# $T$-violation tests for relativity principles

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**Abstract.** We consider the implications of a violation of the equivalence principle or of Lorentz invariance in the neutrino sector for the $T$-asymmetry $\Delta P_T \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$ in a three-flavour framework. We find that additional mixing due to these mechanisms, while obeying all present bounds, can lead to a substantial enhancement, suppression, and/or sign change in $\Delta P_T$ for the preferred energies and baselines of a neutrino factory. This in turn allows for the possibility of improving existing constraints by several orders of magnitude.

## 1. Introduction

The phenomenon of flavour oscillations follows from non-degenerate neutrino states and mixing. The "standard" mechanism stipulates that the degeneracy be broken by means of small neutrino masses. An alternative scenario that does not require neutrino masses is provided by a violation of the equivalence principle (VEP) in the neutrino sector [1, 2].

Phenomenologically, the standard and the VEP mechanisms differ in their energy dependences. In the two-flavour case, a formal transformation from the former to the latter scenario is accomplished by replacing in the oscillation probability $\delta m^2/2E \to 2E|\phi|\delta\gamma$, where $\delta\gamma \equiv \gamma_2 - \gamma_1$ is the difference in the neutrino–gravity couplings, and $\phi$ is the gravitational potential. Note that a violation of Lorentz invariance also contributes to lifting the neutrino degeneracy [3]. This scenario, however, has the same phenomenology as VEP [4].

The purpose of this work is not to promote VEP as a solution to the solar, atmospheric, and LSND neutrino puzzles—this has been shown by many to be futile (see [5] for a review), and the VEP breaking scale is severely constrained by null oscillation experiments, $|\phi|\delta\gamma < 10^{-22}$ [6]. Rather, we examine the possibility of further constraining $|\phi|\delta\gamma$ with a new generation of experiments, namely, the measurement of the $T$-asymmetry $\Delta P_T \equiv P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$ at a neutrino factory.

## 2. $T$-violation

Consider three-flavour oscillations governed by the Hamiltonian (in flavour basis)

$$\widetilde{H} = \frac{1}{2E} U_M M U_M^\dagger + 2E\ U_G G U_G^\dagger + V \equiv H^M + H^G + V, \tag{1}$$

where $M = \mathrm{Diag}(-\delta m^2_\odot, 0, \delta m^2_\oplus)$ is the mass term, $G = |\phi| \times \mathrm{Diag}(\gamma_1, \gamma_2, \gamma_3)$ comes from VEP, $V = \sqrt{2}\ G_F N_e \times \mathrm{Diag}(1, 0, 0)$ represents matter effects, $U_M$ is the MNS matrix

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with three angles and a $CP$ phase $\delta$, and $U_G$ describes mixing in the VEP sector. We assume the solar and atmospheric neutrino anomalies to be completely explained by the square mass differences $\delta m^2_\odot$ and $\delta m^2_\oplus$, together with the angles $\theta_\odot$ and $\theta_\oplus$ in $U_M$.

The asymmetry between the $T$-conjugate oscillation probabilities is given by

$$\Delta\widetilde{P}_T = 16\widetilde{J}\ \sin\frac{\widetilde{\Delta}_{12}L}{2}\ \sin\frac{\widetilde{\Delta}_{23}L}{2}\ \sin\frac{\widetilde{\Delta}_{31}L}{2}, \tag{2}$$

with $\widetilde{\Delta}_{ij} = \widetilde{\lambda}_i - \widetilde{\lambda}_j$, where $\widetilde{\lambda}_{1,2,3}$ are the eigenvalues of $\widetilde{H}$, and $\widetilde{J}$ is the effective Jarlskog factor. We wish to compare $\Delta\widetilde{P}_T$ with the intrinsic $T$-asymmetry $\Delta P_T$ due to $\delta$. Using the identity $\Delta_{12}\Delta_{23}\Delta_{31}J = \mathrm{Im}(H_{e\mu}H_{\mu\tau}H_{\tau e})$ [7, 8], and by restricting the neutrino energy and the experimental baseline to $\mathcal{O}(10)\to\mathcal{O}(30)$ GeV and $\mathcal{O}(3000)\to\mathcal{O}(6000)$ km respectively, we find the ratio $\Delta\widetilde{P}_T/\Delta P_T$ to be well approximated by

$$\frac{\Delta\widetilde{P}_T}{\Delta P_T} \simeq \frac{\mathrm{Im}(\widetilde{H}_{e\mu}\widetilde{H}_{\mu\tau}\widetilde{H}_{\tau e})}{\mathrm{Im}(H_{e\mu}H_{\mu\tau}H_{\tau e})}, \tag{3}$$

where $H \equiv H^M + V$ is the Hamiltonian (1) without VEP contribution, assuming that $|\phi|\delta\gamma$ satisfies the condition $2E|\phi\delta\gamma| < |\delta m^2_\oplus|/2E$.

## 3. An example

The exact consequence of VEP on the $T$-asymmetry depends on the nature of $H^G$. We provide here the simplest example of pure $\nu_e \leftrightarrow \nu_\tau$ mixing in the VEP sector, with VEP breaking scale $|\phi|\delta\gamma$ and mixing angle $\varphi$. Equation (3) can now be written as

$$\frac{\Delta\widetilde{P}_T}{\Delta P_T} \simeq 1 - \frac{4E^2}{\delta m^2_\odot}\frac{\sin 2\theta_\oplus}{\sin 2\theta_\odot}|\phi|\delta\gamma\sin 2\varphi. \tag{4}$$

Using the best fit values $\delta m^2_\odot \simeq 5\times 10^{-5}$ eV$^2$, $\sin 2\theta_\odot \simeq 0.8$, and $\sin 2\theta_\oplus \simeq 1$, we see that at $E = 13$ GeV, the VEP parameters $|\phi|\delta\gamma\sin 2\varphi = \pm 10^{-25}$ can offset the $T$-asymmetry by $\mp 100$ %, i.e., $\Delta P_T$ is enhanced or suppressed depending on the sign of $|\phi|\delta\gamma\sin 2\varphi$. Furthermore, if the offset exceeds 100 %—because of a large VEP breaking scale and/or a high neutrino energy, $\Delta P_T$ flips sign.

## 4. Conclusion

Extra neutrino mixing from VEP can enhance, suppress, and/or flip the sign of the intrinsic asymmetry between two $T$-conjugate oscillation processes, depending on the VEP breaking scale and the neutrino energy. Thus $T$-violation experiments run at two or more energies will allow us to either establish, or further constrain VEP and improve present bounds on $|\phi|\delta\gamma$ by at least two orders of magnitude.

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