Breaking 8–Fold Neutrino Parameter Degeneracies

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Abstract.

Parameter degeneracies are a generic problem in the three–neutrino analysis of long–baseline neutrino appearance measurements. Degeneracies can lead to different inferred values for the neutrino mixing angle θ_{13}, and they often mix CP violating and CP conserving solutions. Appropriate experimental strategies can reduce or eliminate degeneracies and/or the CPV/CPC confusion.

1. Introduction

With the strong evidence for neutrino oscillations from solar [1] and atmospheric [2] neutrino experiments, the focus is now changing from establishing the existence of neutrino oscillations to obtaining a complete understanding of the parameters in the neutrino mass matrix. Solar neutrino data suggest $|\delta m^2_{31}| \sim 3\times10^{-3}$ eV^2 and $\sin^2 2\theta_{23} \simeq 1.0$ (in the standard three–neutrino description), while atmospheric neutrino data provide give $\delta m^2_{21} \sim 5\times10^{-5}$ eV^2 and $\sin^2 2\theta_{12} \simeq 0.8$. Reactor experiments [3] put an upper bound on $\sin^2 2\theta_{13}$ of about 0.1. In the future, KamLAND [4] will likely provide a more precise measurement of the solar neutrino parameters and $\nu_\mu \to \nu_\mu$ survival in long–baseline experiments will provide a more precise measurement of the atmospheric neutrino parameters (to $\leq 10\%$).

The heretofore unknown neutrino parameters are the CP phase δ that can appear in three–neutrino oscillations, the mixing angle θ_{13}, and the sign of δm^2_{31}, which determines the ordering of the neutrino mass eigenstates. In this talk I discuss the difficulties in making a unique determination of these neutrino parameters, and how experiments might be designed to overcome them.

2. Neutrino parameter degeneracies

The standard method for determining the parameters θ_{13} and δ is to measure the rates for $\nu_\mu \to \nu_e$ and $\bar\nu_\mu \to \bar\nu_e$ in a long–baseline neutrino experiment. For our analysis we use approximate expressions for the the oscillation probabilities similar to those derived in Ref. [5]. For $\delta m^2_{31} > 0$

$$P(\nu_\mu \to \nu_e) = Ax^2 + Bxy(\cos\delta\cos\Delta - \sin\delta\sin\Delta) + Cy^2\,, \tag{1}$$

$$\bar P(\bar\nu_\mu \to \bar\nu_e) = \bar A x^2 + \bar B xy(\cos\delta\cos\Delta + \sin\delta\sin\Delta) + Cy^2\,, \tag{2}$$

where $x \equiv \sin\theta_{23}\sin 2\theta_{13}$, $y \equiv |\delta m^2_{21}/\delta m^2_{31}|\cos\theta_{23}\sin 2\theta_{12}$, and $\Delta = 1.27|\delta m^2_{31}/\mathrm{eV}^2|(L/\mathrm{km})/(E_\nu/\mathrm{GeV})$ is the oscillation argument. The coefficients A, B, C, etc., depend on $|\delta m^2_{31}|$, Δ, and the density of matter. Matter effects lead to a suppression of $\bar A$ ($\bar B$) relative to A (B). The probabilities for $\delta m^2_{31} < 0$ are given by making the exchanges $A \leftrightarrow \bar{A}$, $B \leftrightarrow -\bar{B}$, and $\delta \to -\delta$. More details, including a discussion of their range of validity, are given in Ref. [6]. These approximations have a similar form to those for low energy [7] and the exact expressions for constant density [8].

The standard method for determining θ_{13} and δ is to measure P and $\bar{P}$ at one L and E_ν. If θ_{13} is held fixed and δ is varied from 0 to 2π, an ellipse is traced out in the $(P, \bar{P})$ plane [9] (it is assumed that the other oscillation parameters are well–measured from other experiments, as mentioned above). The measured values of P and $\bar{P}$ then in principle should determine θ_{13} and δ. However, there are three possible two–fold degeneracies that can occur:[6]

(i) The ellipses for two different values of θ_{13} can overlap, so that the same P and $\bar{P}$ are described by two different sets of parameters (δ, θ_{13}) and (δ', θ'_{13}) (the (δ, θ_{13}) ambiguity). The values of θ_{13} and θ'_{13} can be very different, and if δ is CP violating, then δ' can be CP conserving (CPV/CPC confusion). In some cases solutions with maximal CP violation can give the same predictions for P and $\bar{P}$ as a solution with CP conserved.

(ii) Ellipses for $\delta m^2_{31} > 0$ are shifted towards the P axis, while those for $\delta m^2_{31} < 0$ are shifted towards the $\bar{P}$ axis, due to different matter effects. These ellipses can overlap, so that for many values of P and $\bar{P}$, the parameters (δ, θ_{13}) for $\delta m^2_{31} > 0$ and a different set of parameters (δ', θ'_{13}) for $\delta m^2_{31} < 0$ can both reproduce the data (the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity). Here θ_{13} and θ'_{13} are not as far apart as they can be for the (δ, θ_{13}) ambiguity, but a large CPV/CPC confusion is possible.

