Neutrino Oscillations, Rare Decays and Warped Geometry

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Abstract. We discuss how the solar and atmospheric neutrino anomalies can be satisfactorily resolved via neutrino oscillations in a five dimensional framework with warped geometry. In one particularly significant example, no new fields such as singlet (right handed) neutrinos have to be introduced. We also consider rare processes including proton and neutrinoless double beta decays. The Kaluza-Klein excitations of the standard model particles are predicted to possess masses in the 10 TeV range.

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1. Introduction

The subject of extra dimensions has created much recent excitement for a variety of reasons. Before discussing them, let us recall some earlier motivations:

- Extra dimensions may lead to 'deeper' unification than grand unification. For instance, through inclusion of gravity, and unification of 'matter', gauge and 'Higgs' sectors.
- Superstring theories for their consistency require 10 space-time dimensions.
- A particularly promising approach is based on the presumed existence of M-theory, whose low energy limit , it is speculated, is 11D supergravity.

Alas, the predictive power of these higher dimensional theories is quite limited. Indeed, it is far from clear how this top-down approach leads to the SM (or MSSM) at low energies. It may greatly help matters if a bridge can be developed between the low energy world of the SM and MSSM and physics at the highest energies, such that the presence of the extra dimensions is somehow clearly exploited.

A number of recent attempts try to show, for instance, how the presence of one or more extra dimension can be exploited to yield possible resolution of the gauge hierarchy problem, the doublet-triplet problem, the apparent stability of the proton, and so on. Three of these approaches will be briefly discussed:

1) Large extra dimension(s). It is assumed that the fundamental (quantum gravity) scale M_f is more or less comparable to the electroweak scale, and this can be made consistent within a $4+N$ dimensional setting through a relation of the type

$$M_P^2 \sim M_f^{2+N} \cdot V_N,$$

where V_N denotes the characteristic volume of the N dimensional compact space, and M_P, as usual, is the reduced Planck scale. Clearly, V_N must be orders of magnitude larger than M_f^{-N}, for $M_P \gg M_f$. For $M_f \sim$ TeV, say, $R \sim 10^{30/N-17}$ cm. Since the gravitational force is currently being tested in the sub-millimeter region, the large extra dimension scenario is still plausible for $N = 2$ from this point of view.

With M_f close to the electroweak scale, it was hoped that the gauge hierarchy problem would simply go away. There are at least two difficulties with this idea:

- A new hierarchy has been introduced, $M_f^{-N}/V_N \ll 1$. How does this arise ?
- Even though laboratory constraints on M_f are still very mild ($\gtrsim$ 1 to 0.6 TeV) as N varies from 2 to 5, far more stringent lower bounds on M_f arise from astrophysical considerations based on supernova and neutron stars. Thus, for N=2 (3), M_f may be at least as high as 1700 (60) TeV [1]. This makes the approach look somewhat less appealing .

2) SUSY GUTS and (flat) extra dimension(s). The presence of a fifth dimension with the topology of $S^1/Z_2 \times Z_2$ has been extensively utilized to construct supersymmetric models that: Achieve doublet-triplet splitting (without many tears!); eliminate dimension five nucleon decay; incorporate fermion mass hierarchies and

fermion mixings without introducing small Yukawa couplings; implement the idea of 'neutrino' democracy which yields large neutrino mixings (without making them bimaximal) [2]; realize inflation and leptogenesis in realistic SUSY GUTS [3].

As far as fermion masses and mixings are concerned, the five dimensional approach is not necessarily more predictive, but it does provide a new and geometric way to think about the hierarchies. The fermion wave functions carry a non-trivial dependence on the fifth dimension, which can be exploited to yield the effective four dimensional Yukawa (determined by their overlap with the Higgs doublets). More interestingly, perhaps, a fifth dimension plays an essential role in realizing the democratic neutrino ansatz in SUSY GUTS [4].

