Beam Dynamics of Muon Acceleration – Beyond Study II

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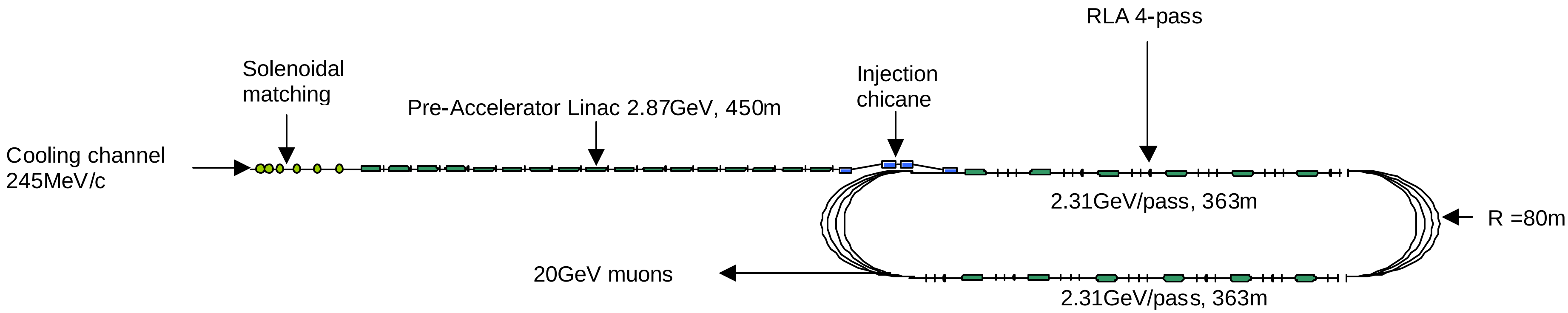
- RLA scheme based on 200MHz superconducting RF
  - Pre-accelerator, Linacs & 'All Arcs' – complete lattice design
  - Optimized beam transport in large acceptance RLA – tracking studies
  - Emittance preservation scheme – chromatic corrections with sextupoles
- Comparison between the RLA and FFAG scenarios – beam dynamics
  - FFAG lattice design challenges (D. Trbojevic et al. Shelter Island, May '02)
  - Longitudinal bunch compression – RLA vs FFAG

20GeV Muon Accelerator Complex – Study II



- Large transverse and longitudinal acceptances drive the design to low RF frequency (200 MHz SRF, 16 MV/m gradient)
  - RF power delivered to the cavities over an extended time – RF source peak power reduced (2-cell cavities, 1MW klystron per cell)
  - Cavity design not limited by a requirement of low shunt impedance – significantly larger apertures.
- Complete lattice design for: solenoidal matching section, linear pre-accelerator (sol.), injection chicane, linacs (tripl.), spreaders/recombiners (hor.) and 7 arcs

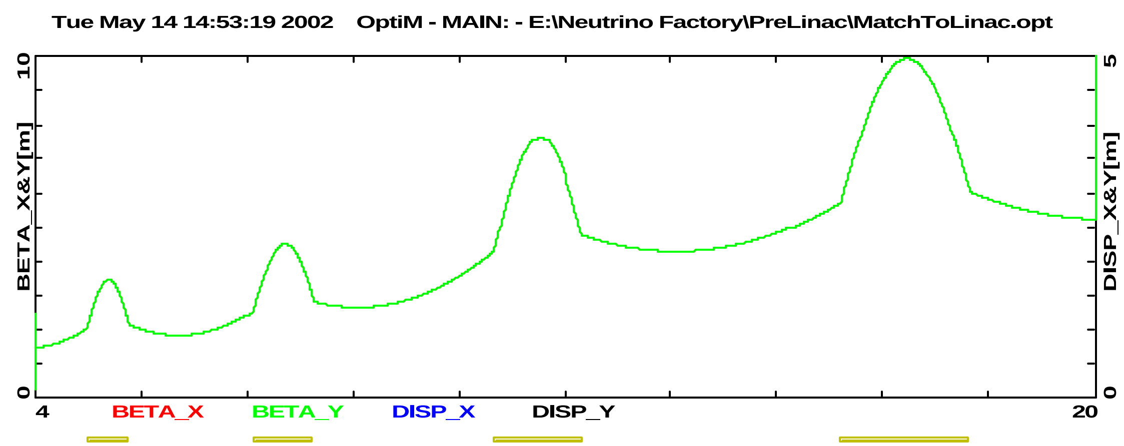
Study II Machine Parameters

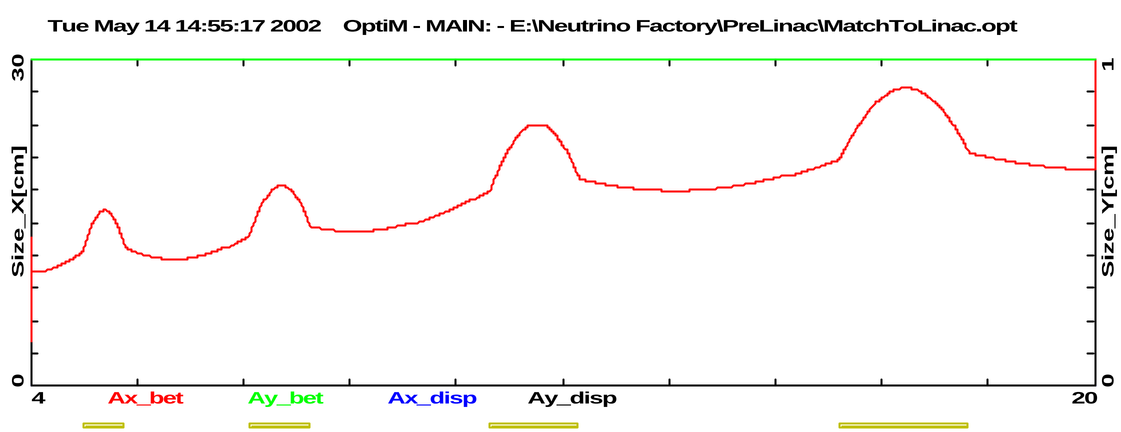
Injection momentum/Kinetic energy	MeV	245/161
Final energy	GeV	20
Initial normalized acceptance rms normalized emittance	mm·rad mm·rad	15 2.4
Initial longitudinal acceptance, $\Delta pL_b/m_\mu$ momentum spread, $\Delta p/p$ bunch length, L_b rms energy spread rms bunch length	mm mm mm	170 ±0.21 ±407 0.084 163
Number of bunches per pulse		67
Number of particles per per pulse		$3 \cdot 10^{12}$
Bunching frequency	MHz	201.25
Average repetition rate	Hz	15
Time structure of muon beam		6 pulses at 50 Hz with 2.5 Hz repetition rate
Average beam power	kW	150

