Modeling power corrections to the Bjorken sum rule for the neutrino structure function F_1

C. Weiss

Institut für Theoretische Physik, Universität Regensburg,
D–93053 Regensburg, Germany
(christian.weiss@physik.uni-regensburg.de)

Abstract. Direct measurements of the the structure functions $F_1^{\nu p}$ and $F_1^{\nu n}$ at a neutrino factory would allow for an accurate extraction of α_s from the Q^2–dependence of the Bjorken sum rule, complementing that based on the Gross–Llewellyn-Smith sum rule for F_3. We estimate the power $(1/Q^2$–) corrections to the Bjorken sum rule in the instanton vacuum model. For the reduced matrix element of the flavor–nonsinglet twist–4 operator $\bar{u}\, g\widetilde{G}_{\mu\nu}\gamma_\nu\gamma_5 u - (u \rightarrow d)$ we obtain a value of $0.18\,\mathrm{GeV}^2$, in good agreement with the QCD sum rule calculations of Braun and Kolesnichenko. Our result allows to reduce the theoretical error in the determination of α_s.

The precise determination of the strong coupling α_s remains a prime objective of particle physics. One way to measure α_s is through the Q^2–dependence of the Gross–Llewellyn-Smith (GLS) sum rule for the isoscalar neutrino structure function $F_3^{\nu p} + F_3^{\nu n}$ [1]. With the perturbative corrections known exactly up to order α_s^3 [2], scheme and scale ambiguities can be minimized [3], and α_s was extracted from QCD fits to data combined from various experiments (CCFR, CERN, IHEP) [4]. A closely related sum rule is the Bjorken sum rule for the isovector structure function F_1 [5]

$$\int_0^1 dx\ \left[F_1^{\nu n}(x,Q^2) - F_1^{\nu p}(x,Q^2)\right] \;=\; 1 - \frac{2}{3}\frac{\alpha_s(Q^2)}{\pi} + \ldots . \qquad (1)$$

The perturbative corrections have also been computed up to order α_s^3 [6] (the α_s^4 contribution was estimated in Refs.[7]). This sum rule has so far not been tested experimentally. While F_1 is theoretically related to F_2 by the Callan–Gross relation, only recently have experiments begun to extract $F_1^\nu(x,Q^2)$ directly from the cross section. Measurements have been reported by the CHORUS experiment at CERN [8] and the CCFR–NuTeV Collaboration at Fermilab [9]; however, the results cover only a limited range in x. High–statistics experiments at a neutrino factory would allow to separate the various components of the cross section, including $F_1(x,Q^2)$. This would offer the possibility of using the Bjorken sum rule (1) for an independent accurate extraction of α_s, complementing that from the GLS sum rule [10].

An important issue in the determination of α_s from QCD fits to both the GLS and the Bjorken sum rule are power $(1/Q^2$–) corrections. Ideally, one would determine the size of these corrections phenomenologically, from the fit to the data. However, correlations of α_s with the coefficient of the $1/Q^2$ correction increase with the order of the perturbative expansion, as a result of which the accuracy of the extracted α_s does not improve with increasing order [3]. A more promising approach is to rely on "advance knowledge" of the size of the power correction from theoretical estimates.

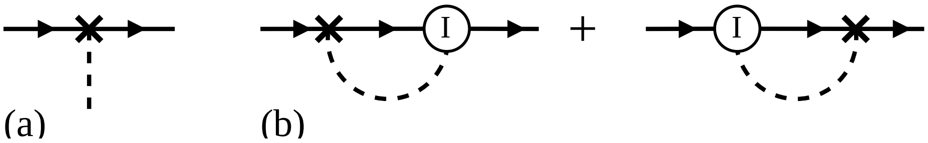

Figure 1. (a) The twist–4 quark–gluon operator, Eq.(2). (b) Quark–gluon correlations induced by single instantons in the instanton vacuum model. The dashed line denotes the gauge field of the instanton, the circle the chirality–flipping fermion vertex induced by the zero mode of the instanton. (Shown is the situation for a single light quark flavor ($N_f = 1$); in general the instanton vertex is a $2N_f$–point fermion vertex.)

Aside from the known target mass corrections the $1/Q^2$ corrections to the GLS and Bjorken sum rules are due to non-perturbative quark–gluon correlations in the nucleon. The coefficients of the $1/Q^2$–corrections are given by $-(8/9)\langle\langle O_{\rm S(NS)}\rangle\rangle$, respectively, where $\langle\langle O\rangle\rangle$ is the reduced proton matrix elements of the twist–4 operator [11]

$$O_\alpha \;=\; \bar{q}\, g\widetilde{G}_{\alpha\beta}\,\gamma_\beta\gamma_5\, q, \qquad \langle p|O_\alpha|p\rangle \;=\; 2p_\alpha\,\langle\langle O\rangle\rangle, \tag{2}$$

and S and NS denote the flavor–singlet and nonsinglet combinations $\bar{u}\ldots u \pm \bar{d}\ldots d$. Here $\widetilde{G}_{\alpha\beta} \equiv \epsilon_{\alpha\beta\gamma\delta}G_{\gamma\delta}/2$ is the dual gluon field strength tensor.