3) Warped geometry. The idea that one or more of the extra dimensions may be warped can be motivated as follows: – It provides possible resolution of the gauge hierarchy problem, namely how $M_W \ll M_P$ [5]. Note that this can be achieved without invoking SUSY; – The wave functions of bulk fields are far more constrained compared to the case of 'flat' geometry. In principle, this could lead to more predictive power; – The observed atmospheric and solar neutrino oscillations can be accommodated within the SM framework without invoking additional fields such as singlet (right handed) neutrinos; – The approach may be consistent with grand unification, which is not the case say when dealing with large extra dimensions; – There exists the intriguing possibility that 'warping' could be exploited to understand the 'tiny' vacuum energy density $\rho_V \sim 10^{-120} M_P^4$ [6]; – There are experimental predictions which can be experimentally tested. For instance, the KK excitations of bulk SM fields are expected to have masses in the several TeV range which, hopefully, can be tested at the LHC. Since there are no large extra dimensions, it is warping which makes these excitations experimentally accessible (without warping the masses would be of order M_P and therefore inaccessible for ever!).

2. Fermions in a warped background

By the Kaluza-Klein (KK) procedure the 5D fields are decomposed into an infinite tower of 4D fields. The wave functions encode information on where the KK states are localized in the extra dimension. Together with the KK masses they are obtained by solving the 5D equations of motion. In five dimensions fermions are vector-like, and we can associate with them a 5D Dirac mass term, parameterized by $m_\Psi = c \cdot k \ \mathrm{sgn}(y)$, where y is the 5th coordinate. Depending on the Z_2 orbifold transformation property of the fermion, $\Psi(-y)_\pm = \pm\gamma_5 \Psi(y)_\pm$, the left-handed (right-handed) zero mode, $f_0 \sim e^{(2-c)k|y|}$, of the KK decomposition is projected out by the boundary conditions [7, 8, 9]. The KK excited states come in left- and right-handed pairs, which are degenerate in mass. Note that the 5D Dirac mass parameter regulates whether the zero mode is localized towards the Planck-brane ($c > 1/2$) or the TeV-brane ($c < 1/2$).

3. The Dirac case

It was suggested in ref. [7] that in warped models small Dirac neutrino masses can arise from a coupling to sterile (right-handed) bulk neutrinos. In order to generate masses in the sub-eV range, the sterile neutrinos have to be localized close to the Planck-brane, while the SM model neutrinos were confined to the TeV-brane. We generalized this scenario to incorporate bulk SM neutrinos [10].

After electroweak symmetry breaking, the Dirac neutrino masses are generated from the Yukawa-type coupling between the SM neutrinos, contained in the SU(2) doublets L_i, and the three sterile neutrinos ψ_i

$$\mathcal{L} = h_{ij}^{(5)} \bar{L}_i \psi_j H + \text{h.c.} \ + \ \ldots \tag{1}$$

where $h_{ij}^{(5)}$ are the 5D Yukawa couplings. The 4D Dirac masses are given by

$$m_{ij} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} h_{ij}^{(5)} e^{-4\sigma} H(y) f_{0iL}(y) f_{0j\psi}(y), \tag{2}$$

where the Higgs field is confined to the TeV-brane, i.e. $H(y) = H_0 \delta(y - \pi R)$, to maintain the solution of the hierarchy problem.

The fermion wave functions and consequently the induced 4D Dirac masses crucially depend on the 5D mass parameters of the left- and right-handed fermions, c_L and c_ψ respectively, which enter eq. (2). As the 5D Dirac mass, i.e. c parameter of the fermion increases, it gets localized closer towards the Planck-brane. Its overlap with the Higgs profile at the TeV-brane is consequently reduced, which is reflected in a smaller 4D fermion mass from electroweak symmetry breaking. In ref. [9] we have shown that this geometrical picture nicely generates the charged lepton and quark mass hierarchies, as well as quark mixings, by employing c-parameters and 5D Yukawa couplings of order unity.

