# Matter profile effect in a neutrino factory: Yet another correlation in the analysis on CP violation at a neutrino oscillation experiment

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**Abstract.** We investigate the effect induced by the variation of the density profile of the Earth's interior in a long baseline neutrino oscillation experiment.

We show that there is a strong correlation between the average density value and the first Fourier coefficient of the matter profile function in the analyses on the oscillation probability. We also point out two facts, the most essential part of the matter profile is the first Fourier mode of the matter profile function, the Earth models based on the study of the seismology include large uncertainty for the first Fourier mode. This means that the matter profile effect induces extra uncertainty for average matter parameter.

Taking into account this extra uncertainty, we make numerical calculations for the sensitivity to CP violation search. The results show that the CP sensitivity is impaired in large $U_{e3}$ region.

## 1. Correlation between the average matter and the matter profile

To pick out the most essential part of the effect induced by the matter profile, we separate the matter effect into two parts, the average matter effect, $a_0$, and the remaining part, $\delta a(x)$, which depends on the position $x$. After that, we expand the matter profile part into the Fourier series:

$$\delta a(x) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} a_n e^{-ip_n x}, \qquad p_n \equiv \frac{2\pi}{L} n. \tag{1}$$

We establish the existence of the correlation between the parameter for the average matter density, $a_0$, and the first Fourier coefficient of the matter profile function, $a_1$. In a high energy experiment, only a first few Fourier modes are essential. This fact can be understood by the resonance conditions [1], and this is consistent with the research using different method [2].

Here, we present one example to understand intuitively this correlation. The oscillation probabilities for $\nu_e \to \nu_\mu$ with the different methods to deal with the matter effect are shown in Fig.1, where the baseline length is 7,332 km. This figure shows that the constant profile with the average density following PREM (dotted line) cannot reproduce the behavior of the oscillation probability with the full-PREM profile (solid line). It reveal that if the shift of the constant value is allowed, then a

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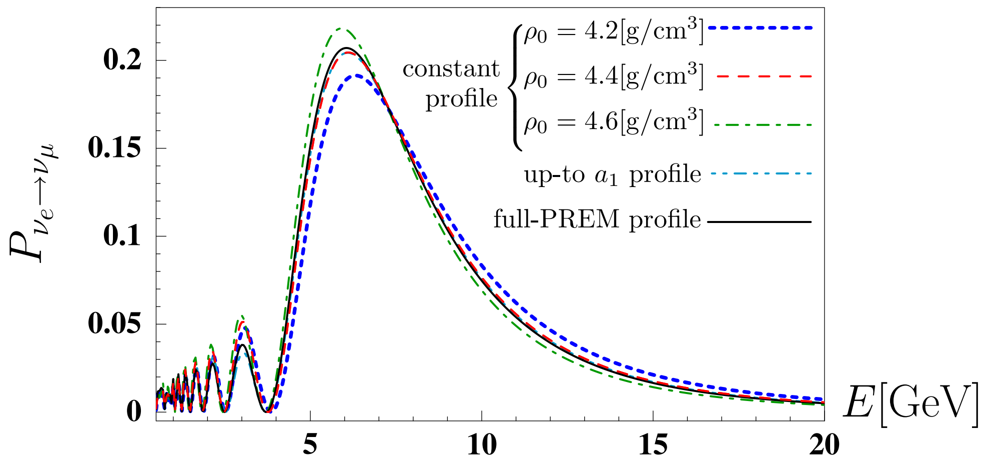


**Figure 1.** Oscillation probability for $\nu_e \to \nu_\mu$ in $L = 7,332$ km. The dotted, dash, dash-dot lines are calculated using the constant matter profile whose values of the matter parameter are 4.2, 4.4, 4.6 g/cm$^3$, respectively. The dash-dot-dot line is adopted the up-to-first mode profile whose values are $\rho_0 = 4.2$ g/cm$^3$ and $\rho_1 = -0.32$ g/cm$^3$. The solid line is used the full-PREM matter profile. We set the oscillation parameters as $\sin\omega = 0.5$, $\sin\psi = 1/\sqrt{2}$, $\sin\phi = 0.1$, $\Delta m^2_{31} = 2.5 \times 10^{-3}\text{eV}^2$, $\Delta m^2_{21} = 5.0 \times 10^{-5}\text{eV}^2$, $\delta = \pi/2$.

good fitting with the full-PREM calculation can be realized at $\rho_0 = 4.4$ g/cm$^3$. This fact means that the shift of the constant matter parameter can pretend the matter profile effect that is almost equal to the effect induced by the first Fourier modes in this baseline length, which can be understood in the comparison of the dash-dot-dot line with solid line.