Beam transport choices

- Solenoidal matching section after the cooling channel (beta~1m to ~5m)
- Solenoidal focusing (SC) in the Linear Pre-accelerator
  - allows one to accommodate very large beam emittance
- Off-crest adiabatic bunch compression in Linear Pre-accelerator
  - prevents head-to-tail 'sag' in acceleration (final: $\Delta p/p=\pm 7.5\%$ or $\Delta\phi = \pm 23$)
- SC dipoles and quads (triplets) in RLA (2 Tesla dipoles/1 Tesla quads)
- Single dipole (horizontal) separation of multi-pass beams in RLA
  - No need to maintain achromatic Spreaders/Recombiners
  - Compact Spreaders/Recombiners – minimized emittance dilution

Solenoidal matching section – constant focal length

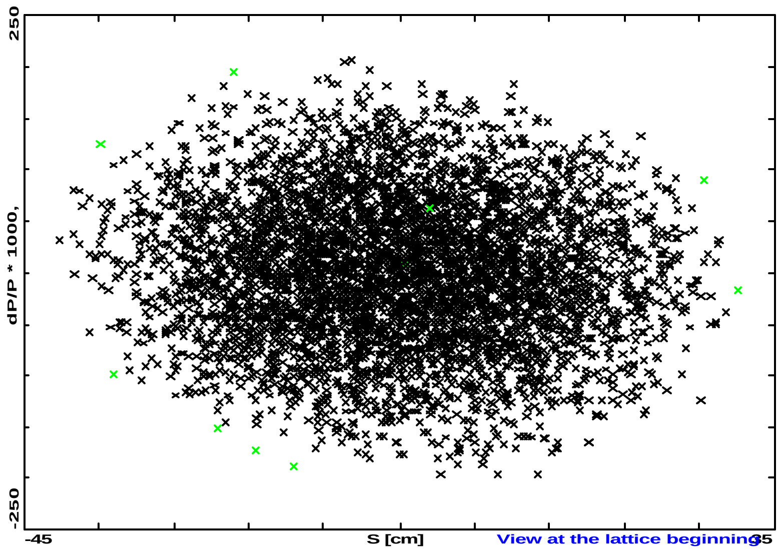


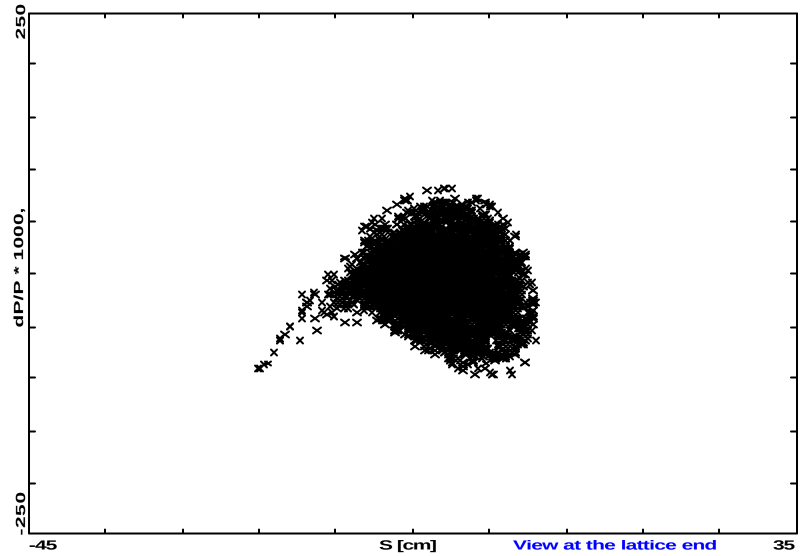


L/A[cm]	B[kG]
64.0973	35.8649
92.9411	24.7344
134.765	17.0582
195.409	11.7643

Beta-functions & beam envelopes (2.5σ) at 245MeV/c

Linear Preaccelerator - adaiabatic bunch compression





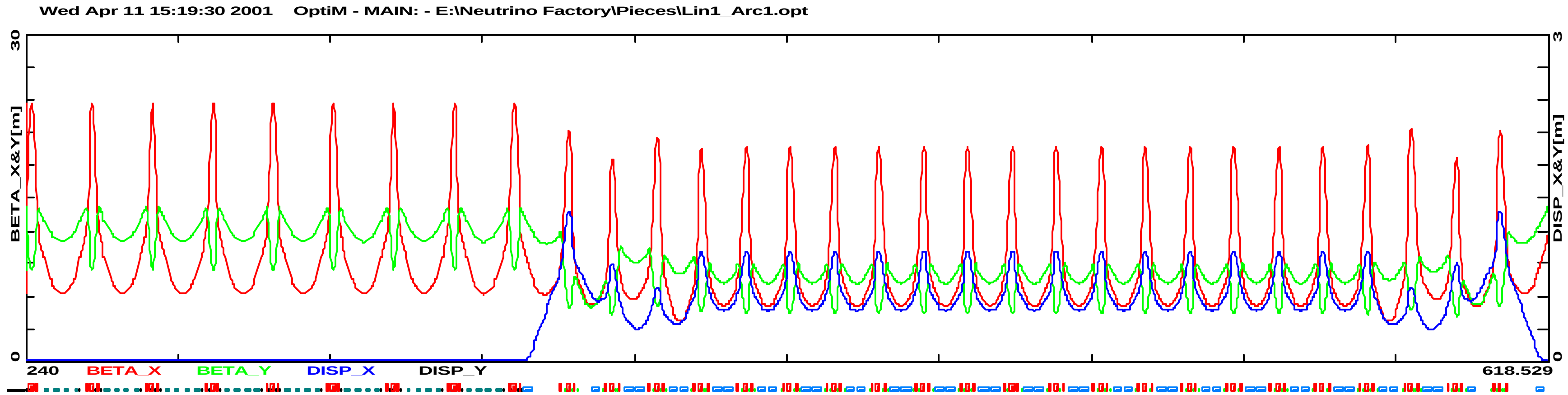
Longitudinal phase space at the beginning and at the end of Linear Pre-accelerator