From the KK reduction of the left-handed neutrino field ν_L we obtain a left-handed zero mode $\nu_L^{(0)}$, corresponding to the SM neutrino, and an infinite tower of left- and right-handed KK excited states $\nu_L^{(i)}$ and $\nu_R^{(i)}$, where we omit flavor indices. The sterile neutrinos decompose into the right-handed zero mode $\psi_R^{(0)}$ and the KK excited states $\psi_L^{(i)}$ and $\psi_R^{(i)}$. After electroweak symmetry breaking the mass matrix takes the form

$$M_\nu = (\nu_L^{(0)}, \nu_L^{(1)}, \psi_L^{(1)}, \ldots) \begin{pmatrix} m^{(0,0)} & 0 & m^{(0,1)} & \cdots \\ m^{(1,0)} & m_{L,1} & m^{(1,1)} & \cdots \\ 0 & 0 & m_{\psi,1} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_R^{(0)} \\ \nu_R^{(1)} \\ \psi_R^{(1)} \\ \vdots \end{pmatrix} \tag{3}$$

where we again suppress flavor indices, i.e. every entry represents a 3×3 matrix in flavor space. The masses $m^{(i,j)}$ are obtained by inserting the relevant wave functions into eq. (2), and $m_{L,i}$ and $m_{\psi,i}$ denote the KK masses of the excited neutrino states. The zeros in (3) follow from the Z_2 orbifold properties of the wave functions.

The squares of the physical neutrino masses are the eigenvalues of the hermitian mass matrix $M_\nu M_\nu^\dagger$. The unitary matrix U_ν, such that $U_\nu^\dagger M_\nu M_\nu^\dagger U_\nu$ is diagonal, relates the left-handed mass eigenstates $N_L^{\rm phys}$ to the interaction eigenstates $N_L = (\nu_L^{(0)}, \nu_L^{(i)}, \psi_L^{(i)})$ via $N_L = U_\nu N_L^{\rm phys}$. The observed neutrinos $\nu_L^{\rm phys}$ correspond to the three lightest states in $N_L^{\rm phys}$. The right-handed mass eigenstates are obtained from a unitary matrix V_ψ which diagonalizes $M_\nu^\dagger M_\nu$. The physical neutrino mixings

$$U = U_l^\dagger U_\nu \tag{4}$$

also depend on the rotations of the left-handed charged leptons U_l. If the Yukawa couplings of the charged leptons are diagonal, the mixing angles of the physical neutrinos can be directly read off from U. Like in the CKM matrix, there are three mixing angles, and a single complex phase δ which induces CP violation in the lepton sector.

The atmospheric neutrino data imply [12]

$$1 \cdot 10^{-3}{\rm eV}^2 < \Delta m^2_{\rm atm} < 5 \cdot 10^{-3}{\rm eV}^2, \quad \sin^2 2\theta_{\rm atm} > 0.85. \tag{5}$$

There are several solutions to the solar neutrino anomaly [11]. Most favored is the LMA solution

$$2.5 \cdot 10^{-5}{\rm eV}^2 < \Delta m^2_{\rm sol} < 4 \cdot 10^{-4}{\rm eV}^2, \quad 0.3 < \sin^2 2\theta_{\rm sol} < 0.93. \tag{6}$$

The SMA solution gives only a poor fit to the data. The CHOOZ reactor experiment constrains $|U_{e3}|^2 \equiv s_2^2$ to be at most a few percent [13]. Nothing is known about the CP violating phase δ.

Various constraints on the scenario with bulk gauge and fermion fields have been discussed in the literature [14, 15, 16]. With bulk gauge fields for instance, the SM relationship between the gauge couplings and masses of the Z and W bosons gets modified. The electroweak precision data then requires the lowest KK excitation of the gauge bosons to be heavier than about 10 TeV [14, 15]. This bound becomes especially important if the fermions are localized towards the Planck-brane ($c > 1/2$). As discussed in ref. [9], this applies to all SM fermions, with the exception of the top-quark, if the fermion mass hierarchy arises from their different locations in the extra dimension. In this case the bounds induced by the contribution of KK excitations of the SM gauge bosons to the electroweak precision observables are weak. They only require the KK masses to be above about 1 TeV [8]. In the following we will therefore assume that the mass of the first KK gauge boson is $m_1^{(G)} = 10$ TeV. The corresponding masses of the KK fermions are then in the range 10 to 16 TeV, for $0 < c < 1.5$. The mass of the lightest KK graviton is 16 TeV.