(iii) Since only $\sin^2 2\theta_{23}$ is measured in atmospheric and $\nu_\mu \to \nu_\mu$ experiments, there are two possible values for θ_{23}, one with $\theta_{23} < \pi/4$ and one with $\theta_{23} > \pi/4$ [10]. The ellipses for these two solutions can overlap, once again with different solutions for θ_{13} and δ (the $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguity). As with the (δ, θ_{13}) ambiguity, the uncertainties in θ_{13} can be large and a large CPV/CPC confusion is also possible. This ambiguity vanishes for $\theta_{23} = \pi/4$, the value preferred by atmospheric data.

Each of these two–fold ambiguities may exist for a given data set, so that it is possible to have an eight–fold ambiguity in some cases, i.e., eight distinct sets of the parameters (δ, θ_{13}) that describe the data.

3. Strategies for breaking parameter degeneracies

Appropriate experimental strategies may be used to either reduce or eliminate the effects of parameter degeneracies. The simplest is to choose your baseline so that the $\cos\delta$ terms in the oscillation probabilities vanish. For a narrow spectrum this can be achieved by choosing $L/\langle E_\nu \rangle$ (where $\langle E_\nu \rangle$ is the average energy) such that $\Delta = (n - 1/2)\pi$, at the peak of the oscillation in vacuum (which also is near the peak after matter is taken into account). Then the ellipses in $(P, \bar{P})$ space collapse to lines, and the line for one value of θ_{13} no longer overlaps the lines for other θ_{13} (for the same $\mathrm{sgn}(\delta m^2_{31})$ and θ_{23}), thereby removing θ_{13} from the (δ, θ_{13}) ambiguity. Since only $\sin\delta$ is being measured, there is a residual $(\delta, \pi - \delta)$ ambiguity; however, CPV and CPC solutions are no longer mixed for the same $\mathrm{sgn}(\delta m^2_{31})$ and θ_{23}. There still can be large CPV/CPC confusion due to the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity, unless $\delta \simeq 3\pi/2$ $(\pi/2)$ for $\delta m^2_{31} > 0$ (< 0), in which case the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity is avoided. On a peak the CPV/CPC confusion due to the $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguity is small, although the uncertainty in $\sin^2 2\theta_{13}$ can be as much as a factor of two for $\sin^2 2\theta_{23} = 0.90$ [6].

Another tactic is to go to longer baseline L to increase the matter effect, thereby splitting apart the predictions for $\delta m^2_{31} > 0$ and $\delta m^2_{31} < 0$. We have found that predictions for the two signs of δm^2_{31} do not overlap for $\sin^2 2\theta_{13} > 0.01(0.005)[0.002]$ for $L = 1290(1770)[2900]$ km, assuming the values of the solar and atmospheric parameters given above [6]. We have also found that the first peak of the oscillation (i.e., $\Delta = \pi/2$) is best for resolving the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity, since higher peaks tend to have more overlap between $\delta m^2_{31} > 0$ and $\delta m^2_{31} < 0$ [6].

Currently there is an experiment planned [12] to go from the Japan Hadron Facility to the Super–Kamiokande detector ($L = 295$ km and $\langle E_\nu \rangle \simeq 0.7$ GeV). This lies approximately on the first peak of the oscillation, but the L is not long enough to resolve the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity except for extreme values of δ. Therefore it is interesting to know what additional measurements are needed to resolve the remaining $(\delta, \pi - \delta)$, $\mathrm{sgn}(\delta m^2_{31})$ and $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguities.

In general, for two measurements, ambiguities occur across wide areas of the (δ, θ_{13}) plane. We have shown [11] that, in principle, (i) a third measurement can resolve the $(\delta, \pi - \delta)$ ambiguity, and the remaining ambiguities occur only along lines in the (δ, θ_{13}) plane, (ii) a fourth measurement may still have $\mathrm{sgn}(\delta m^2_{31})$ and $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguities, but they only occur at isolated points in the (δ, θ_{13}) plane, and (iii) a fifth measurement can remove all of the ambiguities. All of the degeneracies do not necessarily have to be completely removed in a given experiment, since they are only a problem if they occur in the region of parameter space accessible to the experiment. We have found that making the measurements at longer L pushes the degeneracies to lower values of θ_{13}; furthermore, making measurements at different L tends to push the degeneracies to lower θ_{13} than if all of the measurements are at the same L [11]. For example, ignoring experimental uncertainties, the degeneracies for two ν and two $\bar{\nu}$ measurements at the same L lie below $\sin^2 2\theta_{13} = 0.01(0.002)$ for $L = 300(2900)$ km; with only three measurements at different L, they can be pushed to $\sin^2 = 0.001$ or lower. For comparison, the sensitivity of the SuperJHF to Hyper–Kamiokande experiment [12] is expected to be around $\sin^2 2\theta_{13} = 0.002$.

In summary, sitting on an oscillation peak reduces the (δ, θ_{13}) ambiguity to a $(\delta, \pi - \delta)$ ambiguity with no CPV/CPC confusion, which a third measurement can then resolve. Going to long enough L can resolve the $\mathrm{sgn}(\delta m^2_{31})$ ambiguity, and perhaps the $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguity, especially if more than two measurements are made. Even if the $(\theta_{23}, \pi/2 - \theta_{23})$ ambiguity cannot be resolved, the resulting uncertainty in δ is small, although θ_{13} will not be as well determined.

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