Muon acceleration with a very fast ramping synchrotron for a neutrino factory

D J Summers [1], J S Berg [2], A A Garren [3] and R B Palmer [2] §

[1] Dept. of Physics, University of Mississippi–Oxford, University, MS 38677, USA
[2] Brookhaven National Laboratory, Upton, NY 11973, USA
[3] Dept. of Physics, University of California, Los Angeles, CA 90095, USA

Abstract. A 4600 Hz fast ramping synchrotron is explored as an economical way of accelerating muons from 4 to 20 GeV/c for a neutrino factory. Eddy current losses are minimized by the low machine duty cycle plus thin grain oriented silicon steel laminations and thin copper wires. Combined function magnets with high gradients alternating within single magnets form the lattice we describe. Muon survival is 83%.

1. Introduction

Traditionally ramping synchrotrons have provided economical particle acceleration. Here we explore a very fast ramping muon synchrotron for a neutrino factory [1]. The accelerated muons would be stored in a racetrack to produce neutrino beams as they decay ($\mu^- \rightarrow e^- \,\overline{\nu}_e\, \nu_\mu$ or $\mu^+ \rightarrow e^+\, \nu_e\, \overline{\nu}_\mu$). Neutrino oscillations [2] have been observed at experiments such as Homestake [3], Super–Kamiokande [4], and SNO [5]. Further exploration using a neutrino factory could reveal effects such as CP violation in the lepton sector which could explain the matter–antimatter asymmetry of the universe.

This synchrotron must accelerate muons from 4 to 20 GeV/c with moderate decay loss. Because synchrotron radiation goes as m^4, muons radiate two billion times ($(105.7/.511)^4$) less power than electrons for any given ring diameter and lepton energy. Magnet eddy current losses are minimized by the low duty cycle of the machine plus thin iron laminations and copper conductors. Grain oriented silicon steel is used to provide a high magnetic field with a high μ to minimize magnetic energy stored in the return yoke. The magnetic energy stored in the gap is minimized by reducing its size. Cool muons [6] with low beam emittance allow this. Stored energy goes as $B^2/2\mu$. The voltage required to drive a magnet is equal to $-L\, di/dt$. Very high voltage is expensive. di/dt must be large because of the 2 μsec muon lifetime, so the main option for lowering voltage is to shrink the volume of stored energy to reduce the inductance, L.

Acceleration to 4 GeV might feature fixed field dogbone arcs [7] to minimize muon decay loss. Fast ramping synchrotrons [7, 8] might also accelerate muons to higher energies for a $\mu^+ \mu^-$ collider [9].

§ To whom correspondence should be addressed (summers@relativity.phy.olemiss.edu)

2. Lattices

As a first step, we form arcs with sequences of combined function cells formed within continuous long magnets, whose poles are alternately shaped to give focusing gradients of each sign. An example of such a cell has been simulated using SYNCH [10]. The example has gradients that alternate from positive 20 T/m gradient (2.24 m long), to zero gradient (.4 m long) to negative 20 T/m gradient (2.24 m) to zero gradient (0.4 m), etc. The relatively short zero gradient section is included to approximate a real smooth change in the gradients. Details are given in Table 1.

Table 1. Combined function magnet cell parameters. Five cells make up an arc and 18 arcs form the ring.

Cell length	m	5.28
Combined Dipole length	m	2.24
Combined Dipole $B_{central}$	T	0.9
Combined Dipole Gradient	T/m	20.2
Pure Dipole Length	m	0.4
Pure Dipole B	T	1.8
Momentum	GeV/c	20
Phase advance/cell	deg	72
beta max	m	8.1
Dispersion max	m	0.392
Normalized Trans. Acceptance	π mm rad	4

It is proposed to use 5 such arc cells (possibly all in one magnet) to form an arc segment. These segments are alternated with straight sections containing RF. The phase advance through one arc segment is 5 x 72 = 360 degrees. This being so, dispersion suppression between straights and arcs can be omitted. With no dispersion in the straight sections, the dispersion performs one full oscillation in each arc segment, returning to zero for the next straight as shown in Fig. 1. There will be 18 such arc segments and 18 straight sections, forming the 18 superperiods in the ring.

Straight sections (22 m) without dispersion are used for superconducting RF, and, in two longer straights (44 m), the injection and extraction. To assure sufficiently low magnetic fields at the cavities, relatively long field free regions are desirable. A straight consisting of two half cells would allow a central gap of 10 m between quadrupoles, and two smaller gaps at the ends. Details are given in Table 2. Matching between the arcs and straights is not yet designed. The total circumference of the ring including combined functions magnets and straight sections adds up to 917 m ($18 \times 26.5 + 16 \times 22 + 2 \times 44$).

