# Next-to-next-to-leading order fits to CCFR'97 $xF_3$ data and infrared renormalons

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**Abstract.** We briefly summarize the outcomes of our recent improved fits to the experimental data of CCFR collaboration for $xF_3$ structure function of $\nu N$ deep-inelastic scattering at the next-to-next-to-leading order. Special attention is paid to the extraction of $\alpha_s(M_Z)$ and the parameter of the infrared renormalon model for $1/Q^2$-correction at different orders of perturbation theory. The results can be of interest for planning similar studies using possible future data of Neutrino Factories.

The most precise up to date data for $\nu N$ deep-inelastic scattering cross-sections were provided in 1997 by CCFR collaboration [1]. In view of the necessity of understanding of the place of different theoretical effects in the fits to $xF_3$ non-singlet structure function data, interesting from the point of view of possible future theoretical program of Neutrino Factories (for a review see Ref.[2]), it is worth to summarize some basic results of the most recent higher-order perturbative QCD analysis [3], which updates the results of the previous similar studies of Refs.[4],[5]. The analysis is based on the Jacobi polynomial technique of reconstruction of the structure function from its Mellin moments

$$xF_3(x,Q^2) = w(\alpha,\beta)\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)M_{j+2}^{TMC}(Q^2)+\frac{h(x)}{Q^2} \quad (1)$$

where $w(\alpha,\beta)=x^\alpha(1-x)^\beta$, $\Theta_n^{\alpha,\beta}$ are the Jacobi polynomials, $c_j^{(n)}(\alpha,\beta)$ contain $\alpha-$ and $\beta-$ Euler $\Gamma$-functions with the Jacobi polynomial parameters $\alpha=0.7$ and $\beta=3$ fixed following the detailed studies of Ref.[5].

The moments

$$M_n^{TMC}(Q^2) = M_n(Q^2)+\frac{n(n+1)}{(n+2)}\frac{M_{nucl}^2}{Q^2}M_{n+2}(Q^2) \quad (2)$$

contain the leading order target mass correction, while the contribution of higher terms of order $1/Q^4$ do not affect the results of the fits [5]. Here we will fix the dynamical non-perturbative corrections using infrared renormalon (IRR) model as

$$\frac{h(x)}{Q^2} = x^\alpha(1-x)^\beta\sum_{n=0}^{N_{max}}\Theta_n^{\alpha,\beta}\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)M_{j+2}^{IRR}(Q^2) \quad (3)$$

where $M_n^{IRR}(Q^2) = \tilde{C}(n)M_n(Q^2)A_2^{'}/Q^2$, $\tilde{C}(n) = -n+4+2/(n+1)+4(n+2)+4S_1(n)$ was calculated from the single chain of one-fermion loop insertions into the one- gluon contribution to the Born diagram in Ref.[6] and $A_2^{'}$ is the arbitrary fitted parameter.

The Mellin moments obey the following renormalization-group equation

$$\frac{M_n(Q^2)}{M_n(Q_0^2)} = \exp\left[-\int_{\mathrm{A_s(Q_0^2)}}^{\mathrm{A_s(Q^2)}} \frac{\gamma_{\mathrm{F_3}}^{(\mathrm{n})}(\eta)}{\beta(\eta)}\mathrm{d}\eta\right]\frac{\mathrm{C_{F_3}^{(n)}(A_s(Q^2))}}{\mathrm{C_{F_3}^{(n)}(A_s(Q_0^2))}} \tag{4}$$

where $A_s = \alpha_s/(4\pi)$, while $M_n(Q_0^2)$ is the phenomenological quantity, defined at the initial scale $Q_0^2$ as $M_n(Q_0^2) = \int_0^1 x^{n-2}A(Q_0^2)x^b(Q_0^2)(1+\gamma(Q_0^2))dx$.

The main new ingredient of the analysis of Ref.[3] comes from the recent results of analytical calculations of next-to-next-to-leading order (NNLO) correction to $\gamma_{F_3}^{(n)}(A_s)$ at odd $n = 3,5,7,...,13$ [7] (the NNLO corrections to $\beta(x)$ [8] and $C_{F_3}^{(n)}(A_s)$ [9] were already known for quite a long time). The switch from the NNLO non-singlet contributions to $\gamma_{F_2}^{(n)}(A_s)$ at n=2,4,..,10 calculated in Ref.[10], which numerically are closely related to the identical contributions to $\gamma_{F_3}^{(n)}(A_s)$, and were used in the previous NNLO fits of CCFR'97 data of Refs.[4],[5], is making our new analysis more self-consistent. Note, however, that in spite of fixation of the number of previously existing theoretical ambiguities, our new fits are confirming the main results of Ref.[5], putting them on more solid ground.

