Matter profile effect in a neutrino factory

correlation between the constant matter parameter and the matter profile effect

Toshihiko Ota
and
Joe Sato
Dept. of Phys. Kyushu Univ.

Contents:

1. Introduction: Summary of our former work
   - What is the matter profile effect?
   - How large?
   - When does the effect become significant?

2. **Motivation of this work: New issue**
   - Correlation between a_0 and a_1
   - ▼
   - New source of the uncertainty of a_0.

3. **Examples:**
   - 3-1: $L = 7,000$km
   - 3-2: $L = 3,000$km

   preliminary

4. Conclusions:

■Introduction: Summary of our former work

■We investigated the matter density profile effect.
(Phys. ReV. **D63** (2001) 093004)

▶We showed the conditions to enlarge the matter profile effect, analytically.

Hamiltonian in flavor base: 2-generation

$$H = \frac{1}{2E_\nu}\left\{U\begin{pmatrix}0 & \\ & \Delta m^2\end{pmatrix}U^\dagger + \begin{pmatrix}a_0+\delta a(x) & \\ & 0\end{pmatrix}\right\}$$

$$= \frac{1}{2E_\nu}\left\{\tilde{U}_0\begin{pmatrix}\lambda_- & \\ & \lambda_+\end{pmatrix}\tilde{U}_0^\dagger + \begin{pmatrix}\delta a(x) & \\ & 0\end{pmatrix}\right\}$$

*mass-square eigenvalues in const.(a_0) matter

$$\delta a(x) = \sum_{n=-\infty}^{\infty} a_n e^{-ip_n x}, \quad p_n \equiv \frac{2n\pi}{L}$$

(i) Fourier coefficients are large.

(ii)The resonance conditions are satisfied.

$$\lambda_+ - \lambda_- = 2p_n E_\nu$$

▶ Only first few modes are important.