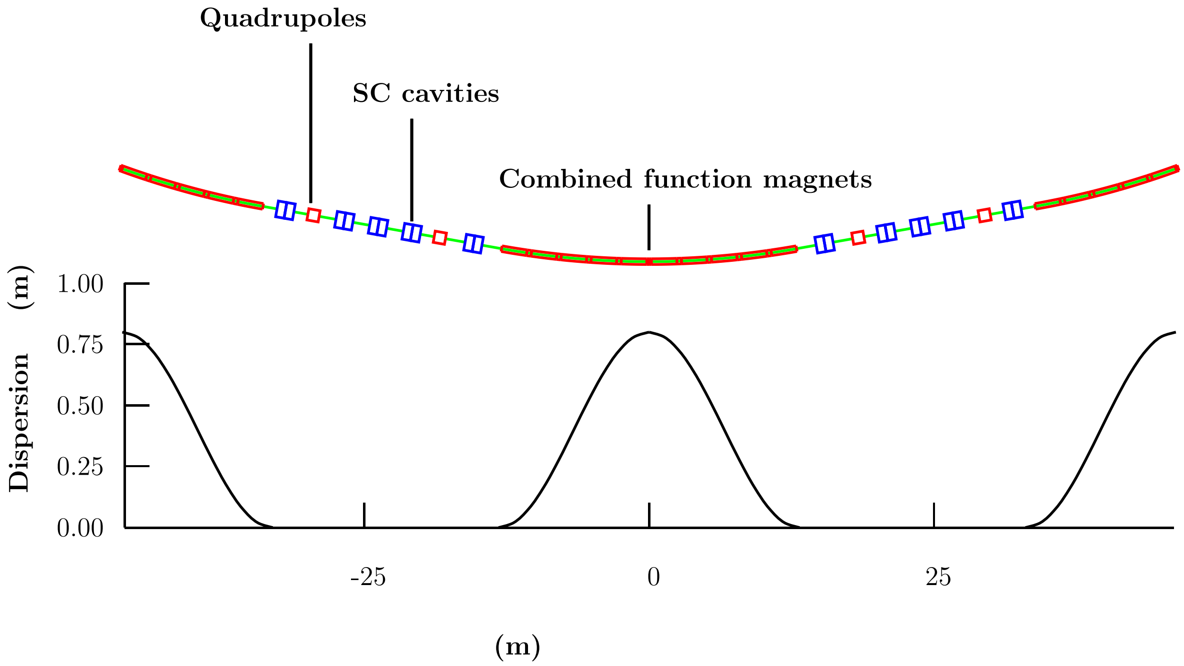

Figure 1. Combined function magnets bend the muons in the arcs. Superconducting RF cavities accelerate muons in the straight sections. Two quadrupoles per straight section provide focusing. The straight sections are dispersion free.

Table 2. Straight section lattice parameters. There are two quadrapoles per straight.

ϕ	$L_{cell}/2$	L_{quad}	dB/dx	a	β_{max}	σ_{max}	B_{pole}	U_{mag}/quad
77^0	11 m	1 m	7.54 T/m	5.8 cm	36.6 m	.0195 m	0.44 T	$\approx$ 3000 J

3. Superconducting RF

The RF must be distributed around the ring to avoid large differences between the beam momentum (which increases in steps at each RF section) and the arc magnetic field (which is increasing continuously). RF parameters are shown in Table 3.

The amount of RF used is a tradeoff between cost and muon survival. Survival is somewhat insensitive to the fraction of stored energy the beam removes from the RF cavities, because the voltage drop is balanced by time dilation. Here, only 8.2% of RF energy is used. One could, in the spirit of Oliver Twist, ask for more. Using more of the RF energy is particularly appealing with a smaller ring. Very cold muons require less focusing and allow a smaller ring. If the muons take a few extra turns at the end to accelerate, only a few extra will be lost. Also, extra acceleration time at the end will translate into less voltage needed to ramp magnets.

Table 3. Superconducting RF parameters.

Frequency	201	MHz
Gap	.75	m
Gradient	15	MV/m
Stored Energy	900	Joules
Muons per train	5×10^{12}	
Orbits (4 to 20 GeV/c)	12	
No. of RF Cavities	160	
RF Total	1800	MV
ΔU_{beam}	110	Joules
Energy Loading	.082	
Voltage Drop	.041	
Muon Acceleration Time	37	μsec
Muon Survival	.83	

4. Combined Function Magnets

The muons accelerate from 4 to 20 GeV. If they are extracted at 95% of full field they will be injected at 19% of full field. For acceleration with a plain sine wave, injection occurs at 11^0 and extraction occurs at 72^0. So the phase must change by 61^0 in 37 μsec. Thus the sine wave goes through 360^0 in 218 μsec, which equals a frequency of 4600 Hz.