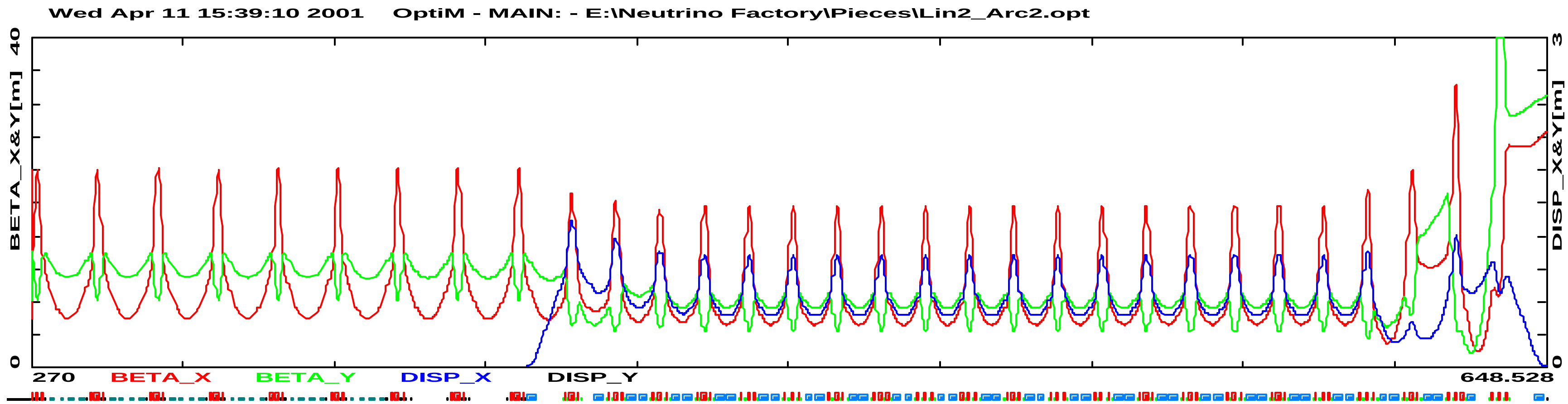
Acceleration parameters for RLA (Arc M_{56} = 1.4 m)

	Kinetic energy [GeV]	Gang phase [deg]	Total energy spread, $2\Delta p/p$ [%]	Horizontal acceptance, [mm mrad]	Vertical acceptance, [mm mrad]
Entrance	2.480	-5	15.0	669	638
Arc 1	4.756	-23	11.3	384	350
Arc 2	6.884	-23	8.9	292	253
Arc 3	9.017	-23	6.7	244	202
Arc 4	11.150	-23	5.8	216	171
Arc 5	13.284	-20	5.0	198	150
Arc 6	15.462	-16	4.4	187	134
Arc 7	17.690	-5	3.4	178	122
Exit	20.000		3.2	157	108

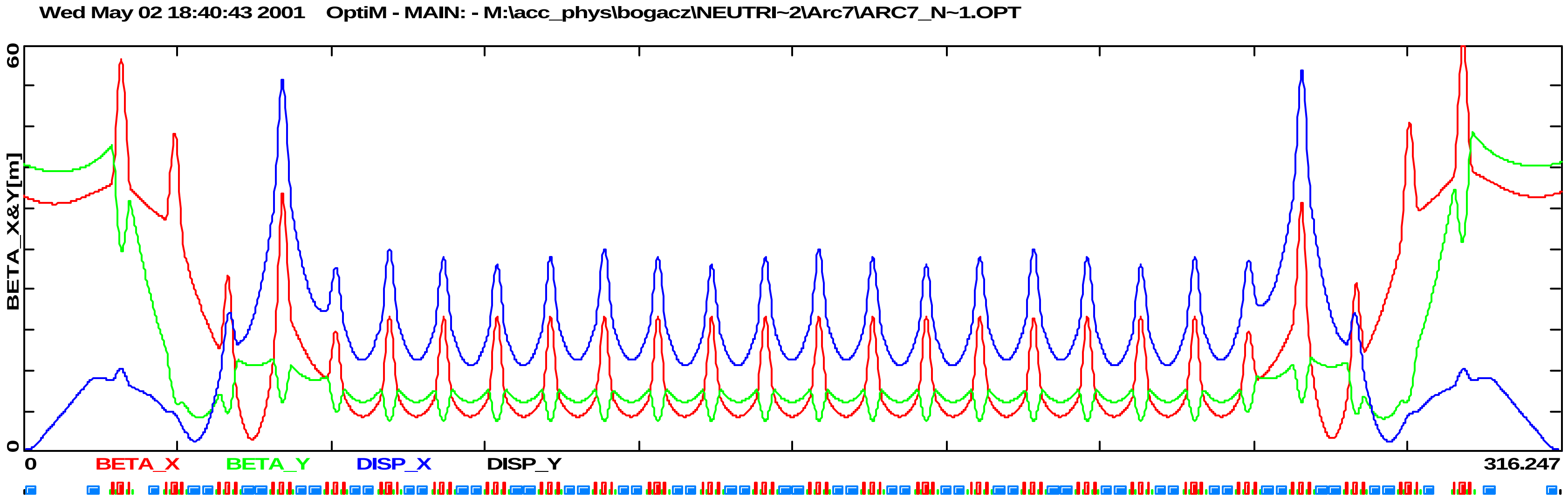
Total voltage per linac = 2.3347 GV at 200 MHz
horizontal/vertical emittance dilution 9%/4% per arc

❖Arc 1 and 2 optics – smooth transition in Spreaders/Recombiners



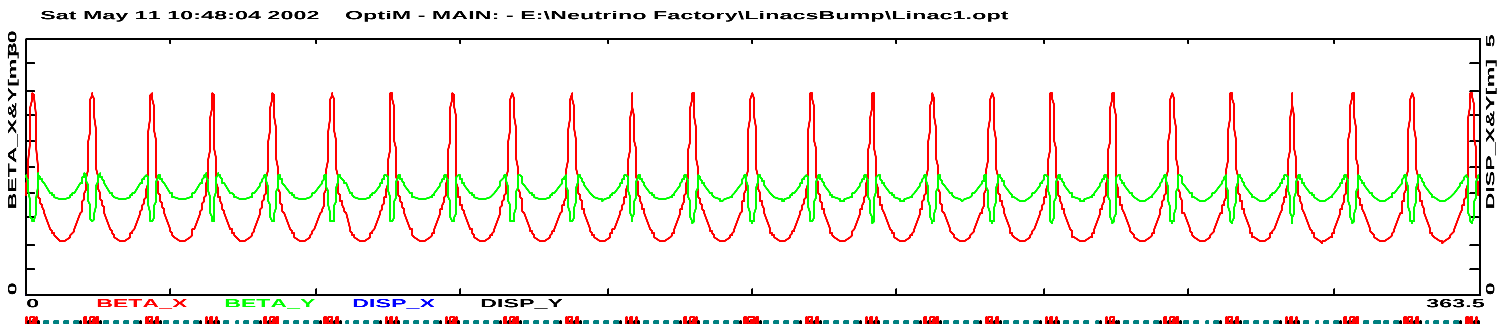


❖Arc 7 Optics - beta-functions and the horizontal dispersion matched to both adjacent linacs, much larger difference (compared to Arc 1)

