QCD effects to Bjorken unpolarized sum rule for νN deep-inelastic scattering

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Abstract. The possibility of the first measurement of Bjorken unpolarized sum rule for F_1 structure function of νN deep-inelastic scattering at Neutrino Factories is commented. The brief summary of various theoretical contributions to this sum rule is given. Using the next-to-leading set of parton distributions functions, we simulate the expected Q^2-behavior and emphasize that its measurement can allow to determine the value of the QCD strong coupling constant α_s with reasonable theoretical uncertainty, dominated by the ambiguity in the existing estimates of the twist-4 non-perturbative $1/Q^2$-effect.

The Bjorken unpolarized sum rule for F_1 structure function of νN DIS, which was theoretically derived quite long ago in the classical work of Ref. [1], is still remaining experimentally unchecked. However, at Neutrino Factories, due to large variation of $y = E_{had}/E_\nu \leq 1/(1 + \frac{xM_W}{2E_\nu})$ it might be possible to extract from the data for differential cross-sections data of νN DIS all 6 structure functions, including namely $xF_1^{\nu N}$, $xF_1^{\overline{\nu} N}$.

The direct measurement of $F_1^{\nu N}$ SF is allowing to perform the first experimental determination of the Bjorken unpolarized sum rule, which has the following theoretical expression

$$I_1(Q^2) = \int_0^1 dx \left[F_1^{\nu n}(x,Q^2) - F_1^{\nu p}(x,Q^2)\right] = 1 - \frac{2}{3}\frac{\alpha_s}{\pi} - ... + O(\frac{1}{Q^2}) \quad (1)$$

The Bjorken unpolarized sum rule is related to the Adler isospin sum rule $I_2(Q^2) = \int_0^1 (dx/2x)[F_2^{\nu n}(x,Q^2) - F_2^{\nu p}(x,Q^2)] = 1$ by the following relation $I_1(Q^2) = I_2 + I_L(Q^2)$, where $I_L(Q^2) = \int_0^1 (dx/x)[F_L^{\nu p}(x,Q^2) - F_L^{\nu n}(x,Q^2)]$ is the Callan-Gross relation for νN DIS, which contains perturbative QCD corrections and higher-twist terms, responsible for difference between the Bjorken unpolarized sum rule and the explicit Adler sum rule. In the $\overline{MS}$-scheme the massless perturbative QCD expression for I_1, namely I_1^{PT}, is known analytically up to order $O(\alpha_s^3)$ corrections. The leading order (LO), next-to-leading order (NLO) and next-to-next-to-leading order corrections were subsequently calculated in Refs.[3], [4] and Ref.[5] respectively. The numerical estimates of the order $O(\alpha_s^4)$-contributions are also available [6, 7]. Moreover, in Ref.[8] the heavy-quark mass dependence of I_1 was calculated to the level of α_s^2-terms. More recently, in Ref.[9] the renormalon contributions to Eq.(1) were evaluated. They indicate that the leading non-perturbative contribution to I_1 should be of order $O(1/Q^2)$.

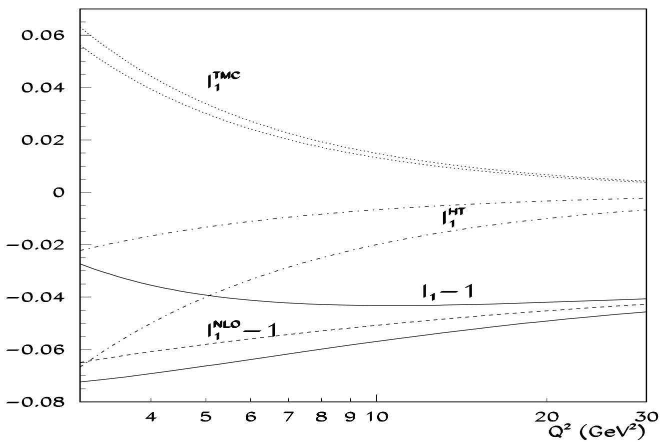

Figure 1. The perturbative contribution (dashes), the target mass correction (dots), and to the high-twist contribution (dashed dots) to $I_1(Q^2)$; full curve give the sum of all these terms.