In order to induce large mixings, the neutrinos have to be located at similar positions in the extra dimension. Therefore, we take $c_L^{(1)} = c_L^{(2)} = c_L^{(3)} \equiv c_L$. The charged lepton masses are, for instance, reproduced by $c_L = 0.567$, $c_E^{(1)} = 0.787$, $c_E^{(2)} = 0.614$ and $c_E^{(3)} = 0.50$, where for simplicity we have assumed diagonal Yukawa couplings for the charged leptons. The neutrino mixings are then solely governed by the neutrino mass matrix M_ν. We take $k = M_5 = M_{\rm Pl}$. From $m_1^{(G)} = 10$ TeV we determine the brane separation $kR = 10.83$. A parameter set which implements the large angle MSW solution is

$$c_\psi^{(1)} = 1.43, \ c_\psi^{(2)} = 1.36, \ c_\psi^{(3)} = 1.30,$$

$$\frac{h_{ij}^{(5)}}{g_2^{(5)}} = \begin{pmatrix} -2.0 & 1.5 & -0.5 \\ -1.8 & -1.1 & 1.9 \\ 0.5 & 1.9 & 1.7 \end{pmatrix}. \tag{7}$$

We obtain the light neutrino masses $m_{\nu 1} = 1.0 \cdot 10^{-3}$ eV, $m_{\nu 2} = 1.0 \cdot 10^{-2}$ eV and $m_{\nu 3} = 7.1 \cdot 10^{-2}$ eV. For the neutrino oscillation parameters we find $\Delta m^2_{\rm atm} = 4.9 \cdot 10^{-3}$ eV^2, $\sin^2 2\theta_{\rm atm} = 0.99$, $\Delta m^2_{\rm sol} = 1.0 \cdot 10^{-4}$ eV^2 and $\sin^2 2\theta_{\rm sol} = 0.90$. Also, $|U_{e3}|^2 = 0.036$ is close to the experimental bound which is typical for the solutions we find.

The SM neutrinos mix not only with each other but also with the left-handed KK states of the sterile neutrinos $\psi_L^{(i)}$ [7]. This effect diminishes the effective weak charge of the light neutrinos. Thus, the effective number of neutrinos contributing to the width of the Z boson is reduced to $n_{\rm eff} = 3 - \delta n$, where δn is obtained from summing the relevant squared entries of U_ν. Measurements of the Z width induce the constraint δn0.005. For the parameter set (7) we find the result $\delta n = 2 \cdot 10^{-5}$, well below the experimental sensitivity.

4. The Majorana case

In the warped SM the suppression scale for non-renormalizable operators can be anywhere between a few TeV and the Planck scale, depending on the localization of the fermions [8, 9]. Majorana masses for the left-handed neutrinos are generated by the dimension-five operator [8, 9, 17],

$$\int d^4x \int dy \sqrt{-g} \frac{l_{ij}}{M_5^2} H^2 \Psi_{iL} C \Psi_{jL} \equiv \int d^4x \; M_{ij}^{(\nu)} \Psi_{iL}^{(0)} C \Psi_{jL}^{(0)}, \tag{8}$$

where l_{ij} are dimensionless couplings constants and C is the charge conjugation operator. The neutrino mass matrix reads

$$M_{ij}^{(\nu)} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \frac{l_{ij}}{M_5^2} e^{-4\sigma(y)} H^2(y) f_{0iL}(y) f_{0jL}(y). \tag{9}$$

By localizing f_{0L} close to the Planck-brane ($c_L \gg 1/2$), $M^{(\nu)}$ can be made very small. However, at the same time, the masses of the charged leptons, residing in SU(2) multiplets with the neutrinos, can become very small as well [8, 9]. Since the tau is the heaviest lepton, it has to reside closest to the TeV-brane, and induces the most stringent constraint on l. In ref. [17] we considered the somewhat favorable case $\lambda_\tau^{(5)} = 10 g_2^{(5)}$ and $k/M_5 = 0.01$, obtaining $m_{\nu,\tau} = l \cdot 25$ eV. A neutrino mass of 50 meV can then be generated by $l = 0.002$, which is only a moderately small number.