In Ref.[3], we show the existence of this correlation analytically and conclude that there is a strong correlation between $a_0$ and $a_1$ in the case that the baseline length is less than 7,500 km. We also show that the reasonable error assumed by the seismological Earth models can induce huge, a few hundred percent, uncertainty of $a_1$. This correlation suggests that the uncertainty in $a_1$ gives an extra uncertainty to $a_0$, so the uncertainty of $a_1$ affects the CP sensitivity.

## 2. Source of an extra uncertainty for the matter effect

Figure 2 indicates the lower boundary of $\Delta m^2_{21}$ and $|U_{e3}|$ ($= \sin\phi$) to reject the hypotheses that $\delta$ are 0 and $\pi$ at 99% level of significance when nature takes $\delta = \pi/2$ in the case that the baseline length is 3,000 km and muon energy is 30 GeV. Including the matter profile effect and its uncertainty induces an extra uncertainty to the parameter of the average matter. This fact can be explained by the correlation between $a_0$ and $a_1$. This extra uncertainty gives rise to an extra absorption of the signal of the CP violating effect. We know that the effect of the fake CP signal induced by the matter effect becomes more serious as $|U_{e3}|$ is larger, so if $|U_{e3}|$ will be measured just under the current boundary, we will have to take the effect induced by the density profile of the Earth more seriously. If we can lower the detection threshold the shorter baseline and

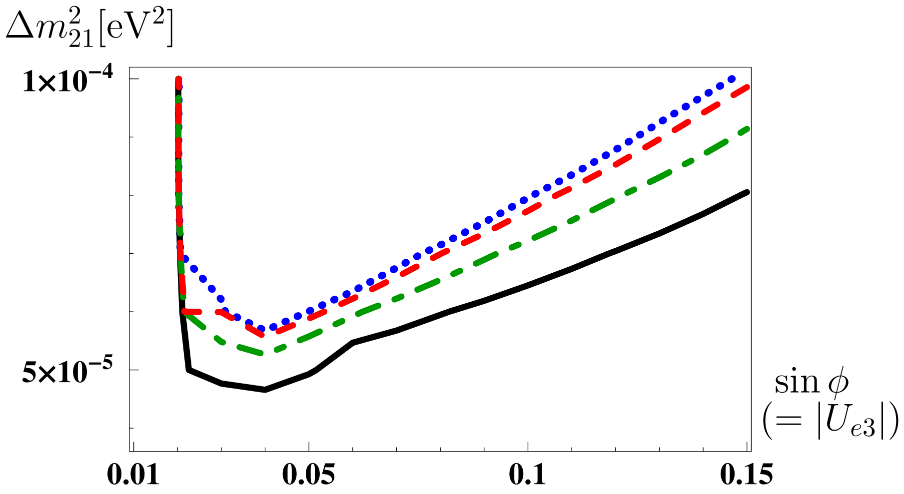


**Figure 2.** Sensitivity reach for CP violation with $\delta = \pi/2$, the baseline length 3,000km, and the muon energy 30 GeV, where we assume $10^{21}$ muon decays and 40 kt detector. The dotted curve is calculated using the up-to-fist-mode profile where the parameter uncertainty, $\Delta(a_0) = 3\%$, $\Delta(a_1) = 200\%$, $\sin\phi = 0 \sim 0.16$, $\Delta(\sin\psi) = 1\%$, $\Delta(\Delta m^2_{31}) = 3\%$, $\Delta(\sin\omega) = 5\%$, and $\Delta(\Delta m^2_{21}) = 5\%$ are assumed. The reference values for the oscillation parameters except $\Delta m^2_{21}$ and $\sin\phi$ are the same as Fig.1. The solid, dash-dot and dash curves are calculated using the constant matter profile whose uncertainties are $\Delta a_0 = 0\%, 3\%$, and 5%, respectively and uncertainty for the other parameters is the same as the dotted line.

the lower energy option can avoid this disadvantage. On the contrary, if small $|U_{e3}|$ will be established, the long baseline and high energy option is more advantageous because of large statistics.

We would like to stress that the uncertainty of the matter effect is not so small when we introduce the matter profile effect. Therefore, we should regard it more seriously whatever the statistics we use, especially in the case that the main part of the path of the neutrino beam is the upper mantle and transition zone (up to 670 km in depth) which include large uncertainty for the density profile. We should deal with the matter effect much more cautiously.

[1] T. Ota and J. Sato Phys. Rev. **D63** (2001) 093004 [arXiv:hep-ph/0011234].
[2] W. Winter, in these proceedings.
[3] T. Ota and J. Sato, in preparation.