Indeed, with taking into account the non-perturbative twist-4 contributions, one can write down theoretical expression for I_1 as

$$I_1(Q^2) = I_1^{PT}(Q^2) - \frac{8}{9}\frac{<< O >>}{Q^2} + O(\frac{1}{Q^4}) \tag{2}$$

where $2p_\mu << O >>=< p|O_\mu|p >$, $O_\mu = \overline{u}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5 u - \overline{d}\tilde{G}_{\mu\nu}\gamma_\nu\gamma_5 d$ and $\tilde{G}_{\mu\nu} = (\epsilon_{\mu\nu\alpha\beta}/2)G^a_{\mu\nu}(\lambda^a/2)$ [10]. The numerical value of the twist-4 term was calculated in Ref.[11] with the help of 3-point functions QCD sum rules formalism. The result of this work is $<< O >>= 0.15 \pm 0.07$ GeV^2, where we take for the theoretical error the conservative estimate 50%.

The Q^2-behavior of I_1^{PT} in the NLO is given in Fig.1 together with the high-twist contribution of Eq.(2) and the target mass (TMC) correction term calculated using the parton distributions functions (PDFs) of Ref.[12]. One can see that the main uncertainty in $I_1(Q^2)$ comes from the high-twist contribution, which rises at low Q^2. The theoretical error in the $\alpha_s(Q)$ value determined from the measurement of $I_1(Q^2)$ reads

$$\Delta\alpha_s(M_Z) = \Delta I_1(Q^2)\left[\frac{dI_1(Q^2)}{d\alpha_s(Q^2)}\frac{d\alpha_s(Q^2)}{d\alpha_s(M_Z)}\right]^{-1}, \tag{3}$$

where $\Delta I_1(Q^2)$ is the corresponding theoretical uncertainty in $I_1(Q^2)$. The errors in $\alpha_s(Q^2)$ due to uncertainties in the high-twist term and the TMC are given in Fig.2 (the latter is determined by the PDFs uncertainties). At $Q^2 = 4$ GeV^2 and $Q^2 = 10$ GeV^2 the total theoretical error in α_s is

$$\Delta^{HT}\alpha_s(M_Z) = 0.012 \quad \textit{and} \quad \Delta^{HT}\alpha_s(M_Z) = 0.007 \tag{4}$$

correspondingly.

To conclude, the first experimental determination of the Bjorken unpolarized sum rule, which will be possible at Neutrino Factories, can be considered as an additional source of precise determination of α_s, provided the twist-4 contributions are known

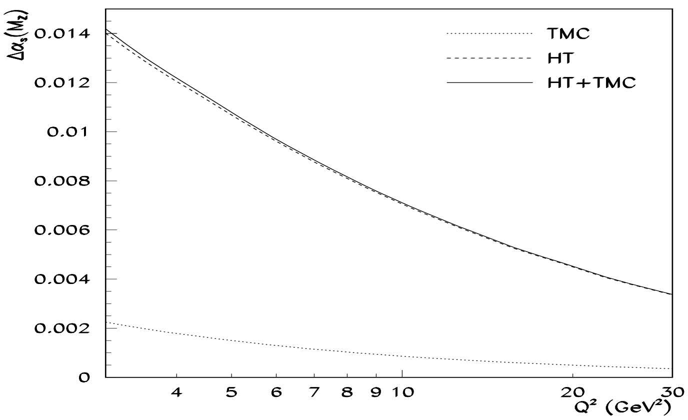

Figure 2. The theoretical errors in α_s extracted from the Bjorken sum rule (dashes: the error due to high-twist contribution; dots: the error due to target mass correction; full curve: combination of both).

with less ambiguities. Moreover, turning the problems around and fixing $\alpha_s(M_Z)$-value in the available perturbative expression for the Bjorken unpolarized sum rule, one can try to extract from the analysis of low Q^2-data of Neutrino Factories the independent estimates of the corresponding twist-4 contributions. Another problem, which can be of interest for the future program of Neutrino Factories, is the study of nuclear corrections to this still unmeasured sum rule of νN DIS (for the discussions see Ref.[2]).

Acknowledgments We are grateful to S.A. Kulagin for discussions. The work was supported by RFBR Grant N 00-02-17432. One of us (ALK) wishes to thank the OC of NuFact'02 Workshop for hospitality in London. His research was also supported by RFBR Grant N 02-01-00601.

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