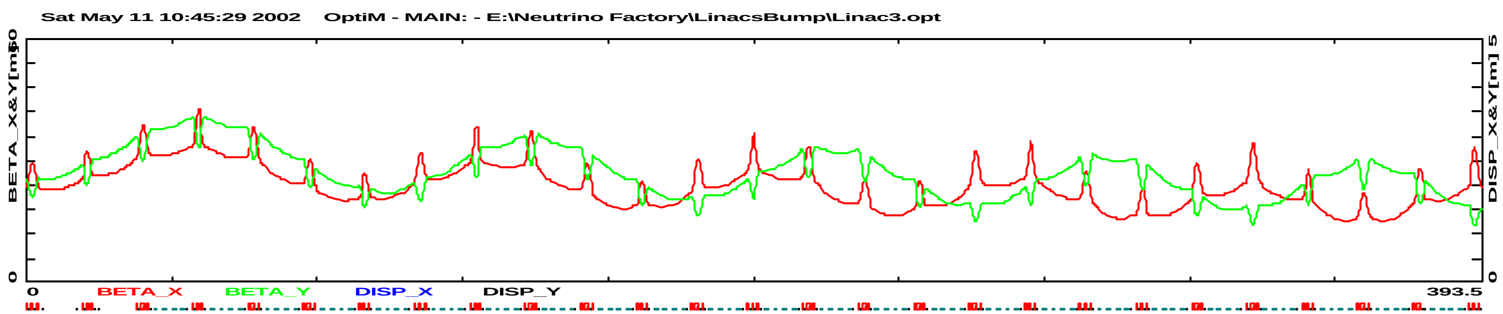


❖Optimized linac optics for multi pass beams – smooth transition Arc-linac

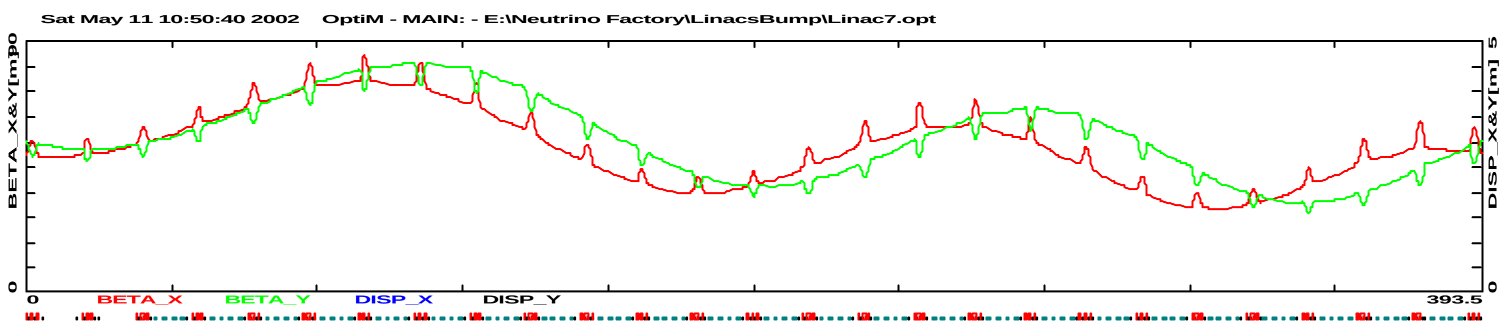
Pass 1