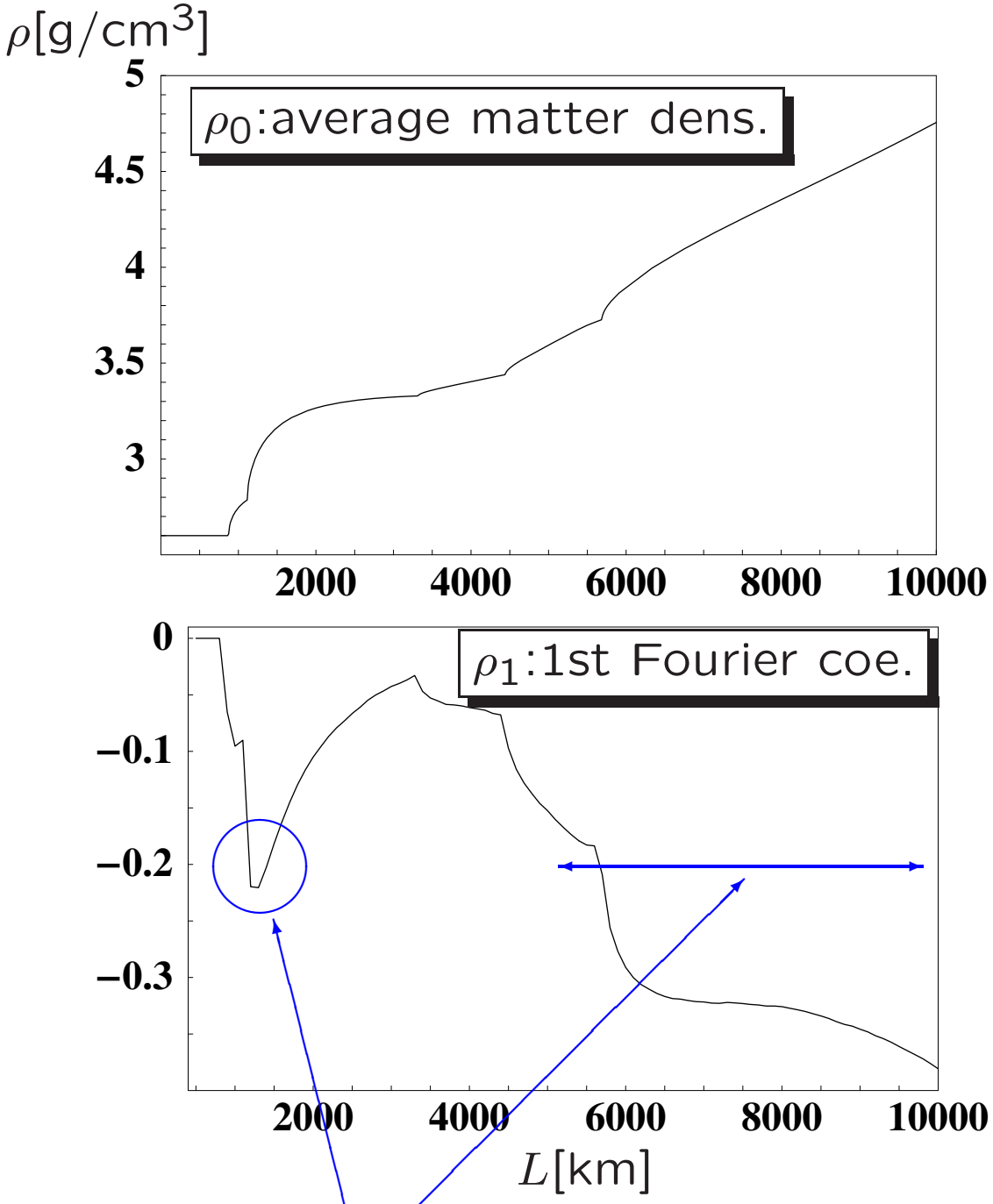
▼

The conclusion was

In $L \lesssim$5,000km,
the matter profile effect is not significant,
if PREM is trustworthy.

■How large is the matter profile effect?

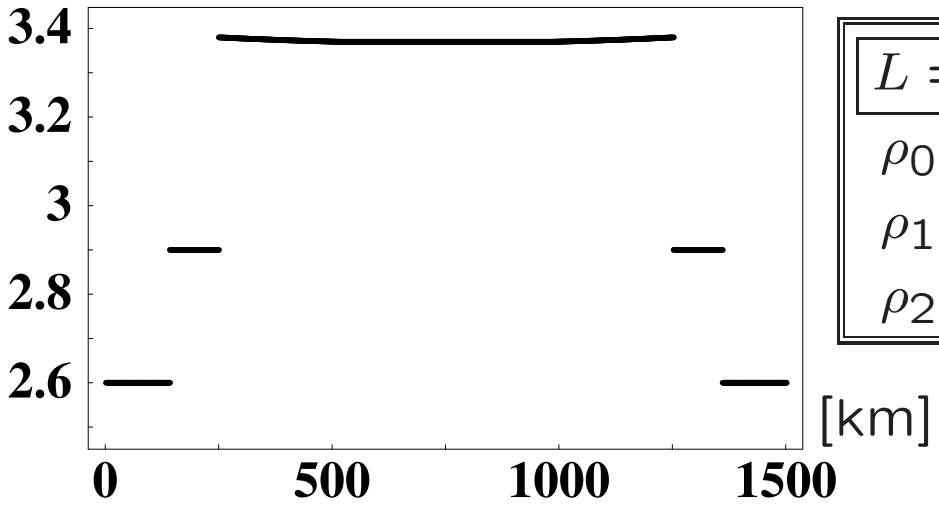
Based on PREM



In these regions, the matter profile effect is large in terms of the comparison with the average density.