Estimate the energy stored in each 26.5 m long combined function magnet. The gap is about .14 m wide and has an average height of .06 m. Assume an average field of 1.1 Tesla. The permeability constant, μ_0, is $4\pi \times 10^{-7}$. $W = B^2/2\mu_0[\text{Volume}] = 110\,000$ Joules. Next given one turn, an LC circuit capacitor, and a 4600 Hz frequency; estimate current, voltage, inductance, and capacitance.

$$B = \frac{\mu_0 \, NI}{h} \;\rightarrow\; I = \frac{Bh}{\mu_0 \, N} = 52\,\text{kA}; \qquad W = .5\, L\, I^2 \;\rightarrow\; L = 2\, W/I^2 = 80\,\mu\text{H} \tag{1}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \;\rightarrow\; C = \frac{1}{L\,(2\pi f)^2} = 15\,\mu\text{F}; \;\; W = .5\, C\, V^2 \;\rightarrow\; V = \sqrt{2W/C} = 120\,\text{kV} \tag{2}$$

Separate coils might be put around each return yoke to halve the voltage as illustrated in Fig. 2. The stack of SCRs driving each coil might be center tapped to halve the voltage again. Four equally spaced coil slots could be put in each side yoke to cut the voltage by five, while leaving the pole faces continuous. 6 kV is easier to insulate than 120 kV. It may be useful to shield or chamfer [12] magnet ends to avoid large eddy currents where the field lines typically do not follow laminations. A DC offset power supply [13] could be useful. Neutrino horn power supplies look promising [11].

Grain oriented silicon steel is chosen for the return yoke due to its high permeability at high field at noted in Table 4. This minimizes the energy stored in the yoke which goes as $B^2/2\mu$. The skin depth [14] of a 100 micron thick lamination is given by

$$\text{Skin Depth} = \delta = \sqrt{\rho \,/\, \pi\, f\, \mu} = \sqrt{47 \times 10^{-8} \,/\, \pi\, 4600\, 1000\, \mu_0} = 160\mu\text{m} \quad (3)$$

Take $\mu = 1000\mu_0$ as a limit on magnetic saturation and hence energy storage in the yoke. Next estimate the fraction of the inductance of the yoke that remains after eddy currents shield the laminations [15]. The lamination thickness is t.

$$\mathrm{L}/\mathrm{L}_0 = (\delta/t)\,(\sinh(t/\delta) + \sin(t/\delta)) \,/\, (\cosh(t/\delta) + \cos(t/\delta)) = 0.995 \quad (4)$$

So it appears that magnetic fields can penetrate 100 micron thick laminations at 4600 Hz. If allowable, thicker 175 micron thick laminations would be half as costly and can achieve a somewhat higher packing fraction.

Table 4. Approximate permeabilities of soft magnetic materials. The permeability is $B/\mu_0 H$. Grain oriented silicon steel has a much higher permeability parallel ($\|$) to its rolling direction than in the perpendicular ($\perp$) direction [16, 17].

Material	1.0 Tesla	1.5 Tesla	1.8 Tesla
1008 Steel	3000	2000	200
Grain Oriented ($\|$)	40000	30000	3000
Grain Oriented ($\perp$)	4000	1000	
NKK Super E-Core	20000	300	50
Metglas 2605SA1	300000	10000	1

Calculate the resistive energy loss in the copper coils, which over time is equal to 1/2 the loss at the maximum current of 52 000 amps. The 1/2 comes from the integral of cosine squared. Table 5 gives the resistivity of copper. Four 5 cm square copper conductors each 5300 cm long have a total power dissipation of 130 kilowatts/magnet. Eighteen magnets give a total loss of 2340 kilowatts. But the neutrino factory runs at 30 Hz. Thirty half cycles of 109 μsec per second gives a duty factor of 300 and a total I^2R loss of 8000 watts. Muons are orbited in opposite directions on alternate cycles. If this proves too cumbersome, the duty cycle factor could be lowered to 150.

$$R = \frac{5300\ (1.8\,\mu\Omega\text{-cm})}{(4)\,(5^2)} = 95\,\mu\Omega; \qquad P = I^2 R \int_0^{2\pi} \cos^2(\theta)\, d\theta = 130\,000 \text{ w/magnet} \quad (5)$$

Find the skin depth of copper at 4600 Hz to see if .25 mm (30 gauge) wire is useable.

$$\text{Skin Depth} = \delta = \sqrt{\rho \,/\, \pi\, f\, \mu_0} = \sqrt{1.8 \times 10^{-8} \,/\, \pi\, 4600\, \mu_0} = 0.97\text{mm} \quad (6)$$

Now calculate the dissipation due to eddy currents in this .25 mm wide conductor, which will consist of transposed strands to reduce this loss [22, 12]. To get an idea, take the maximum B-field during a cycle to be that generated by a 0.025m radius conductor carrying 26000 amps. The eddy current loss in a rectangular conductor made of transposed square wires .25 mm wide (sometimes called Litz wire [23])

Table 5. Resistivity, magnetic saturation, and coercivity of conductors, cooling tubes, and soft magnetic materials. The magnetic materials include 50, 100 [18], and 175 μm [16, 19] thick grain oriented silicon steel, NKK Super E-Core [20], and Metglas [21].

