The effects of matter density uncertainties on neutrino oscillations in the Earth

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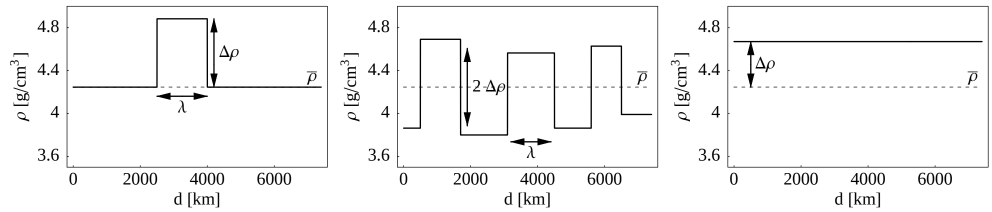
Abstract. We compare three different methods to evaluate uncertainties in the Earth's matter density profile, which are relevant to long baseline experiments, such as neutrino factories.

It is generally believed that neutrino oscillations are influenced by the presence of matter [1, 2, 3]. Especially, for very long baseline experiments, such as often proposed for neutrino factories, matter effects can be quite substantial. The knowledge on the matter density profile along a certain baseline through the Earth's mantle is limited to about 5% precision (for a summary, see, e.g., Ref. [4]). Thus, matter density uncertainties can affect the measurement of the neutrino oscillation parameters, such as the CP phase δ_{CP} [5]. There have been several approaches to model the matter density uncertainties and check their influence on neutrino oscillations [5, 6, 7, 8]. In this talk, we especially focus on three different techniques and show their advantages and disadvantages where applicable. Based on these techniques, we will finally comment on the relevance of matter density uncertainties compared to other problems. Figure 1 shows the models for matter density uncertainties used in this presentation.

Figure 1. The different models for matter density uncertainties used in this presentation for a baseline length of 7400 km: a single perturbation (left), random fluctuations (middle), and the mean density as measured quantity (right).



The left-hand plot illustrates a single perturbation of the average matter density, such as it could come from a mine, mountain, or lake. The middle plot shows random fluctuations, which can be used to model realistic fluctuations known from geophysics.

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The right-hand plot illustrates the technique of measuring the average matter density, i.e., the density difference $\Delta\rho$ between the average matter density from the PREM profile [9] and the actual average density is measured as an additional quantity by the neutrino oscillation experiment.

As indicated in Fig. 1 (left), a single perturbation could be modeled as a bump with a length scale λ and an amplitude $\Delta\rho$. A general result is that the relative error in the appearance probability of a neutrino factory is smaller than about 1% for applications such as coal mines and lakes with a length scale $\lambda \sim 10\,\mathrm{km}$ and a density contrast $\Delta\rho/\rho \lesssim 100\%$ [6] (for the oscillation parameters, we choose, if not otherwise stated, the LMA solution, $L = 7400\,\mathrm{km}$, and $E = 30\,\mathrm{GeV}$). Analytically, one can show with perturbation theory that the change in the probabilities is proportional to the product $\lambda\,\Delta\rho$ [6].

In geophysics, uncertainties in the matter density profile up to 5 % have been documented. Unfortunately, there is no general agreement among the geophysics results. However, one can easily observe common characteristics of measurements in certain depths (see, e.g., Ref. [4]), such as the length scales and amplitudes of the fluctuations. With the random density fluctuations method we model the matter density uncertainties based on their characteristics and average over a large number of random matter density profiles in order to estimate the average effects [6]. As indicated in Fig. 1 (middle plot), we introduce two different parameters, which are again the length scale λ and the amplitude $\Delta\rho$. These we allow to vary around their average values with Gaussian distributions with standard deviations σ_λ and $\sigma_{\Delta\rho}$, respectively. From geophysics maps, we can estimate the parameter values: $\lambda \sim 2000\,\mathrm{km}$, $\sigma_\lambda \sim 1500\,\mathrm{km}$, $\Delta\rho \sim 3\%\,\bar{\rho}$, and $\sigma_{\Delta\rho} \sim 1\%\,\bar{\rho}$. As the most interesting result, we find that the errors in the appearance probabilities qualitatively behave as the ones for a single perturbation, i.e., they are essentially proportional to the product of the length scale and the amplitude. However, because of averaging effects, they are suppressed by a factor of about two to three. Note that they are not necessarily completely averaging out because of interference effects: It is easy to show that the quantum mechanical evolution operators in different density layers do not commute. Analytically, one can again show with perturbation theory that fast fluctuations on length scales much shorter than the oscillation length in matter average out for limited amplitudes [10].

A different approach is to assume the mean density to be known with about $\pm 5\%$ precision and to measure it, within these limits, together with the other neutrino oscillation parameters by the experiment (cf., Fig. 1, right plot). This method can be applied to a full statistical neutrino factory analysis, such as it is done in Ref. [8] for an initial and advanced stage neutrino factory, respectively, at a baseline length of 3000 km (0.75 MW and 4 MW target power, 5 yr and 8 yr running time, 10 kt and 50 kt detector mass, respectively). However, it should be noted that this method in many cases only allows a quite conservative estimate, since it does not take into account averaging effects among more than one density layer. As the main result of this method, the effects of the matter density uncertainties are most important for the measurement of the CP phase at the advanced stage at a neutrino factory. For the measurement of $\sin^2 2\theta_{13}$, many other experimental issues, such as the efficiencies at low energies, are of comparable magnitude.

Let us summarize the individual approaches. The single perturbation method can be evaluated analytically on the level of oscillation probabilities. The relative errors are proportional to the product $\lambda\,\Delta\rho$ and no averaging effects enter this result. The errors are small for lakes, mountains, mines, etc., and the method has only limited applications. The random fluctuations approach can be used numerically on the level of neutrino oscillation probabilities. The errors on the neutrino oscillation probabilities are qualitatively proportional to the product of λ and $\Delta\rho$. Since there are averaging effects, they are, however, suppressed by a factor of about two or three compared to the single perturbation. For large enough structures, such as for tectonic plates or realistic fluctuations in the Earth's mantle, the impact on the results can be much larger than the one of a single perturbation. One problem is the high computational effort, which means that it is hard to apply this method in a complete statistical analysis. However, in the limit of large enough $\lambda \sim L$, the next method provides a good approximation: The mean density as measured quantity approach can be used in a complete statistical analysis, which needs to be performed numerically. The errors on the neutrino oscillation probabilities can then be directly translated into the errors on the quantities to be measured by the experiment. Especially, a CP phase measurement can be substantially affected by matter density uncertainties. Averaging and interference effects of matter density fluctuations are, however, not directly modeled by this method. In this context, assuming a 5% error on the average matter density should be a rather conservative estimate, since averaging effects are completely neglected.

We have seen that there can be quite substantial effects of matter density uncertainties in a complete statistical analysis. However, we have demonstrated that these are partially reduced by averaging in a more realistic random fluctuations model. Thus, we may expect that the effects in a complete analysis are smaller than for the conservative estimate of the measured average matter density. In addition, the information on the matter density profile along a specific baseline from geophysics should be much better than assumed here, since at least a part of the terrain in lower depths should be well-known and the existing information could be combined for higher depths. Furthermore, matter density uncertainties are only relevant for a very advanced neutrino factory experiment to be built at a time when geophysics research has also been advancing. We conclude that matter density uncertainties will probably not be the bottleneck of the statistical analysis of a planned experiment, though somewhat more effort should be spent on improving the results from geophysics.

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