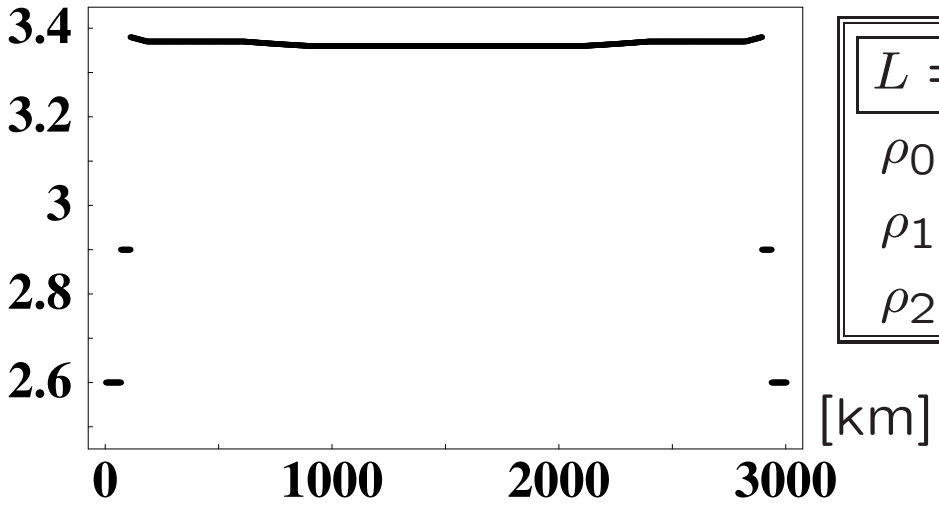
Matter profile based on PREM: Examples

$\rho[\mathrm{g/cm^3}]$



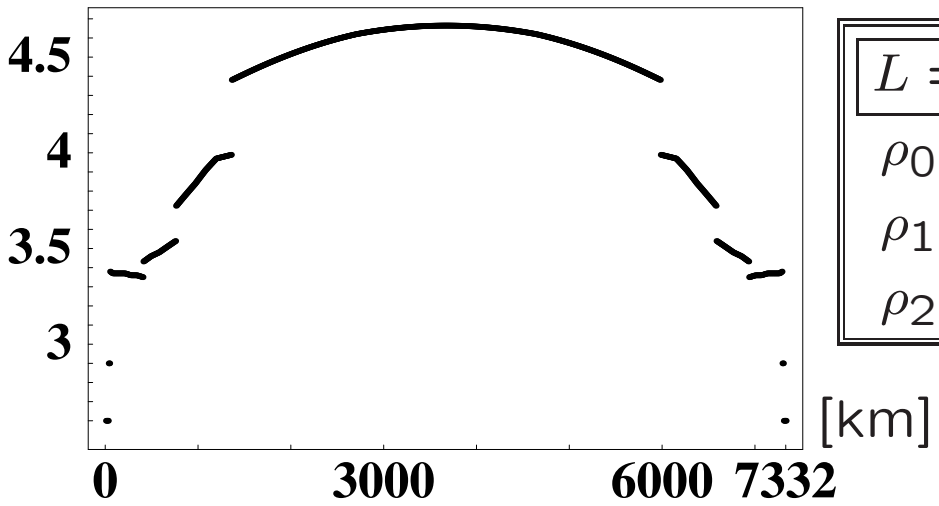
$L = 1,500\mathrm{km}$
$\rho_0 = 3.16$
$\rho_1 = -0.182$
$\rho_2 = -0.111$

$\rho[\mathrm{g/cm^3}]$



$L = 3,000\mathrm{km}$
$\rho_0 = 3.32$
$\rho_1 = -0.0426$
$\rho_2 = -0.0448$

$\rho[\mathrm{g/cm^3}]$



$L = 7,332\mathrm{km}$
$\rho_0 = 4.21$
$\rho_1 = -0.322$
$\rho_2 = -0.131$

■Alternative choices to treat the matter effect

To get the best-fit values and/or to test the theory, we have used

method 1. fit $N(\theta s, \Delta m^2 s, a)$ to $N(\bar{\theta} s, \overline{\Delta m^2} s, \bar{a})$

or

method 2. fit $N(\theta s, \Delta m^2 s)$ with PREM to $N(\bar{\theta} s, \overline{\Delta m^2} s)$ with PREM.

method 2'. using PREM but with overall uncertainty.

▼

1. Is the constant approximation OK?

2. Is a disregard of uncertainty OK?

2'. We deal with a_0 and a_1 separately.

To resolve these questions, we will show

★ the region where the *const.* approximation breaks down and *"up-to-*a_1*"* approximation works well,

and

★ the effects which the matter profile and its error induce.

■Conclusion:

In $L \lesssim 10,000$km,

★ The profile including up to a_1 can reproduce the event number using full PREM profile.

★ There is a strong correlation between a_0 and a_1.

▼

■ To ignore the matter profile effect $\overset{★}{=}$ $a_1 \to 0$

$\overset{★}{=}$ To shift a_0 from *the average* based on PREM

■ To introduce uncertainty of a_1: $a_1 \to a_1$+error

$\overset{★}{=}$ To introduce extra uncertainty of a_0

▼

These might affect the sensitivity for CP violation.

The effect of a_1 cannot be estimated by realistic data-size.