Material	Composition	ρ	B_{Max}	H_c	Thicknesses
		$\mu\Omega$-cm	Tesla	Oe	μm
Copper	Cu	1.8	—	—	—
Stainless 316L	70 Fe, 18 Cr, 10 Ni, 2 Mo, .03 C	74	—	—	—
Titanium 6Al–4V	90 Ti, 6 Al, 4 V	171	—	—	—
1008 Steel	99 Fe, .08 C	12	2.09	0.8	—
Grain Oriented	3 Si, 97 Fe	47	1.95	.1	50, 100, 175
NKK Super E-Core	6.5 Si, 93.5 Fe	82	1.8	.2	50, 100
Metglas 2605SA1	81 Fe, 14 B, 3 Si, 2 C	135	1.6	.03	30

with a perpendicular magnetic field is as follows. The width of the wire is w and $B = \mu_0\, I/2\pi r = 0.2$ Tesla.

$$P = [\text{Volume}]\frac{(2\pi\, f\, B\, w)^2}{24\rho} = [4\ .05^2\ 53]\ \frac{(2\pi\ 4600\ .2\ .00025)^2}{(24)\,1.8\times 10^{-8}} = 2800 \text{ kilowatts} \qquad (7)$$

Multiply by 18 magnets and divide by a duty factor of 300 to get an eddy current loss in the copper of 170 kilowatts. Stainless steel water cooling tubes will dissipate a similar amount of power [7]. Alloy titanium cooling tubes would dissipate less.

Do the eddy current losses [22] in the 100 micron thick iron laminations. Take a quarter meter square area, a 26.5 meter length, and an average field of 1.1 Tesla.

$$\text{P} = [\text{Vol}]\frac{(2\pi\, f\, B\, t)^2}{24\rho} = [(26.5)\ (.5^2)]\ \frac{(2\pi\ 2600\ 1.1\ .0001)^2}{(24)\,47\times 10^{-8}} = 5900 \text{ kw} \qquad (8)$$

Multiply by 18 magnets and divide by a duty factor of 300 to get an eddy current loss in the iron laminations of 350 kilowatts or 700 watts/m of magnet. So the iron will need some cooling. The ring only ramps 30 time per second, so the $\int \mathbf{H}\cdot d\,\mathbf{B}$ hysteresis losses will be low, even more so because of the low coercive force, H_c, of grain oriented silicon steel.

5. Conclusions

The low duty cycle of the neutrino factory leads to reasonable eddy current losses in a 4600 Hz ring. Muon survival is 83%. The high permeability of grain oriented silicon steel permits high fields with little energy stored in the yoke. Gradients are switched within dipoles to minimize eddy current losses in ends. Time dilation allows extra orbits with little muon decay at the end of a cooling cycle. This allows one to use more of the stored RF energy. Much of the magnetic field in our lattice is used for focusing rather than bending the muon beam. More muon cooling would lead to less focusing, more bending, and an even smaller ring.

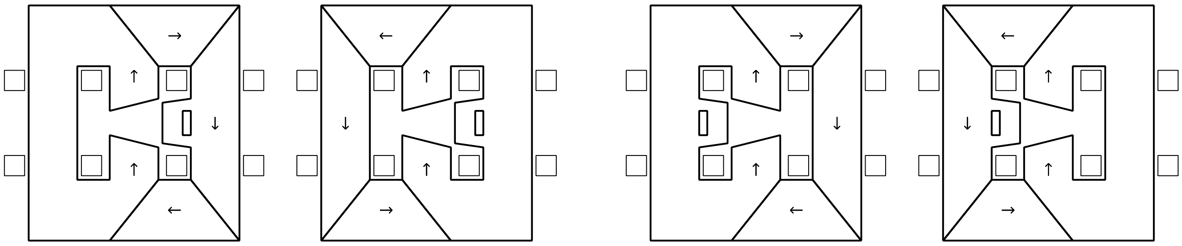

Figure 2. Alternating gradient magnet laminations with grain oriented silicon steel. The arrows show both the magnetic field direction and the grain direction of the steel. If needed, four pieces might be used per layer as shown to fully exploit the high permeability and low hysteresis in the grain direction [12, 22, 24, 25] noted in Table 4. The "C" pieces provide rigidity. Simpler solutions with one or two pieces per layer are under investigation. The horizontal tab increases the gradient by lowering the field to roughly zero on the wide side of the gap. The four coils (□) are wired in parallel.

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