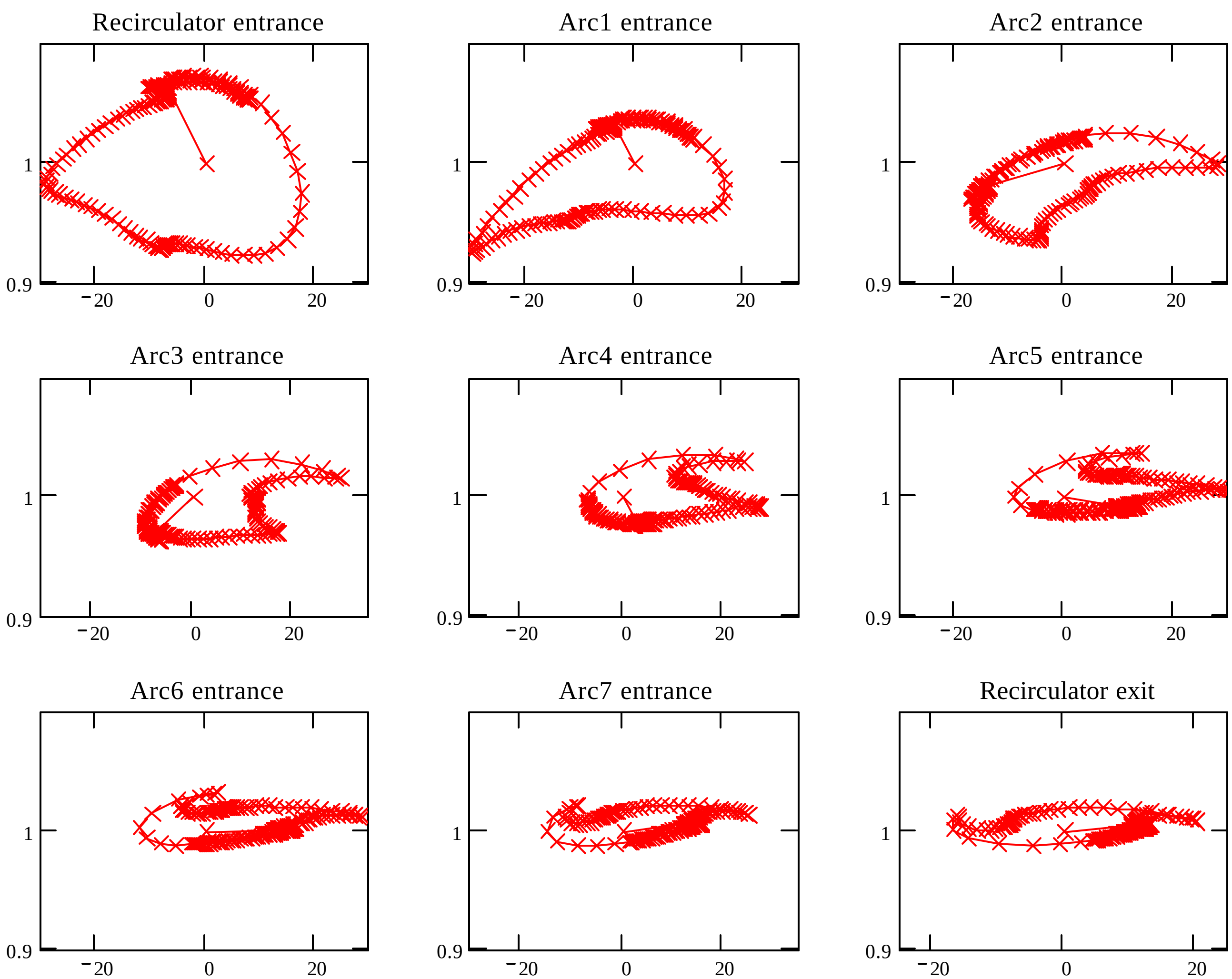
Pass 2



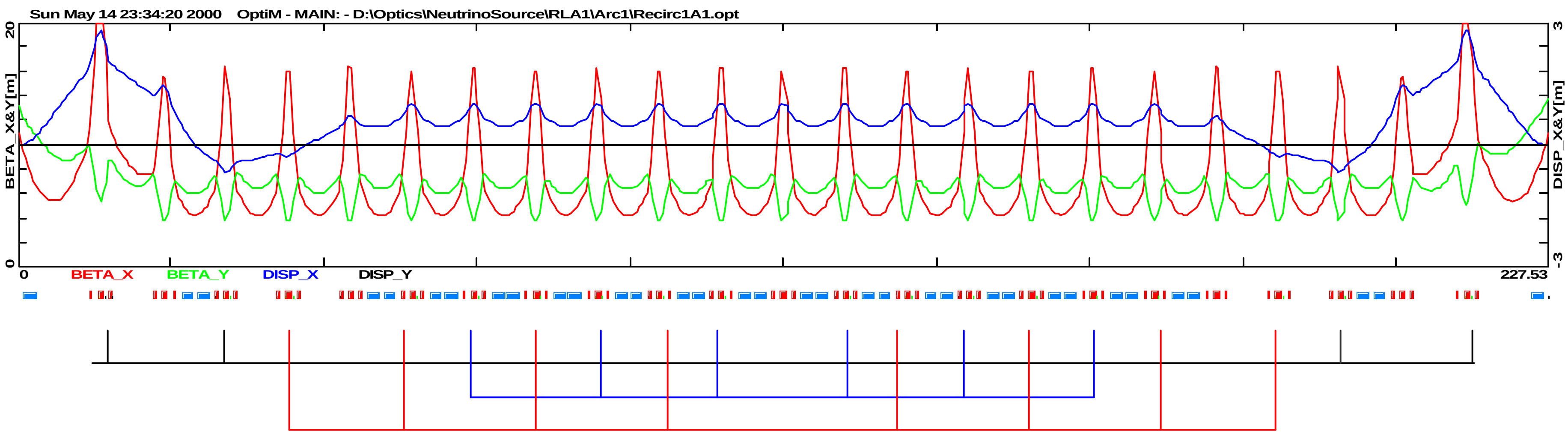
Pass 4



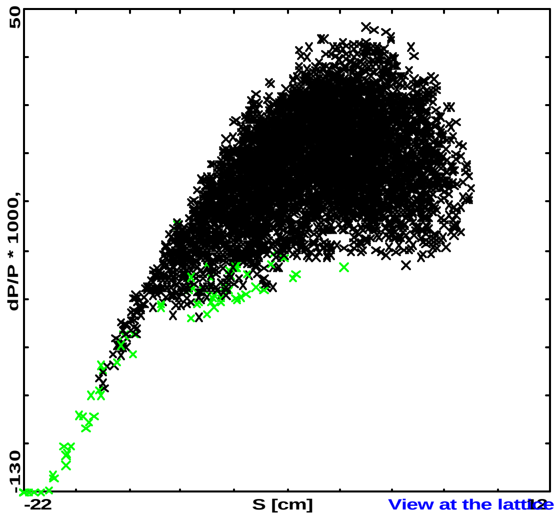
❖Longitudinal dynamics in the RLA, $M_{56} = 1.4$ m

