Determination of α_s from Gross-Llewellyn Smith sum rule by accounting for infrared renormalon

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Abstract. We recapitulate the method which resums the truncated perturbation series of a physical observable in a way which takes into account the structure of the leading infrared renormalon. We apply the method to the Gross-Llewellyn Smith (GLS) sum rule. By confronting the obtained result with the experimentally extracted GLS value, we determine the value of the QCD coupling parameter which turns out to agree with the present world average.

The infrared (IR) renormalons are singularities at positive values of the Borel variable b in the Borel transforms of QCD physical observables. These renormalons represent the contributions of loop integrations in the low energy regimes [1]. They represent obstacles for and lead to ambiguities in the Borel integration. This integration then gives discontinuities (cuts) of the observables in the complex coupling parameter plane. It is thus important to eliminate these unphysical cuts from the Borel integration. This approach was proposed in [2] and applied to the GLS sum rule in [3]. Here we recapitulate the method and the results of [3], and emphasize some physical aspects. The GLS sum rule is the quantity

$$M_3(Q^2) \equiv 3\left(1-\Delta(Q^2)\right) = \int_0^1 dx\ F_3(x;Q^2)\left[1+\mathcal{O}\left(\frac{m_N^2}{Q^2}\right)\right] \ , \quad (1)$$

where $F_3 = (F_3^{\nu p} + F_3^{\nu n})/2$ is the nonsinglet structure function in the νN DIS, m_N is the nucleon mass. Theoretical knowledge about the GLS $\Delta(Q^2)$ is the following. The truncated perturbation series (TPS) up to $\sim a^3$ $[a(Q) \equiv \alpha_s(Q)/\pi]$ is known [4]

$$\Delta(Q^2) \sim a(1 + w_1 a + w_2 a^2) \ , \quad (2)$$

as is also the structure of the first IR renormalon singularity of its Borel transform

$$\mathrm{BT}(b) = \frac{C}{(1-b)^{1+\nu}}\left[1 + \kappa_1(1-b) + \cdots\right] \ . \quad (3)$$

Here $\nu = (\beta_1/\beta_0 - \gamma_2)/\beta_0$, where β_0 and β_1 are the one- and two-loop QCD beta function coefficients and γ_2 is the one-loop coefficient of the anomalous dimension of the twist-four ($d=2$) operator appearing in the Operator Product Expansion (OPE) for $\Delta(Q^2)$. When the number of active quark flavors is $n_f = 3$, we have $\beta_0 = 9/4$, $\beta_1 = 4$, and $\gamma_2 = 8/9$ (the latter value obtained in [5]), giving $\nu = 32/81$. The TPS of $\mathrm{BT}(b)$ in powers of b is known to $\sim b^2$, as a consequence of (2). A consequence of the IR singularity (3) at $b \geq 1$ is that the Borel integral $\Delta_{\mathrm{BI}}(z)$, which for the complex coupling $z = |z|\exp(i\phi)$ is defined as

$$\Delta_{\mathrm{BI}}(z) = \frac{1}{\beta_0}\int_0^{+\infty \exp(i\phi)} db\ \exp\left[-\frac{b}{\beta_0 z}\right]\mathrm{BT}(b) \ , \qquad (b = |b|e^{i\phi}) \ , \quad (4)$$

has a discontinuity (cut) at the positive real axis $z = a(Q) \geq 0$. Namely, for z near the positive real axis, it can be shown from (4) that

$$\Delta_{\rm BI}(z)\big|_{z=a(Q)\pm i\varepsilon} = \frac{1}{\beta_0} \int_{\pm i\varepsilon}^{\infty \pm i\varepsilon} db \ \exp\left[-\frac{b}{\beta_0 a(Q)}\right] {\rm BT}(b) \ . \qquad (5)$$

This quantity is not even real; the discontinuity shows up in its imaginary part

$$\begin{aligned} {\rm Im}\Delta_{\rm BI}(a(Q) \pm i\varepsilon) &= \frac{1}{\beta_0}{\rm Im}\int_{\pm i\varepsilon}^{\infty \pm i\varepsilon} db \exp\left[-\frac{b}{\beta_0 a(Q)}\right] \frac{C}{(1-b)^{1+\nu}}\left[1+\kappa_1(1-b)+\cdots\right] \\ &= \mp C{\beta_0}^{-1-\nu}\left[e^{-1/(\beta_0 a(Q))}\right] a(Q)^{-\nu}\Gamma(-\nu)\sin(\pi\nu)\left[1+\kappa_1\beta_0 a\nu+\cdots\right]. \qquad (6)\end{aligned}$$