Following the spirit of "anarchy", we parametrize the coefficients in (9) by

$$l_{ij} \equiv \Lambda \cdot \tilde{l}_{ij}. \tag{10}$$

The supposedly fundamental theory responsible for the effective interaction (8) is represented through the order unity coefficients $\tilde{l}_{ij}$. In order to generate large mixings we locate the left-handed doublets at the same position in the extra dimension,

$$c_{e,L} = c_{\mu,L} = c_{\tau,L} = 0.72. \tag{11}$$

	$\Delta m^2_{\text{atm,sol}}$	$+\sin^2 2\theta_{\text{atm,sol}}$	$+\vert U_{e3}\vert^2 < 0.05$
LMA	44.4 (28.1)	5.8 (5.9)	5.0 (0.7)
SMA	1.3 (0.04)	0.3 (< 0.001)	0.3 (< 0.001)
LOW	0.008 (< 0.001)	0.002 (< 0.001)	0.002 (< 0.001)

Table 1. Probability in percent that a randomly generated set of coefficients $1/2 < |\tilde{l}_{ij}| < 2$ satisfies the constraints from $\Delta m^2_{\text{atm,sol}}$ (first column) and $\sin^2 2\theta_{\text{atm,sol}}$ (second column) and $|U_{e3}|^2 < 0.05$ (third column). The results are given for the case $c_{e,L} > c_{\mu,L} = c_{\tau,L}$ ($c_{e,L} = c_{\mu,L} = c_{\tau,L}$).

To accommodate the tau, muon and electron masses we are led to $c_{\tau,R} = 1/2$, $c_{\mu,R} = 0.64$ and $c_{e,R} = 0.85$. Taking the order unity coefficients to be homogeneously distributed $1/2 < |\tilde{l}_{ij}| < 2$ with random phases from 0 to 2π, we find the most favorable value for the overall scale to be $\Lambda = 1.9 \cdot 10^{-3}$. We randomly generate parameter sets for $\tilde{l}_{ij}$, calculate the neutrino mass matrix from (9) and compute the neutrino masses and mixings. The left-handed charged lepton rotations are similar to the neutrino rotations because both fields sit together in the extra dimension. We include them by generating random Yukawa couplings for the charged leptons as well.

Focusing on the LMA solution of the solar neutrino anomaly, which turns out to be clearly favored, we find the following picture [17]. From our neutrino parameter sets about 70 % reproduce Δm^2_{atm} (5). Imposing in addition the constraint from Δm^2_{sol} (6), we are left with about 28 % of the parameter sets. The solar and atmospheric mixings angles bring this number down to about 6 %, which is still a considerable fraction given the number of constraints. The most stringent constraint turns out to come from the CHOOZ experiment. Adding the requirement $|U_{e3}|^2 < 0.05$, the fraction of viable parameter sets shrinks to about 0.7 %. This result is closely related to an average $\langle |U_{e3}|^2 \rangle = 0.22$, which is considerably above the experimental bound. Averaging only over the sets which pass all experimental constraints, we find $\langle |U_{e3}|^2 \rangle = 0.022$. This means that $|U_{e3}|^2$ is probably close to the experimental bound and can likely be tested at future neutrino experiments, such as MINOS, ICARUS and OPERA which are sensitive down to $|U_{e3}|^2 \sim 0.01$ [18]. Since we typically start with large phases in the neutrino and charged lepton mass matrices, the complex phase δ in the mixing matrix U is found to be most likely of order unity. The SMA, LOW and VAC solutions to the solar neutrino anomaly can be realized only with severe fine-tuning. Our results are summarized in table 1.