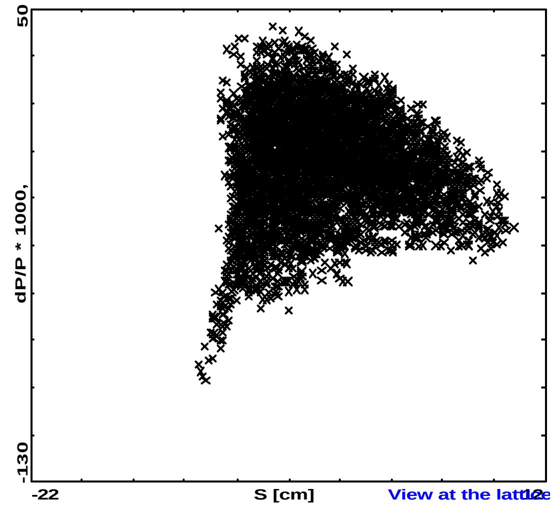


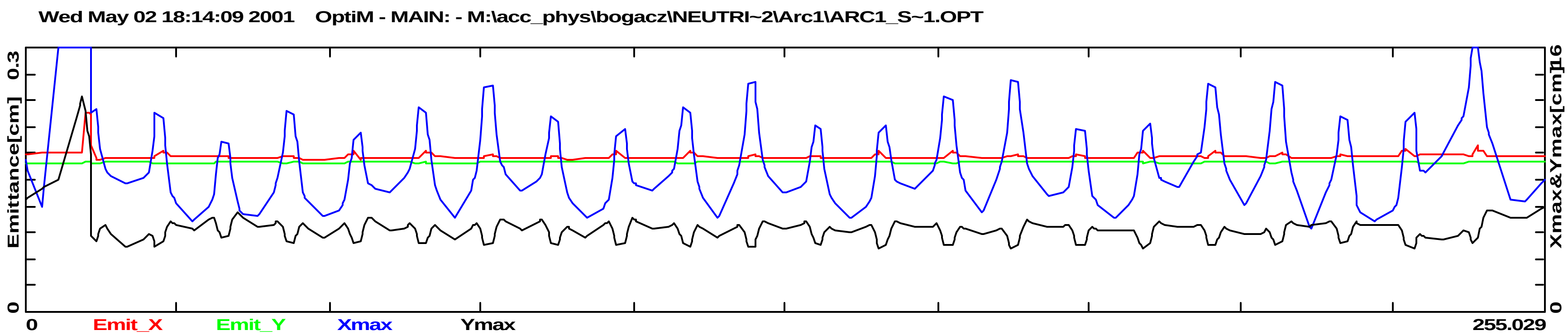
❖Arc optics - Scheme of chromatic correction with 3 families of sextupoles

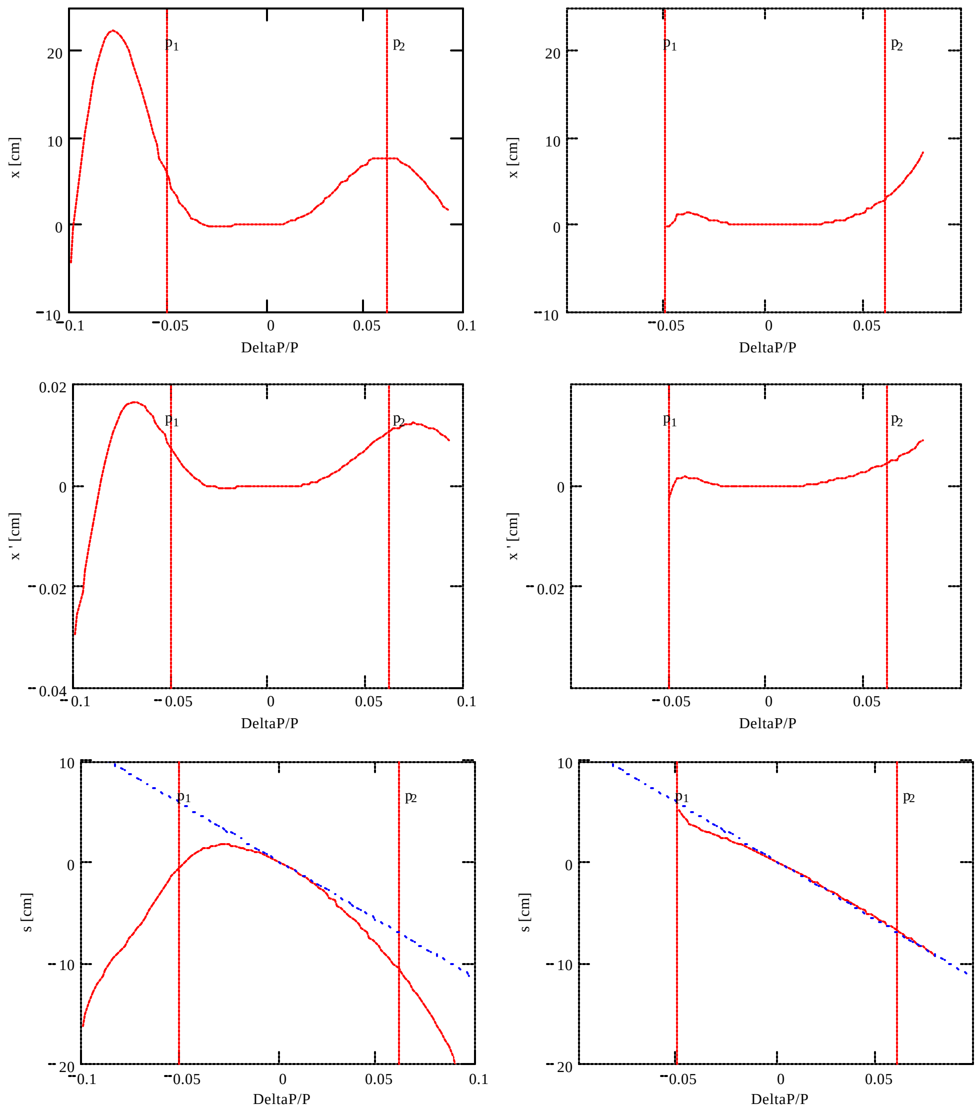


❖Emittance preservation via chromatic corrections – no emittance dilution









Chromaticity correction

Non-linearity for

- Dispersion
- Dispersion-prime
- M_{56}

For the cases of

- no sextupole corrections
- 3 sextupole families

Muon Acceleration in FFAG

- FFAG ring (10-20 GeV) – **'Shelter Island Lattice'** (D. Trbojevic et al.)
  - lattice without opposing bends – compact ring, R~50 m
  - full acceleration in ~10 turns – muon lifetime
  - small dispersion D~6 cm
    - $\Delta x < D\ \Delta p/p = 6\ \text{cm} \times (\pm ½) = \pm 3\ \text{cm}$
    - magnet aperture: $10 \times \pm 3$ cm = ± 30 cm (60 cm vs 4-pass RLA, 15 cm arc apertures)
  - large $M_{56} = \alpha\ C = \Delta C/\Delta p/p \sim$ 90 cm
    - $B(r) = B_0\ (r/r_0)^k$, large k~300, $\alpha \sim 1/k$ $\Rightarrow$ $C \sim 300$ m
  - FODO lattice (100 cells) with distributed RF (every 3-rd cell)
    - Small betas ~15 cm