▼

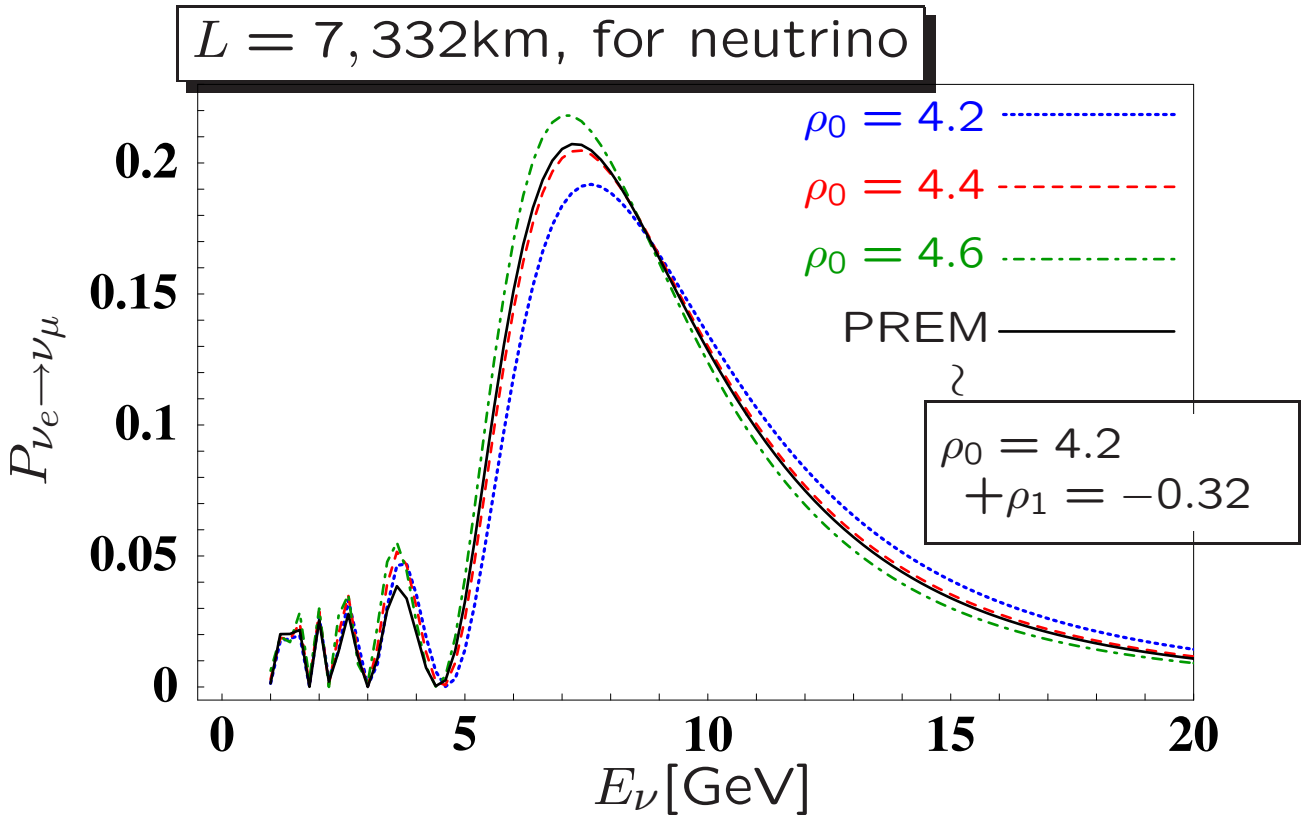
An extra error of a_0 still exists, even if using up-to-a_1 method.

▼

This error is essential to analysis.

Now, we are making a numerical calculation.

■Correlation between a_0 and a_1: Example



The probability calculated by constant profile with $\rho_0 = 4.42$[g/cm^3] coincides with that of PREM (and $\rho_0 + \rho_1$), perfectly!

$$P_0(a_0 + \Delta a_0) \simeq P_0(a_0) + P_1(a_1).$$

This correlation is not accident in this example.
It holds in wide energy and baseline region.

■Analytical interpretation of the correlation

Oscillation probability

$$P_{\nu_e\to\nu_\mu} = \overbrace{A\cos\delta + B\sin\delta + C}^{\text{K. Kimura et.al.,}} + \sum_n a_n P_n$$

Main part: $C \simeq P_0 \equiv s^2_{23} s^2_{2\times\widetilde{13}} \sin^2 \dfrac{\lambda_+ - \lambda_-}{4E} L$

Matter profile effect:

$$P_n \equiv 2 s^2_{23} s^2_{2\times\widetilde{13}} c_{2\times\widetilde{13}} \frac{\lambda_+ - \lambda_-}{(\lambda_+ - \lambda_-)^2 - (2Ep_n)^2} \sin^2 \frac{\lambda_+ - \lambda_-}{4E} L$$

The shift of a_0 can pretend the matter profile effect.
It means

$$\frac{\partial P_0}{\partial a_0} \Delta a_0 = 2\mathrm{Re}[a_1] P_1.$$

Actually, this relation holds in wide energy and baseline region.