So far the constraint from the CHOOZ experiment is the most stringent one. Its inclusion reduces the probability that a parameter set realizes the LMA solution from about 6 % to less than one percent. With all entries in the neutrino mass matrix being of similar magnitude, the ensuing mixing angles are typically large. The fit to the neutrino data improves considerably if the electron neutrino is somewhat separated from the other two neutrino species. Shifting the electron neutrino closer to the Planck-brane induces small elements in the neutrino mass matrix. As a result, small values of Δm^2_{sol}

and $|U_{e3}|^2$ become more probable. The neutrino mass matrix (9) acquires the following structure

$$M^{(\nu)} \sim \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \tag{12}$$

where $\epsilon \approx f_{0,e}^{(\nu)}(\pi R)/f_{0,\mu}^{(\nu)}(\pi R) \approx \exp(-(c_{e,L} - c_{\mu,L})\pi k R)$.

We keep the muon and tau neutrinos at the previous locations $c_{\mu,L} = c_{\tau,L} = 0.72$. A separation of these two would make the fit to the atmospheric neutrino data more difficult since the corresponding mixing is reduced. For the electron neutrino the most favorable choice turns out to be $c_{e,L} = 0.79$ leading to $\epsilon = 0.15$. The positions of the right-handed leptons are found to be $c_{e,R} = 0.79$, $c_{\mu,R} = 0.62$ and $c_{\tau,R} = 0.49$.

Let us again focus on the LMA solution. Assuming $1/2 < |\tilde{l}_{ij}| < 2$ for the order unity coefficients, we find the best value $\Lambda = 2.6 \cdot 10^{-3}$. The correct values of the Δm^2's are fitted by 44 % of the parameter sets. Taking into account also the constraints from the solar and atmospheric mixing angles, this fraction shrinks to 5.8 %. This reduction is mostly due to the solar mixing angle which is suppressed if ϵ is small. Most important, however, the CHOOZ constraint $|U_{e3}|^2 < 0.05$ is now satisfied almost automatically, and we are finally left with 5.0 % of the parameter sets. Compared to the case of $\epsilon = 1$, the probability for a parameter set to satisfy all observational constraints is enhanced by almost an order of magnitude. The distribution of $|U_{e3}|^2$ is now peaked at small values. Averaging only over parameter sets which satisfy all constraints, we find a moderately small value $\langle |U_{e3}|^2 \rangle = 0.010$. As a consequence, the next generation neutrino experiments still has a chance to detect a non-vanishing $|U_{e3}|^2$ [18].

Taking even smaller values of ϵ, it is possible to implement the SMA solution to the solar neutrino problem. The LOW and VAC solutions are very difficult to accommodate because of their small solar Δm^2. Thus, the LMA solution is by far the most favored scenario. A collection of our results is given table 1.

5. Rare processes

The most direct evidence for a Majorana mass of neutrinos would be the discovery of neutrinoless double beta decay ($0\nu\beta\beta$). The non-observation of this process in the Heidelberg-Moscow experiment implies an upper bound on $|M_{11}^{(\nu)}| \equiv m_{ee} < 0.35$ eV [19]. There are plans to bring this limit down to about 0.01 eV. If all neutrinos are at the same position in the extra dimension, we expect $m_{ee} \sim 0.02$ eV, which is far below the current experimental sensitivity but within reach of future experiments. Once the electron neutrino is localized closer towards the Planck-brane $0\nu\beta\beta$ becomes drastically suppressed proportional to ϵ^2. For the clearly favored LMA scenario we find $m_{ee} \sim 0.001$ eV, which is too small to be detected in the near future. For the SMA, LOW and VAC solutions m_{ee} is even smaller. It is therefore questionable if our model induces $0\nu\beta\beta$ at a detectable level. In any case the Majorana masses we find are far below the recently claimed evidence of about 0.4 eV [20].

The low cut-off scale dramatically amplifies the impact of non-renormalizable operators on rare processes, such as proton decay and flavor violation. With bulk fields localized towards the Planck-brane the corresponding suppression scales can be significantly enhanced [8, 9]. However, there are limits because the SM fermions need to have sufficient overlap with the Higgs field at the TeV-brane to acquire their observed masses. In the case of proton decay, typical dimension-six operators still have to be suppressed by small couplings of order 10^{-8} to be compatible with observations. One may be tempted to completely eliminate these small numbers by imposing additional symmetries. In ref. [10] we used lepton number conservation to make the proton stable and to forbid the potentially large Majorana neutrino masses of eq. (8). In the case of Majorana neutrinos proton stability can be ensured by lepton parity. In both cases baryon number violation may still be present, and processes like neutron anti-neutron oscillations and double nucleon decay could be close to their experimental bounds.

