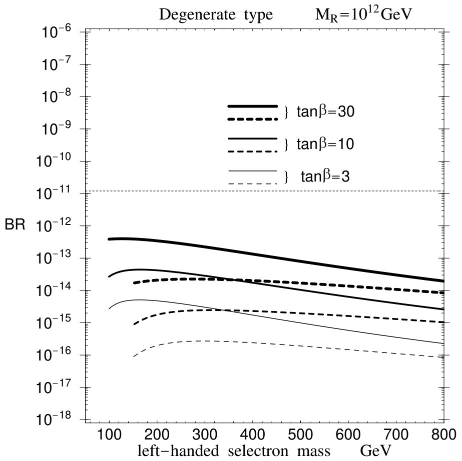

Figure 1. Predicted branching ratio BR($\mu \to e + \gamma$) versus the left-handed selectron mass for $\tan\beta = 3,\ 10,\ 30$ in the case of the degenerate neutrino masses. Here $M_R = 10^{12}$GeV and $U_{e3} = 0.2$ are taken. The solid curves correspond to $M_2 = 150$GeV and the dashed ones to $M_2 = 300$GeV.

sides of eqs.(6) can be estimated by putting the experimental data. For the case of the degenerate neutrino masses, $(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{21}$ depends on the unknown neutrino mass scale m_ν. As one takes the smaller m_ν, one predicts the larger branching ratio. In our calculation, we take $m_\nu = 0.3$eV, which is close to the upper bound from the neutrinoless double beta decay experiment, and also leads to the smallest branching ratio. It is easy to see the fact that the second terms in eqs.(6) are dominant as far as U_{e3} 0.01(degenerate), 0.01(inverted) and 0.07(hierarchical), respectively. The comparison between the cases of the inverted masses and the normal hierarchical masses depends on the magnitude and the phase of U_{e3}.

We present numerical results in the case of the degenerate neutrino masses assuming $\mathbf{M_R} = M_R\mathbf{I}$. In Fig.1, the branching ratio of $\mu \to e + \gamma$ is given versus the left-handed selectron mass $m_{\tilde{e}_L}$ for each $\tan\beta = 3,\ 10,\ 30$ and a fixed wino mass M_2 at the electroweak scale. Our predictions depend on M_R strongly, because the magnitude of the neutrino Yukawa coupling is determined by M_R. If M_R increases to 10^{14}GeV, the branching ratio becomes 10^4 times larger since it is proportional to M_R^2 approximately. In general, we expect the relation BR(degenerate)$\ll$BR(inverted)$\cong$ BR(hierarchical)[2].

The branching ratio of $\mu \to e + \gamma$ will be improved to the lebel 10^{-14} in the PSI. Therefore, future experiments can probe the framework for the neutrino masses.

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