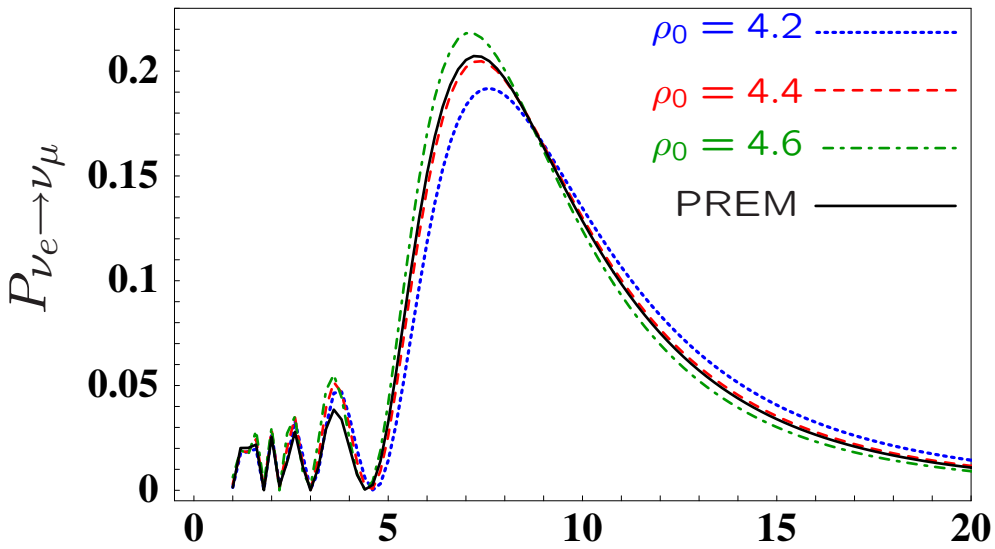
*Strictly speaking, the condition is

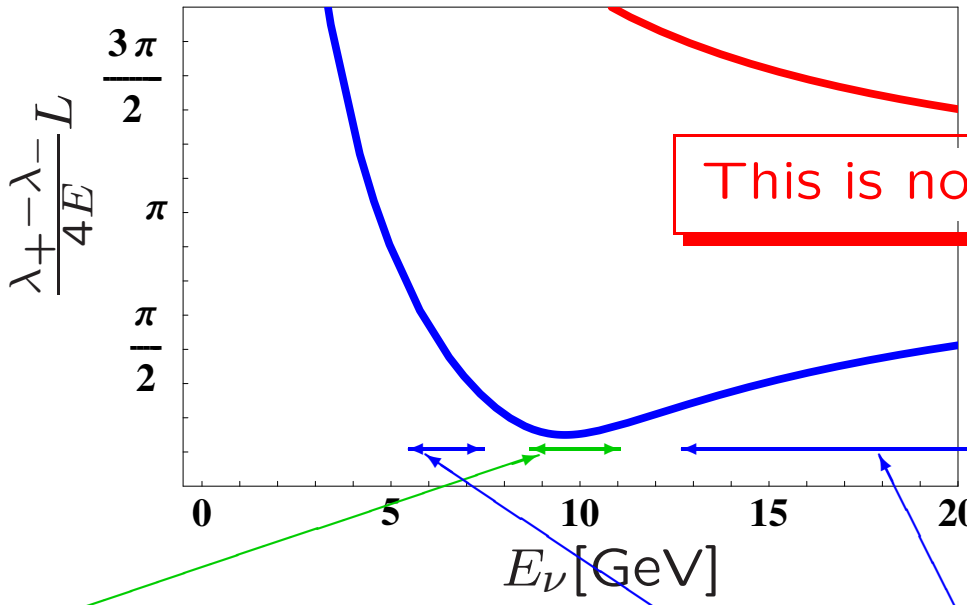
$$\frac{\partial}{\partial a_0}(C + Ac_\delta + Bs_\delta)\Delta a_0 = 2\mathrm{Re}[a_1] P_1.$$

However, fitting of a_0 almost only depends on P_0.

* The condition for anti-neutrino is the same.

