Calculations of Energy Loss and Multiple Scattering (ELMS) in Molecular Hydrogen

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Abstract. To show that the principle of Ionisation Cooling will work for muon beams we must be able to simulate energy loss and scattering in media reliably. We have three choices: we can use traditional calculations with their uncertainties; we can make measurements (MUSCAT); or, we can calculate the phenomena more carefully, looking afresh at the phenomena from first principles. In this paper we report on work following this third approach. We derive the double differential cross section for a collision with transverse momentum transfer P_t and longitudinal momentum transfer P_ℓ from a knowledge of the UV and X-ray photoabsorption cross section of the medium, together with the known kinematics and dynamics of the scattering of point charges with screening. Distributions in energy loss and scattering may then be found by Monte Carlo techniques which take into account both correlations in scattering and energy loss, and the true effects of non-gaussian tails in distributions of interest. Preliminary results are reported for molecular hydrogen. Further work is in progress.

1. Introduction

Traditional methods of calculating energy loss and multiple scattering date from days when data on media were poor, computers were rare and the priority was on quick back-of-the-envelope results using simple parameters, *eg* radiation length and mean ionisation potential. Today, given that the underlying physics is understood and that good atomic and molecular data are available, reliable calculations can be made. The ELMS (Energy Loss and Multiple Scattering) program is intended to do that.

2. The double differential cross section

Collisions of the incident muon with the medium comprise:

(i) Coulomb collisions with nuclei,

(ii) Coulomb collisions with constituent electrons

(iii) collisions with atoms as a whole.

Some years ago we described calculations of energy loss due to processes (ii) and (iii), depending only on the photoabsorption of the medium, but including δ-rays, ionisation, excitation and Cherenkov Radiation in a unified way [Allison & Cobb, 1980]. From the same starting point we have derived the cross section for transfer $(\hbar\mathbf{k}_t,\ \hbar k_\ell)$ to the muon:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}^2\mathbf{k}_t\mathrm{d}k_\ell} = \frac{\alpha\varepsilon_2}{\pi^2\beta N(k_\ell^2+k_t^2)}\left[\frac{\beta^4 k_\ell^2 k_t^2}{\left(k_\ell^2(1-\varepsilon_1\beta^2)+k_t^2\right)^2+\left(\varepsilon_2\beta^2k_\ell^2\right)^2}-\frac{1}{|\varepsilon|^2}\right]$$

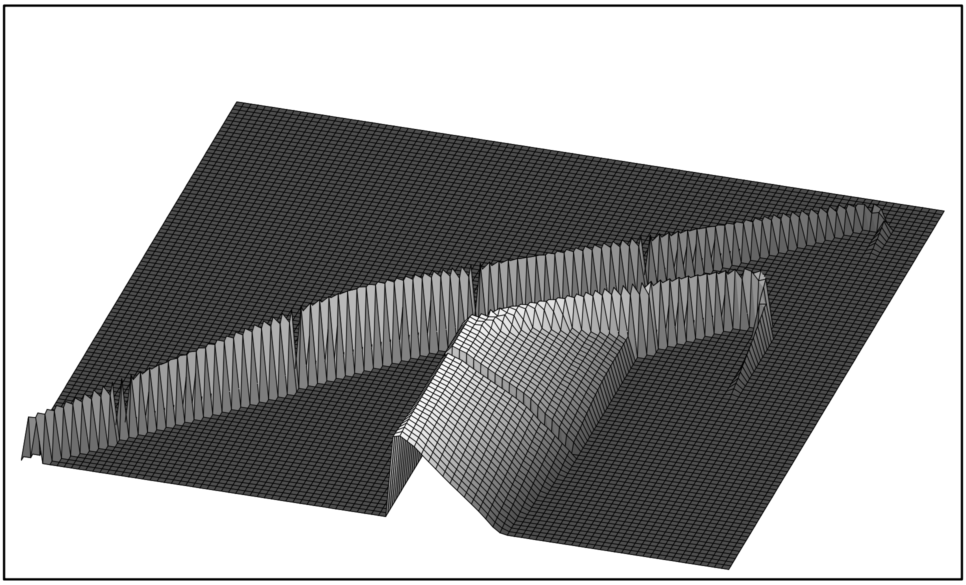

Figure 1. The collision probability for 180 MeV/c muon incident on molecular hydrogen at liquid density. The left-right log scale is $10^{-8} \to 10^{9}$ eV in energy transfer. The front-back log scale is $10^{1} \to 10^{9}$ eV/c in transverse momentum transfer, P_t. The vertical log scale covers more than 8 decades. The lines of nuclear and electron constituent scattering are clearly separated. The latter joins on to the band of P_t-independent atomic collisions. (Note that some of the finer structures are plotting artefacts.)

