Polarized parton distribution functions of AAC analysis

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Abstract. This paper reports recent AAC analysis results and uncertainty estimation of polarized parton distribution functions (polarized PDF's) by the Hessian method. The SLAC-E155 proton target data, which are precise measurements, affect the polarized PDF uncertainty.

1. Introduction

PDF uncertainty estimation has been developed in the last several years (see a brief review [1]). It is important to show that phenomenological ambiguity of a predicted observable with parameterized PDF's and their uncertainties. We have so far pursued a global analysis of the polarized PDF's and estimated these uncertainties from polarized deep inelastic scattering (polarized DIS) experimental data [2], However, the antiquark and gluon distributions still had large uncertainty. A purpose of this analysis is to investigate influence of precise data on the polarized PDF uncertainty. We re-analyzed polarized PDF's including the SLAC-E155 proton data [3], and a new result is compared with the previous AAC result.

2. Uncertainty estimation

Polarized PDF uncertainty comes from several error sources, e.g., experimental errors, unpolarized PDF, Λ_{QCD}, and so on. However, it is difficult to include these errors in uncertainty estimation all at once. In the present analysis, the polarized PDF uncertainty is estimated from experimental errors by using the Hessian matrix, and we neglect systematic error correlation. The systematic error is added in quadrature, so that it could be overestimation.

The Hessian H_{ij} is defined as a second order derivative matrix in the expanded $\chi^2(a_i)$ function around its minimum point. The PDF uncertainty $\delta\Delta f(x)$ can be obtained by the inverse matrix of the Hessian and linear error propagation:

$$[\delta\Delta f(x)]^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial\Delta f(x)}{\partial a_i} H_{ij}^{-1} \frac{\partial\Delta f(x)}{\partial a_j} \ , \tag{1}$$

where $\Delta\chi^2 (= \chi^2(a_i) - \chi^2_{min})$ is defined as the difference from the minimum χ^2. It determines a confidence level of the PDF uncertainty, and it depends on the χ^2

Table 1. First moments of the polarized antiquark, gluon, and spin content with these uncertainties at 1 GeV^2.

	$\Delta\bar{q}$	Δg	$\Delta\Sigma$
New	-0.062 ± 0.023,	0.499 ± 1.268,	0.213 ± 0.138
AAC00	-0.057 ± 0.038,	0.532 ± 1.949,	0.241 ± 0.228

distribution $K(s)$ with N degrees of freedom. Here, N is the number of optimized parameters. In our estimation, the value of $\Delta\chi^2$ is obtained by the following equation:

$$\int_0^{\Delta\chi^2} K(s)\ ds = \sigma\ , \tag{2}$$

where $\sigma(= 0.683)$ corresponds to 1 σ error of a standard distribution in order to compare with general experimental errors.

3. Parameterization of AAC

In the AAC analysis, the polarized PDF $\Delta f(x)$ is defined at initial Q^2 by the weight function form: $\Delta f(x) = Ax^\alpha(1+\lambda x^\gamma)f(x)$, where $f(x)$ is the unpolarized PDF from the GRV98 set [5]. A, α, λ, and γ are free parameters. Optimized PDF's are four distributions; $\Delta u_v(x)$, $\Delta d_v(x)$, $\Delta\bar{q}(x)$, and $\Delta g(x)$. These distributions are evolved from Q_0^2=1 GeV^2 to the same Q^2 of experimental data by the DGLAP equation [4]. Polarized PDF's are determined by the χ^2 analysis with following two conditions.

First, the positivity condition is used to restrict large-x behavior of the polarized PDF's. This condition needs not to be satisfied strictly in the NLO analysis. However, the antiquark and gluon distributions tend to badly break the positivity limit: $|\Delta f(x)|/f(x) \gg 1$. Such excessive behavior is due to the large experimental errors in the large-x region. Hence, this behavior should be limited by this condition.

Next, the SU(3)$_f$ flavor symmetry is assumed: $\Delta\bar{u}(x) = \Delta\bar{d}(x) = \Delta\bar{s}(x) = \Delta s(x)$. In order to decompose antiquark flavor dependence, we need to determine a shape and sign of each distribution. The current polarized DIS data, however, do not have enough accuracy. Thus, most of the polarized PDF analyses assume the SU(3)$_f$ symmetry at this stage. By this assumption, one can fix first moments of the valence quarks with hyperon decay constants, and $\Delta u_v = 0.926$, and $\Delta d_v = -0.341$ are obtained.