■Correlation between a_0 and a_1: Example





anti-neutrino

This is not significant

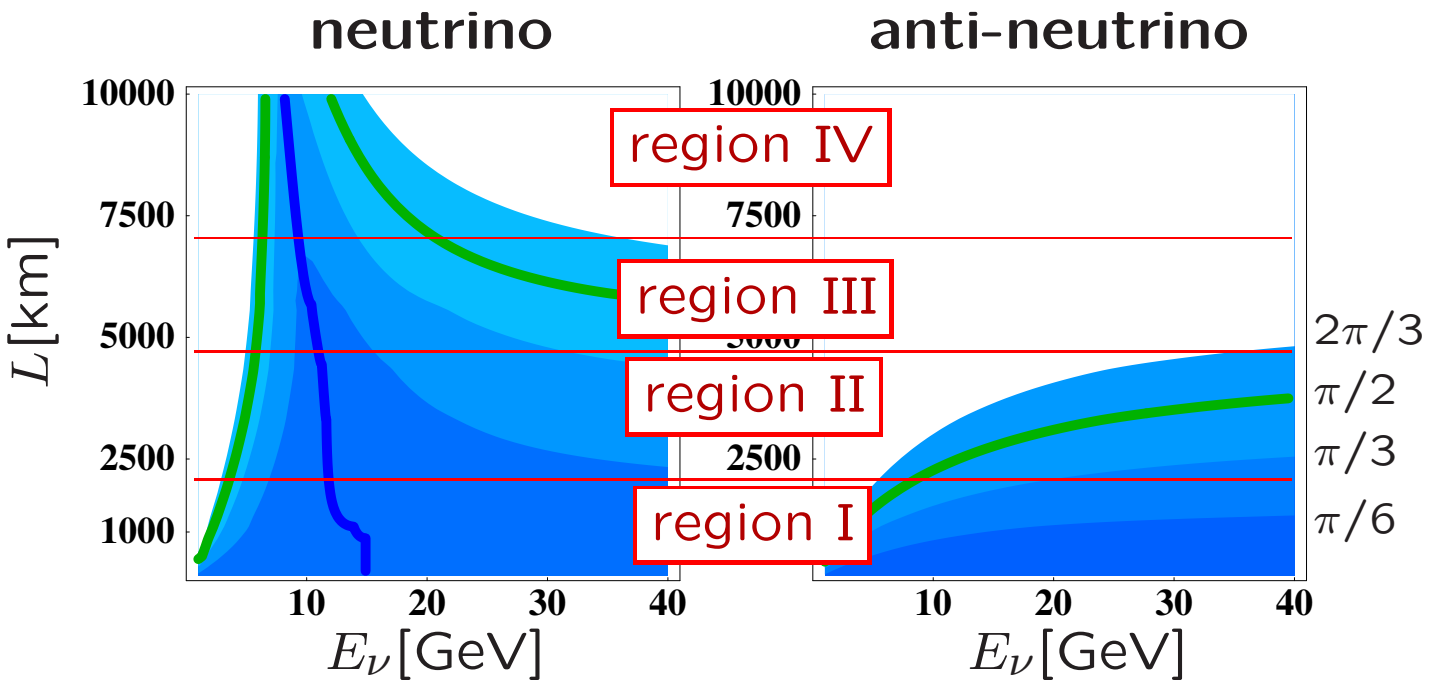
neutrino

The condition is satisfied at $\Delta a_0 = -6/\pi^2 \mathrm{Re}[a_1]$.
"around 0 expansion"

The condition is satisfied at $\Delta a_0 = -2/3 \mathrm{Re}[a_1]$.
"around $\pi/2$ expansion"

■The region where the correlation exists

Contour plots of $\frac{\lambda_+ - \lambda_-}{4E_\nu} L$



	neutrino	anti-neutrino	correlation
region I	0	0	○
region II	0	$\pi/2$	○
region III	0, $\pi/2$	—	○
region IV	0, $\pi/2$ ...	—	—

*In these regions, anti-neutrino events are not significant in statistical sense.

preliminary

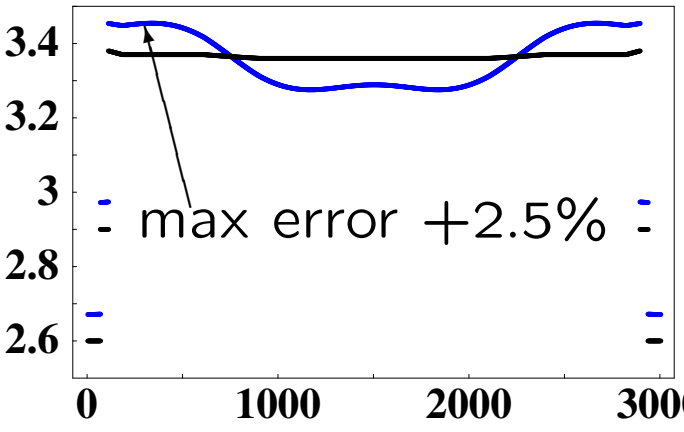
Example: $L = 3,000$km, CP phase search

According to PREM, the matter profile effect might be small in 3,000km.

In $\not{CP}$ search, is it small enough to be ignored?

If there are 5% error in each point, the error of a_1 can become more than 100% without tuning of a_0.

Example:



$$\Delta\rho_0 = 0$$
$$\Delta\rho_1 = 0.047 \sim -\rho_1$$
$$\Delta\rho_2 = 0$$
$$\Delta\rho_3 = -1/4 \times \Delta\rho_1$$

Does not really the error affect the CP sensitivity?

▼

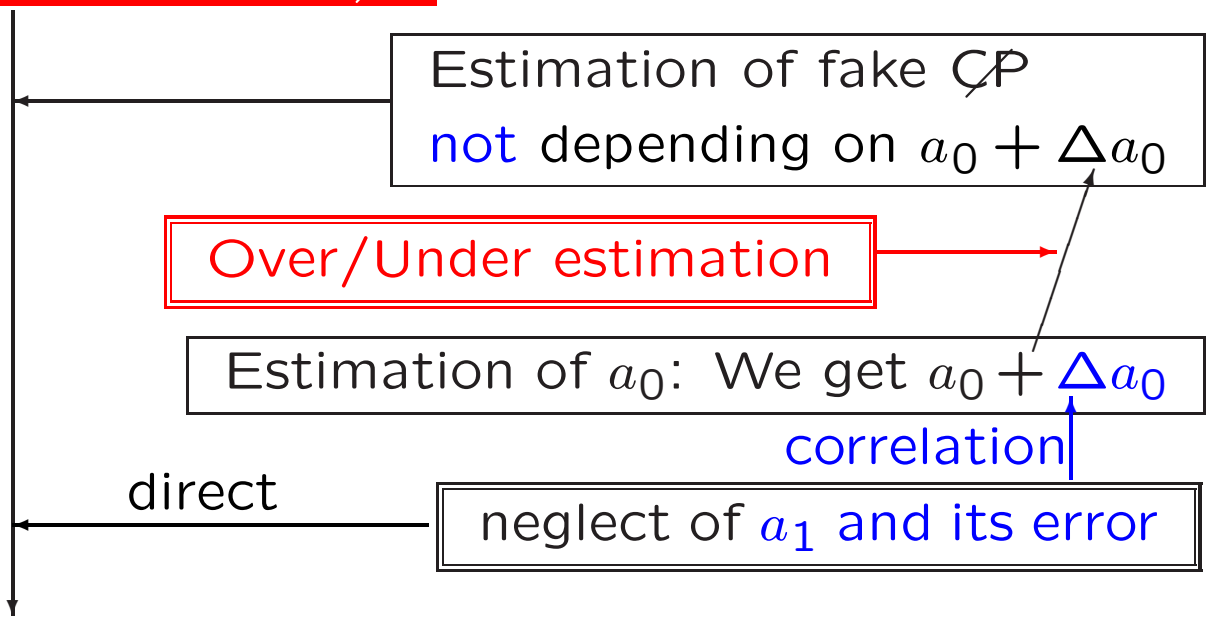
These effects can be understood by the correlation.