where N is the atomic density, ε_1 and ε_2 are the real and imaginary parts of the dielectric response function $\varepsilon(k,\omega)$. Following Allison & Cobb $\varepsilon_2(k,\omega)$ is given in terms of the photoabsorption cross section for real photons $\sigma(\omega)$ by

$$\varepsilon_2(k,\omega) = \frac{Nc}{\omega}\left[\sigma(\omega)\times H\left(\omega - \frac{\hbar k^2}{2m}\right) + \int_0^{\omega}\sigma(\omega')\mathrm{d}\omega' \times \delta\left(\omega - \frac{\hbar k^2}{2m}\right)\right]$$

where m is the electron mass. ε_1 is given in terms of ε_2 by causality (Kramers-Kronig Relations [Jackson 1999]). The first term describes atomic collisions as a whole, the second collisions with constituent electrons and the two are coupled by the Thomas-Reiche-Kuhn Sum Rule.

The cross section (ii) for hard collisions of an incident muon on a target electron is described Dirac Scattering [Perkins 1987] which involves a small spin-dependent modification to Rutherford Scattering. There is some rewriting to include the kinematics of relativistic recoil.

For scattering by nuclei (i) we have the same $1/Q^4$ constituent scattering cross section, with form factors given by atomic screening and nuclear elastic scattering, both well known for hydrogen.

The contribution of each process may be seen qualitatively in fig. 1. Collisions with constituent electrons give correlated contributions to scattering and energy loss, while nuclear and atomic collisions give rise predominantly to scattering and energy loss respectively.

Multiple scattering		Energy loss over density				
θ (proj.) mrad.		MeV g^{-1} cm^2				
RMS	RMS98%	Mean	RMS	Median	Most probable	Mean, atomic H
7.55	6.94	4.64	0.65	4.46	4.25	4.32

Table 1. Multiple scattering and specific energy loss of 180 MeV/c muons in 10cm Molecular Hydrogen with $\rho = 0.0708$ g cm^{-3}. The last column is Atomic Hydrogen for comparison.

3. The ELMS program

Modern data on the photoabsorption spectrum of molecular and atomic hydrogen are available [Berk2002]. These have been modified marginally to take account of some broadening and shifting of discrete transitions at liquid densities. Small effects on electron constituent scattering to include electron binding and fermi momentum have been taken into account. The different effect of nuclear screening by an electron with a Thomas Fermi distribution instead of the ground state atomic wavefunction has also been considered. These effects are small but they indicate the extent of systematic uncertainties.

From these cross sections the ELMS program generates by Monte Carlo a database of energy-loss and momentum transfer pairs in thin absorbers for a range of incident momenta. 'Thin' means that the cross section and the path length in the absorber are constant (1mm, or much less at low momenta). From this database a second ELMS code generates single particle histories through finite absorbers, allowing for both changes of direction and momentum.

4. Preliminary results

Tracks are input to the code as 6-D phase space variables on the front side of an absorber; these variables are then generated at absorber exit. Table 1 shows some findings for an H_2 absorber of thickness 10cm, 0.708g cm^{-2}. Preliminary results of a detailed study of possible systematic errors suggest that errors on scattering are of order 1% and on energy loss of order 2%. We note that the extreme case of ignoring molecular binding altogether gives a 7% shift in energy loss. However the shape, tails and correlations of the distributions are also important. For example, the expected positive energy-scattering correlation is found: the mean $|P_t|$ of tracks in the smallest energy-loss decile is 1.55 MeV/c, while in the highest decile it is 1.77 MeV/c. A fuller report will be published latter.

References

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