4. Results

The total χ^2 was minimized by the CERN subroutine `MINUIT`, and it became $\chi^2(/d.o.f.) = 346.33(0.90)$. First moments of the polarized PDF's are shown in Table. 1. New results are compared to the AAC pervious results (AAC00, NLO set2) [2]. These uncertainties of the new results become smaller than those of the previous results. The gluon first moment still has large uncertainty. Furthermore, the spin content $\Delta\Sigma$ is obtained by $\Delta\Sigma_{N_f=3} = \Delta u_v + \Delta d_v + 6\Delta\bar{q}$. The fixed first moments Δu_v and Δd_v do not have uncertainty. The $\Delta\Sigma$ uncertainty is six times as large as the antiquark uncertainty, and it is affected by small-x behavior of the antiquark distribution. Uncertainties of the antiquark and gluon distributions are shown in Figure. 1. The antiquark uncertainty becomes rather large in the small-x region. It indicates that small-x behavior of the antiquark distribution is difficult to be clarified from the current polarized DIS data. It is because the experimental data do not exist,

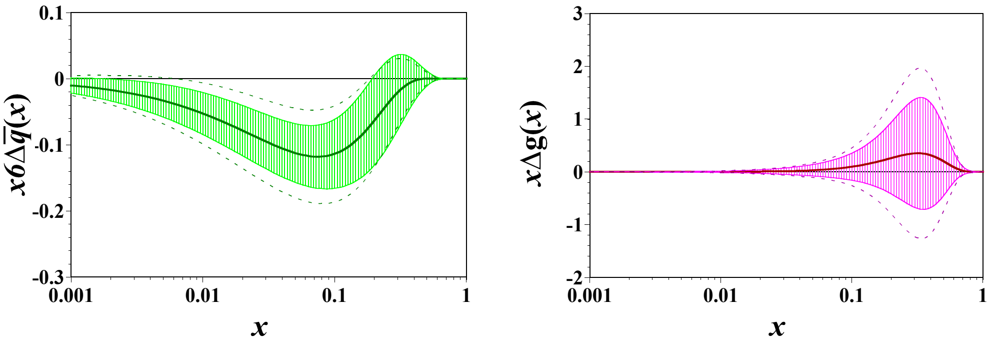

Figure 1. The polarized antiquark and gluon distributions with their uncertainties at $Q^2 = 1$ GeV^2. Dashed curves are uncertainties of previous results (AAC00 NLO set2) [2]

and the spin asymmetries $A_1^{p,d}(x)$ approach rapidly to zero in the rang $x < 0.004$. Of course, the $\Delta\Sigma$ uncertainty includes such extrapolating ambiguity from the antiquark distribution.

Additionally, new polarized PDF uncertainties are compared to those of the previous results in Figure. 1. These uncertainties are reduced in the range $0.01 < x < 0.5$, where the E155 proton data exist. The precise data of polarized DIS experiments can reduce the antiquark uncertainty. On the other hand, the gluon uncertainty changes in response to antiquark uncertainty reduction due to a strong correlation between two distributions. It is because the gluon contribution to the structure function $g_1(x)$ is smaller than the antiquark contribution. Actually, the large gluon uncertainty indicates the difficulty of extracting this contribution from the polarized DIS experimental data. Therefore, unexpected uncertainty reduction of the gluon distribution is due to the strong correlation rather than the NLO contribution.

5. Summary

In this analysis, polarized PDF's were optimized from the polarized DIS data, and these uncertainties were estimated by the Hessian method. The antiquark and gluon uncertainties are reduced by the E155 precise measurements, and there is a complementary relation due to the strong correlation between two distributions. Spin content determination has the extrapolating issue of the antiquark behavior at small x. Both distributions are still wrapped in large uncertainty. The gluon distribution will be unveiled by prompt photon production in RHIC-Spin at BNL. The neutrino DIS experiment has a possibility to reveal the details of polarized antiquark distributions.

References

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