* This correlation exists only in the relation between main term(P_0) and matter profile term(P_1). Therefore, if there are (unrealistic) huge statistics, the correlation between a_0 and a_1 will break when we observe CP-violating effect ($\cos\delta$, $\sin\delta$ terms).

preliminary

■Effect for CP sensitivity: rough explanation

Observation of $\not{CP}$



Genuine $\not{CP}$

direct effect

Fake $\not{CP}$ from a_1 term

correlation effect

The matter profile effect brings an extra uncertainty and a shift of a_0.

In $L = 3,000$km, neglect of a_1 introduce "1% shift" into a_0. Furthermore, "200%" error for a_1 also introduce "$2 \sim 3$% extra error" into a_0.

■Fit a_0 (a_1) to PREM

■Oscillation parameters that we take here:

$$\sin\theta_{12} = \frac{1}{2}, \quad \sin\theta_{23} = \frac{1}{\sqrt{2}}, \quad \sin\theta_{13} = 0.1,$$
$$\delta m^2_{21} = 5 \times 10^{-5}, \quad \delta m^2_{31} = 3 \times 10^{-3},$$
$$\delta = \frac{\pi}{2}.$$

■The figures indicate the necessary data size (muon number $\times$ detector mass) to distinguish the PREM matter profile with the profile include only a_0 or up to a_1 at 90% confidence level.

▶Concretely,

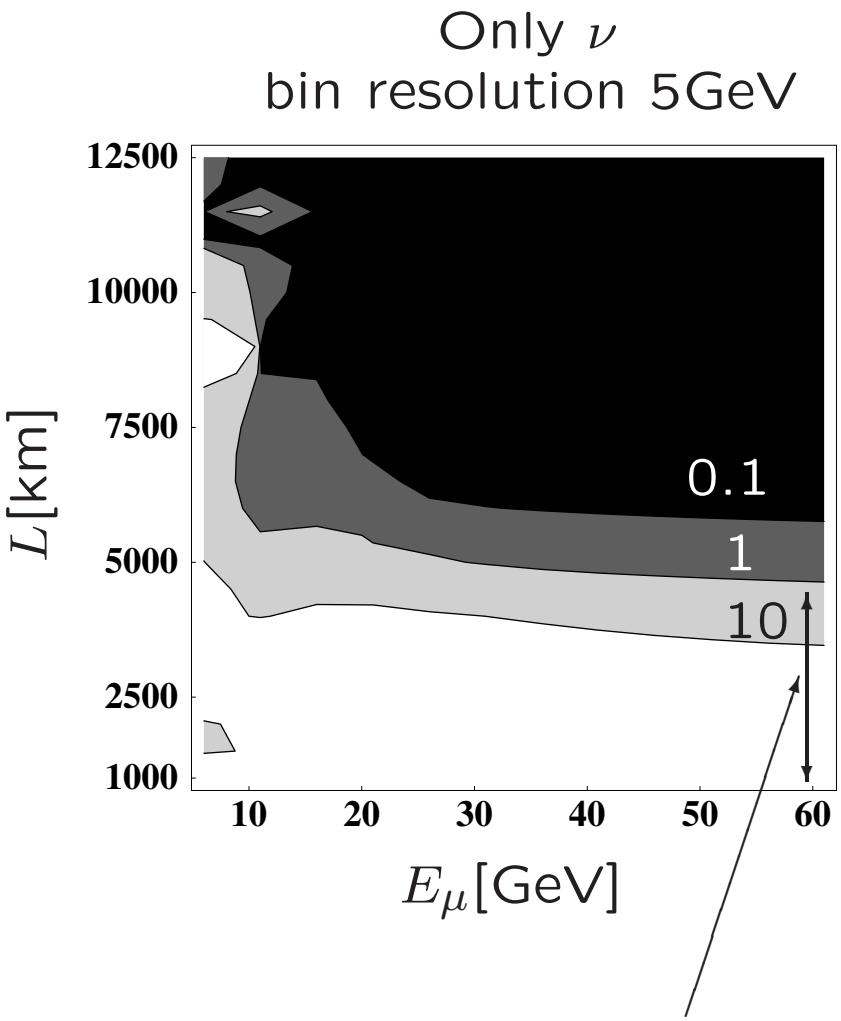
$$\chi^2_{90\%} < \chi^2 \equiv \sum_i^{\text{bin}} \frac{\left| N_i^{\text{PREM}} - N_i^{a_0(a_1)} \right|^2}{N_i} = N_\mu M_{\text{det}} X^2$$