The matrix elements (2) were estimated by Braun and Kolesnichenko using QCD sum rules [12], see Table 1. An alternative approach is based on the picture of the QCD vacuum as a "medium" of instantons — topological fluctuations of the gauge fields. This picture explains the dynamical breaking of chiral symmetry in QCD and a host of phenomenological data on hadronic correlation functions [13, 14], and is supported by lattice simulations. Our estimate of higher–twist matrix elements is based on the analytic approach to the instanton vacuum by Diakonov and Petrov [14]. The average size of the instantons in the vacuum is $\bar{\rho} \approx 0.3\,\mathrm{fm}$, while their average distance is $\bar{R} \approx 1\,\mathrm{fm}$. The fact that the instanton medium is dilute, $\bar{\rho}/\bar{R} \approx 1/3 \ll 1$, is of crucial importance for this picture. It allows for a systematic classification of non-perturbative effects generated by instantons. In leading order of $\bar{\rho}/\bar{R}$ quark–gluon correlations as measured by the operator (2) are induced by *single* instantons, see Fig. 1. By coupling to the instanton the quark–gluon QCD operator (2) (normalized at the scale $\mu \sim \bar{\rho}^{-1} = 600\,\mathrm{MeV}$) turns into a chirality–flipping "effective quark operator", whose matrix element is to be evaluated in the low–energy effective theory derived from the instanton vacuum, characterized by a dynamical mass of the quarks and the appearance of Goldstone boson modes, the pions (for details see Refs.[15, 16]). This effective theory has extensively been tested and shown to reproduce the "chiral phenomenology" of strong interactions at low energies. In particular, it describes the nucleon as a "soliton" of the pion field in the large–N_c limit [17].

The flavor–singlet nucleon matrix element was estimated in the instanton vacuum model in Ref. [16], see Table 1. Here we report about an estimate of the flavor–nonsinglet matrix element, which determines the $1/Q^2$ corrections to the Bjorken sum rule. The "effective quark operator" obtained from the single–instanton contribution of Fig. 1 has the quantum numbers of the isovector vector current of the effective low–energy theory, and the twist–4 nucleon matrix element is proportional to the nucleon vector charge, with a coefficient of the order of the square of the inverse instanton size, $\bar{\rho}^{-2}$ (details will be given elsewhere). We find a numerical value of $\langle\langle O_{\rm NS}\rangle\rangle = 0.5\,\bar{\rho}^{-2} = 0.18\,\mathrm{GeV}^2$. The instanton vacuum results for both the flavor–

	$\langle\langle O_{\mathrm{S}}\rangle\rangle/\mathrm{GeV}^2$	$\langle\langle O_{\mathrm{NS}}\rangle\rangle/\mathrm{GeV}^2$
QCD sum rules [12]	0.33	0.15
Instanton vacuum [16]	0.32	0.18

Table 1. Theoretical estimates of the twist–4 matrix element (2), determining $1/Q^2$ corrections to the GLS (S) and Bjorken (NS) sum rules.

singlet and nonsinglet matrix elements are in good agreement with the QCD sum rule predictions of Ref.[12], which is very encouraging, given the general difficulties with modeling higher–twist matrix elements. The accuracy of the QCD sum rule results was estimated at about $\pm 30\%$ [12]. The theoretical error of the instanton predictions is difficult to quantify; we expect it to be not larger than $\pm 50\%$.

A recent simulation of the Q^2–dependence of the Bjorken sum rule incorporating twist–4 corrections as estimated from QCD sum rules [12] found the theoretical error of the extracted α_s to be dominated by the uncertainty of the twist–4 matrix element [18]. Our result indicates that one can be more confident about the magnitude of this matrix element. This makes the idea of an accurate measurement of this sum rule at a neutrino factory even more attractive [10]. Note also that analyses of power corrections to $x(F_3^{\nu N} + F_3^{\bar{\nu} N})$ within the infrared renormalon model, based on the IHEP and CCFR data [20], suggest a large negative twist–4 contribution to the GLS sum rule [21], which would be consistent with the theoretical estimates of $\langle\langle O_{\mathrm{S}}\rangle\rangle$ quoted in Table 1. Finally, the instanton vacuum can be used to model also power corrections to polarized electron/muon structure functions [16, 19]. First results of a comparison of the instanton predictions with the higher–twist contribution to g_1 extracted from QCD fits are very encouraging [22].

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