In order to save space, we will avoid discussions of the number of technical steps of the analysis of Ref.[3] and will concentrate ourselves on the presentation of the main physical results.

| order/$N_{max}$ | $Q_0^2 =$ | 5 Gev$^2$ | 20 GeV$^2$ | 100 GeV$^2$ |
|---|---|---|---|---|
| LO/9 | $\Lambda_{\overline{MS}}^{(4)}$ | 447±54 | 443±54 | 439±56 |
| | $\chi^2$/nep | 79.8/86 | 80.1/86 | 79.6/86 |
| | $A_2^{'}$ | −0.340±0.059 | −0.337±0.059 | −0.335±0.059 |
| NLO/9 | $\Lambda_{\overline{MS}}^{(4)}$ | 379±41 | 376±39 | 374±42 |
| | $\chi^2$/nep | 78.6/86 | 79.5/86 | 79.0 |
| | $A_2^{'}$ | −0.125±0.053 | −0.125±0.053 | −0.124±0.053 |
| NNLO/9 | $\Lambda_{\overline{MS}}^{(4)}$ | 331±33 | 332±35 | 331±35 |
| | $\chi^2$/nep | 73.1/86 | 75.7/86 | 76.9/86 |
| | $A_2^{'}$ | −0.013±0.051 | −0.015±0.051 | −0.016±0.051 |

**Table 1.** The results of the fits to the CCFR'97 $xF_3$ data with $Q^2 > 5\ GeV^2$ with related statistical uncertainties. The cases of different $Q_0^2$ are considered. The values of $\Lambda_{\overline{MS}}^{(4)}$ are given in MeV, while $A_2^{'}$ is "measured" in GeV$^2$.

In Table 1 we present the results of the fits taking into account leading order (LO), next-to-leading order (NLO) and NNLO perturbative QCD corrections. Due to the incorporation of the recently calculated NNLO corrections to $\gamma_{F_3}^{(n)}(\alpha_s)$ [7] into our new NNLO fits of Ref.[3] the NNLO fitted parameters turn out to be rather stable to the variation of $Q_0^2$-scale from 20 GeV$^2$, used in Ref.[5], to the lower initial scale $Q_0^2 = 5$ GeV$^2$. It should be stressed, that the tendency $|A_2^{'}|_{LO} > |A_2^{'}|_{NLO} > |A_2^{'}|_{NNLO}$ demonstrates the minimization of the role of the power corrections for the description of $xF_3$ data. Moreover, at the NNLO the value of the IRR model parameter $A_2^{'}$ is effectively minimized and within statistical uncertainties is comparable with zero. This property was already observed in the process of our previous NNLO analysis of

CCFR'97 data [4, 5] and in the NNLO fits to the combined data of SLAC,NMC and BCDMS collaboration for $F_2$ structure function of deep-inelastic scattering of charged leptons on nucleons [11].In all these fits the QCD coupling constant was closed to its world average value $\alpha_s(M_Z) \approx 0.118$. For example, in our analysis of Ref.[3] we got

$$NLO\ \alpha_s(M_Z) = 0.120 \pm 0.002(stat) \pm 0.005(syst) \\ \pm 0.002(thresh.)^{+0.010}_{-0.006}(scale) \qquad (5)$$

$$NNLO\ \alpha_s(M_Z) = 0.119 \pm 0.002(stat) \pm 0.005(syst) \\ \pm 0.002(thresh.)^{+0.004}_{-0.002}(scale) \qquad (6)$$

The latter result agree, within errors, with the results of the NNLO fits to CCFR'97 data, performed with the help of Bernstein polynomial technique using different theoretical representations (compare the original publication of Ref.[12] with the recent work of Ref.[13]). Note, however, that if one gets the "small" NNLO value of $\alpha_s(M_Z)$, namely $\alpha_s(M_Z) \approx 0.114$, like in the work of Ref.[14], the $1/Q^2$ contributions to $F_2$ remain visible even at the NNLO [14]. Thus, we can conclude, that there is some correlation between twist-4 terms and NNLO perturbative QCD corrections and NNLO value of $\alpha_s(M_Z)$. One can hope, that the possibility to detect higher-twist terms at the NNLO will be clarified in future using drastically more precise data for $F_2$ and $xF_3$ structure functions, which can be obtained at Neutrino Factories in the lower $Q^2$-region.

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