Because $(-a \mp i\varepsilon)^{-\nu+n} = a^{-\nu+n}(-1)^n[\cos(\pi\nu) \pm i\sin(\pi\nu)]$, (6) leads to

$$\Delta_{\rm BI}(z) = -\frac{C\Gamma(-\nu)}{\beta_0^{1+\nu}}\left[e^{-1/(\beta_0 a(Q))}\right]\left[(-z)^{-\nu}+\kappa_1(-\nu)\beta_0(-z)^{-\nu+1}+\cdots\right] + \tilde{\Delta}_{\rm BI}(z), \quad (7)$$

where $\tilde{\Delta}_{\rm BI}(z)$ is a function without cuts in the complex z-plane since the first expression on the right-hand side absorbs the renormalon-induced cut at $z \geq 0$. This expression, when $z = a(Q)$ is real, has the Q-dependence $Q^{-2}\alpha_s(Q)^{\gamma_2/\beta_0}[1+\mathcal{O}(\alpha_s)]$, the same as the twist-four term in the OPE for the GLS sum rule [5]. The Im part of this expression, i.e. expression (6), can be identified as the Im contribution from the leading IR renormalon (3). The premise of the method is that the full first expression on the right-hand side of (7), i.e., including its real part, represents the unphysical cut contribution which is to be eliminated. This leads to the final result for the resummed GLS value

$$\Delta(a(Q)) = \tilde{\Delta}_{\rm BI}(a(Q)) = [{\rm Re} \mp \cot(\pi\nu){\rm Im}]\int_{\pm i\varepsilon}^{\infty \pm i\varepsilon} db \exp\left[-\frac{b}{\beta_0 a(Q)}\right]\frac{R(b)}{(1-b)^{1+\nu}}. \quad (8)$$

In the Borel integration here, the exactly known IR renormalon singularity has been factored out explicitly. Function $R(b) = (1-b)^{1+\nu}{\rm BT}(b)$, whose TPS up to $\sim b^2$ is known exactly due to (2), has much weaker singularity at $b = 1$ than ${\rm BT}(b)$. The function $R(b)$ has, in principle, singularities from other renormalons, at $b = -1, \pm 2, \ldots$ To accelerate the convergence in the TPS for R further, we introduce in integral (8) the change of variables $b \mapsto w$ according to the conformal mapping

$$b \mapsto w(b) = \left(\sqrt{1+b} - \sqrt{1-b/2}\right)\left(\sqrt{1+b} + \sqrt{1-b/2}\right)^{-1} \ , \qquad (9)$$

which maps all these renormalon locations onto the unit circle $|w| = 1$, except for the leading IR renormalon $w(b=1) = 1/3$ whose structure has been factored out. Similar applications of conformal mappings have been applied in other methods in Refs. [6, 7]. Further, expansion of $R(b(w))$ in powers of w suggests that the N^{3}LO coefficient (at a^4) in the GLS TPS (2), in $\overline{\rm MS}$ and with renormalization scale $\mu = Q$, is $w_3 = 158 \pm 30$, somewhat higher than the TPS PMS prediction $w_3 \approx 130$ [8]. Using $w_3 = 158 \pm 30$, we use for $R(b(w))$ in (8) the TPS up to $\sim w^4$.

Expression (8) represents the massless part of the GLS sum rule. For $Q^2 \approx$ 2-3 GeV^2, where reasonable experimental data are available, the contributions of the massive c-quarks [9] represent corrections contributing about 3-4% to $\Delta(Q^2)$. Nuclear effects of the iron target [10] contribute about 1% to $\Delta(Q^2)$ $[\Delta_{\rm Fe}(Q^2) \approx 4\cdot10^{-3}{\rm GeV}^2/Q^2]$. The "light-by-light" contribution in the GLS TPS (2) (it is $\sim a^3$) is kept apart, it contributes to $\Delta(Q^2)$ less than 0.5%.

We work in the $\overline{\rm MS}$ renormalization scheme. The renormalization scale μ was fixed according to the principle of minimal sensitivity (PMS), obtaining $\mu^2 \approx 3.3Q^2$.

Experimentally extracted values of GLS $\Delta(Q^2)$ at $Q^2 = 3.16$ GeV^2 are given by the Fermilab CCFR Collaboration [11]

$$\Delta(Q^2\!=\!3.16\text{GeV}^2) = 0.149 \pm 0.039 \ , \tag{10}$$

where the nuclear corrections were subtracted out. When we confront (10) with the theoretical GLS expression [Eq. (8) with the conformal mapping (9); plus c-quark corrections; plus "light-by-light;"], the obtained QCD α_s in $\overline{\text{MS}}$ is

$$\alpha_s(Q) = 0.305^{+0.132}_{-0.074}(\text{exp}) \pm 0.006(\text{th}) \ , \ \alpha_{\text{s}}(\text{M}_\text{Z}) = 0.1167^{+0.0128}_{-0.0118}(\text{exp}) \pm 0.0008(\text{th}). \tag{11}$$

Using the central CCFR data value of $\Delta(Q^2)$ at $Q^2 = 2.00$ GeV^2, we obtain almost the same central value: $\alpha_s(M_Z) = 0.1166$. The values of $\alpha_s(M_Z)$ obtained are in agreement with the present world average. The experimental uncertainties are large. The theoretical uncertainties are dominated by the ambiguity with respect to the change of the renormalization scheme.

The term proportional to $\cot(\pi\nu)$ in (8), which eliminates the unphysical cut, was important numerically as it decreases the value of $\Delta(Q^2)$ by about 10%, thus increasing the extracted value of $\alpha_s(M_Z)$ by about 0.003. The CCFR Collaboration [11], by evaluating the NNLO TPS of $\Delta(Q^2)$, corrected by a term $\propto 1/Q^2$ as estimated by QCD sum rules, obtained from the central values of the data $\alpha_s(M_Z) \approx$ 0.113-0.114. An earlier similar (TPS-PMS) evaluation [12] gave $\alpha_s(M_Z) \approx 0.115$.

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