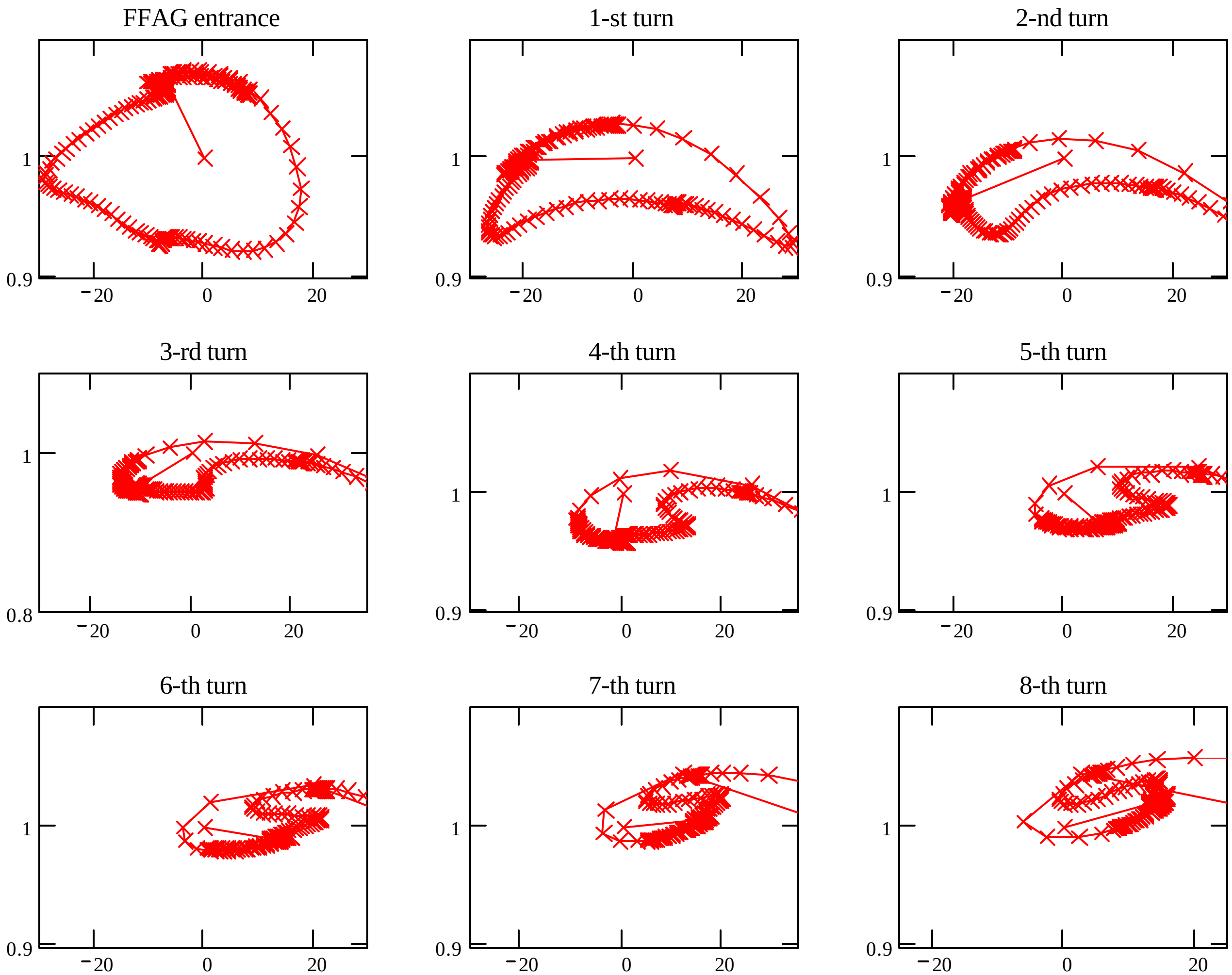
❖Longitudinal dynamics in FFAG with 200MHz RF

Acceleration in FFAG

$$\mathrm{Transport}(r, Np, \Delta E, \phi cav, M56) := \left| \begin{array}{l} \text{for } j \in 0..Np-1 \\ \quad \left| \begin{array}{l} R_{j,0} \leftarrow \phi cav + r_{j,0} - r_{Np,0} - 2\pi \cdot \frac{M56}{\lambda} \cdot \left[\sqrt{\frac{(r_{j,1})^2 - m_\mu^{\,2}}{(r_{Np,1})^2 - m_\mu^{\,2}}} - 1 \right] \\ R_{j,1} \leftarrow r_{j,1} + \Delta E \cdot \cos(R_{j,0}) \\ R_{j,2} \leftarrow \phi cav + r_{j,2} - r_{Np,0} - 2\pi \cdot \frac{M56}{\lambda} \cdot \left[\sqrt{\frac{(r_{j,3})^2 - m_\mu^{\,2}}{(r_{Np,3})^2 - m_\mu^{\,2}}} - 1 \right] \\ R_{j,3} \leftarrow r_{j,3} + \Delta E \cdot \cos(R_{j,2}) - \Delta E \cdot d\Delta E \cdot \cos(R_{j,2} - \phi cav) \end{array} \right. \\ R_{Np,0} \leftarrow \phi cav \\ R_{Np,1} \leftarrow r_{Np,1} + \Delta E \cdot \cos(\phi cav) \\ R_{Np,2} \leftarrow \phi cav \\ R_{Np,3} \leftarrow r_{Np,1} + \Delta E \cdot \cos(\phi cav) \\ R \end{array} \right.$$

- one full synchrotron period per cycle – head-to-tail ‘sag’ in acceleration
- significant head-to-tail ‘sag’ in acceleration (inital: $\Delta p/p = \pm 7.5\%$ or $\Delta\phi = \pm 23$ deg.)