In the scenario of Dirac neutrino masses the rate of $\mu \rightarrow e\gamma$ transitions is considerably enhanced by the presence of heavy sterile neutrino states. If the SM neutrinos are confined to the TeV-brane, its large branching ratio pushes the KK scale up to 25 TeV and thus imposes the most stringent constraint on the model [21]. However, the rate for $\mu \rightarrow e\gamma$ is very sensitive to the mixing between light and heavy neutrino states. With bulk neutrinos the mixing with heavy states is considerably reduced. In the case of the large angle MSW solution (7) we obtain $\mathrm{Br}(\mu \rightarrow e\gamma) \approx 10^{-15}$ [10].

6. Conclusions

We have discussed how the atmospheric and solar neutrino oscillations, especially the bimaximal mixing scenario, are incorporated in models with a small but warped extra dimension. In one scenario Dirac neutrino masses arise from a coupling to right-handed neutrinos in the bulk. Alternatively, dimension five interactions may induce a Majorana mass for the neutrinos. In both cases the mixing angle U_{e3} is expected be close to its present bound. While the direct detection of KK states is still waiting, rare processes, such as proton decay, neutrinoless double beta decay and flavor violation can provide important hints to the higher dimensional theory.

[1] S. Hannestad and G.G. Raffelt, *Phys. Rev. Lett.* **88** (2002) 071301.
[2] Q. Shafi and Z. Tavartkiladze, hep-ph/0208162.
[3] B. Kyae and Q. Shafi, hep-ph/0211059.
[4] Q. Shafi and Z. Tavartkiladze, hep-ph/0210181.
[5] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370; M. Gogberashvili, hep-ph/9812296.
[6] S J. Huber and Q. Shafi, hep-ph/0207232.
[7] Y. Grossman and M. Neubert, *Phys. Lett.* **B474** (2000) 361.
[8] T. Gherghetta and A. Pomarol, *Nucl. Phys.* **B586** (2000) 141.
[9] S.J. Huber and Q. Shafi, *Phys. Lett.* **B498** (2001) 256.
[10] S.J. Huber and Q. Shafi, *Phys. Lett.* **B512** (2001) 365.

[11] V. Barger, D. Marfatia, K. Whisnant and B.P. Wood, hep-ph/0204253; J.N. Bahcall, M.C. Gonzalez-Garcia, C. Pena-Garay, hep-ph/0204314.
[12] T. Toshito [SuperKamiokande Collaboration], hep-ex/0105023.
[13] M. Apollonio *et al.* [CHOOZ Collaboration], *Phys. Lett.* **B466** (1999) 415.
[14] S.J. Huber and Q. Shafi, *Phys. Rev.* **D63** (2001) 045010.
[15] S.J. Huber, C.-A. Lee and Q. Shafi, *Phys. Lett.* **B531** (2002) 112.
[16] J.L. Hewett, F.J. Petriello and T.G. Rizzo, *JHEP* **0209** (2002) 030.
[17] S.J. Huber and Qaisar Shafi *Phys. Lett.* **B544** (2002) 295.
[18] M. Komatsu, P. Migliozzi and F. Terranova, hep-ph/0210043.
[19] The Heidelberg-Moscow collaboration, H.V. Klapdor-Kleingrothaus *el al.* in the proceedings of *DARK 2000*, Heidelberg, Germany, 10-16 July 2000, 520-533 [hep-ph/0103062].
[20] H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney and I.V. Krivosheina., *Mod. Phys. Lett.* **A16** (2001) 2409 [hep-ph/0201231].
[21] R. Kitano, *Phys. Lett.* **B481** (2000) 39 [hep-ph/0002279].