❖FFAG – turn-by-turn longitudinal bunch compression, M_{56} = 90 cm



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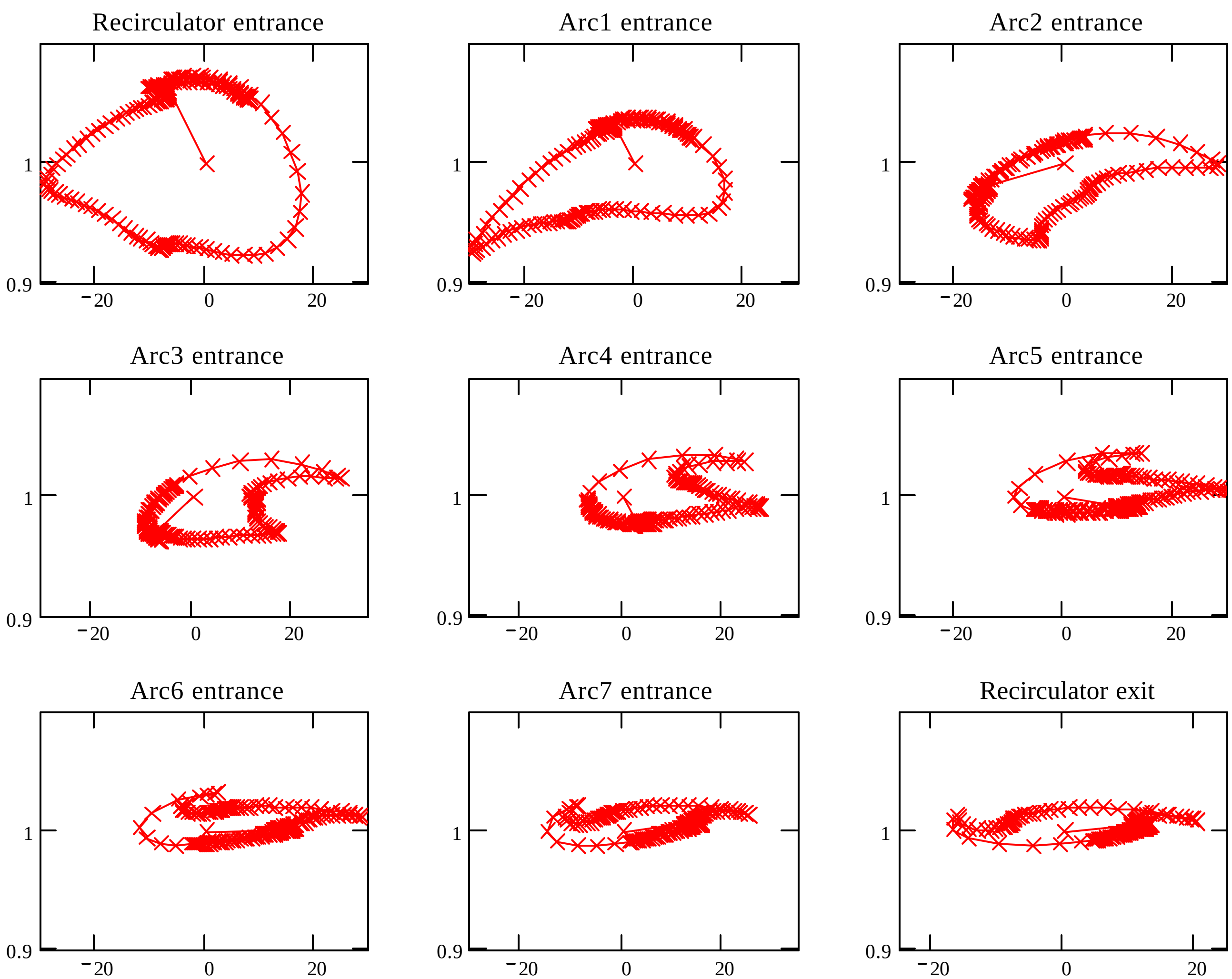
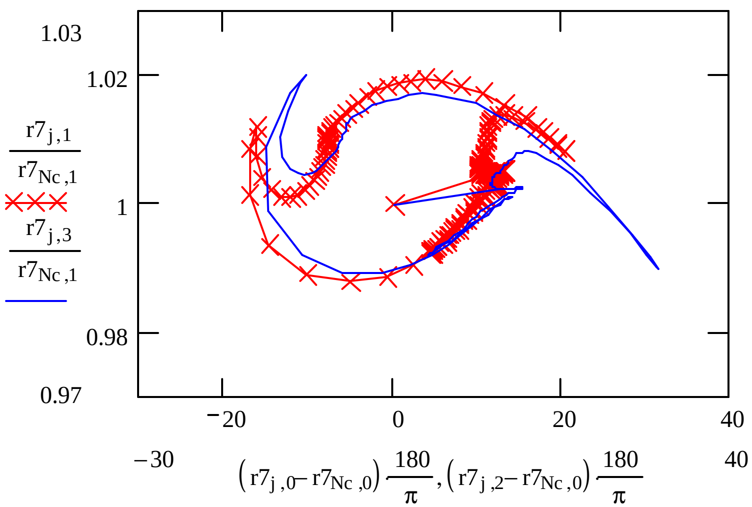
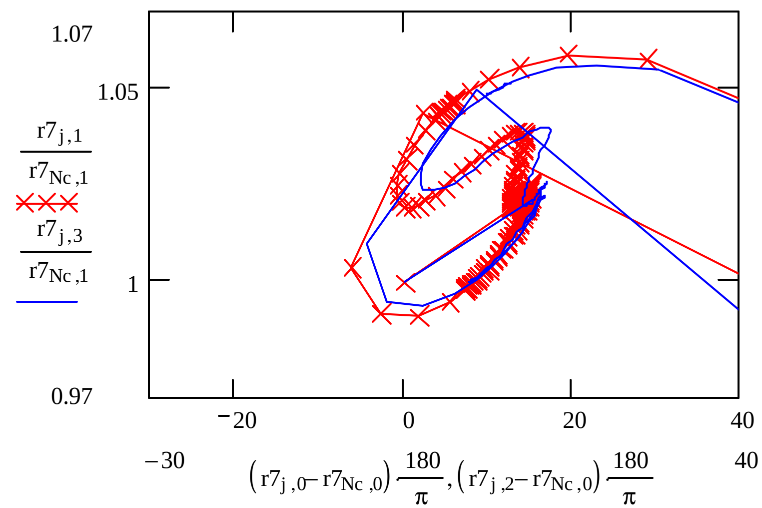


❖Effect of beam loading, $\Delta E/E=6\cdot10^{-3}$, $N_\mu=3\cdot10^{12}$

RLA



FFAG



- The red line shows the longitudinal phase space boundary for the first bunch; and the blue line shows it for the last bunch of $3\cdot10^{12}$ particle train. The difference in comparison with the first bunch is related to the voltage droop (0.6% per pass for 200 MHz) in RF voltage due to beam loading.

Summary

- RLA scheme – already done…
  - Overall accelerator complex architecture (245MeV/c Matching section, 2.8 GeV Linear Pre-accelerator, Injection chicane, 2.8-to-20 GeV 4-pass RLA)
  - Complete lattice designed for the entire complex
  - Multi-particle tracking studies throughout the entire accelerator
  - Chromatic corrections in the Arcs to effectively restore longitudinal space linearity (via families of sextupoles)
- FFAG scheme – initial comparison with the RLA
  - ‘Shelter Island Lattice’ (D. Trbojevic et al.) – FFAG ring with distributed RF
  - Longitudinal compression for 10 turns
  - Beam loading effect for a train of bunches
  - Klystron requirements for operating RF cavities at stored energy