$$N_\mu M_{\text{det}} > \frac{\chi^2_{90\%}}{X^2}$$

In each E_μ and L, this value is ploted.

The contour "10" means if we have 10^{21}muons and 1Mt detector, then we can observe the difference between "a_0" and "PREM" at 90% C.L..

PREM vs a0: No Uncertainty



When we assume the 10^{21}muons·100kt, the difference between a_0 and PREM is not significant in $L \lesssim 5,000$km.

▼

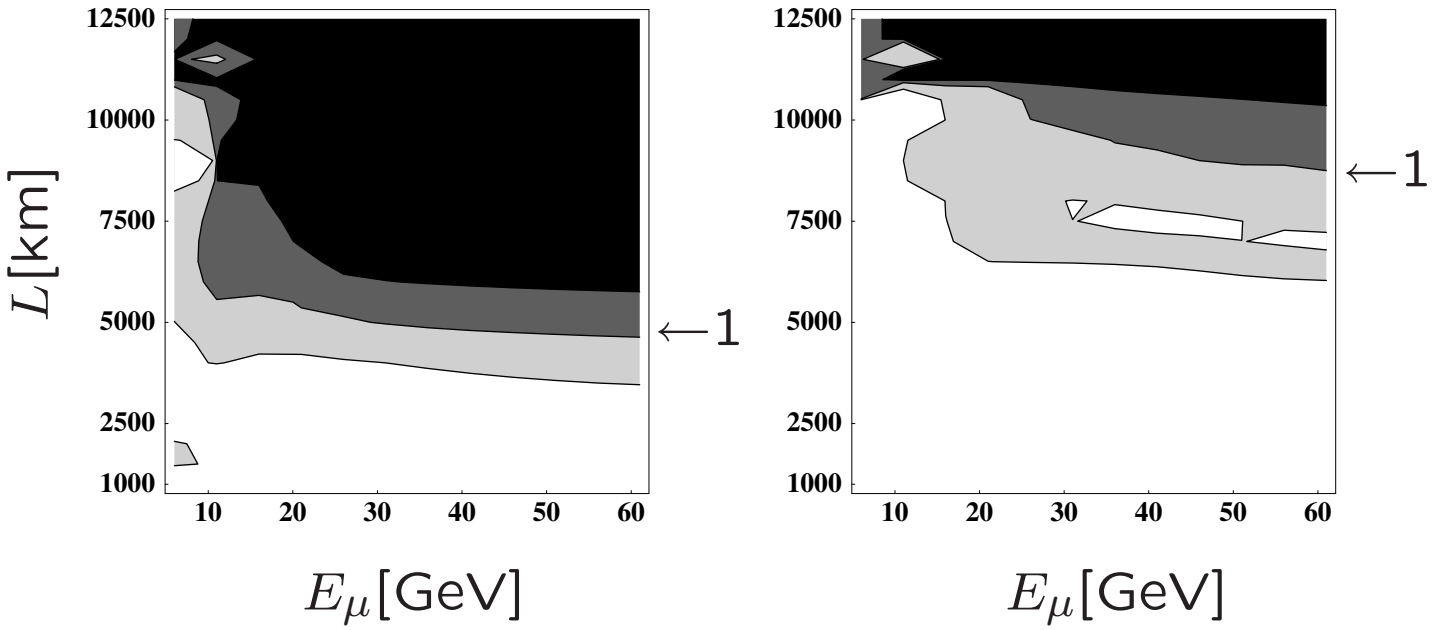
In $L \lesssim 5,000$km, a_0 approximation is OK.

* This result holds if the profile of PREM is trustworthy. If PREM has *some uncertainty for a_1 (not for a_0)*, the situation will change even in 3,000km.

PREM vs a0: Only a_0 has 10% uncertainty

Only ν, bin resolution 5GeV

No Uncertainty ⟶ Only a_0 has 10%



In 5,000∼10,000km, the matter profile effect is significant.

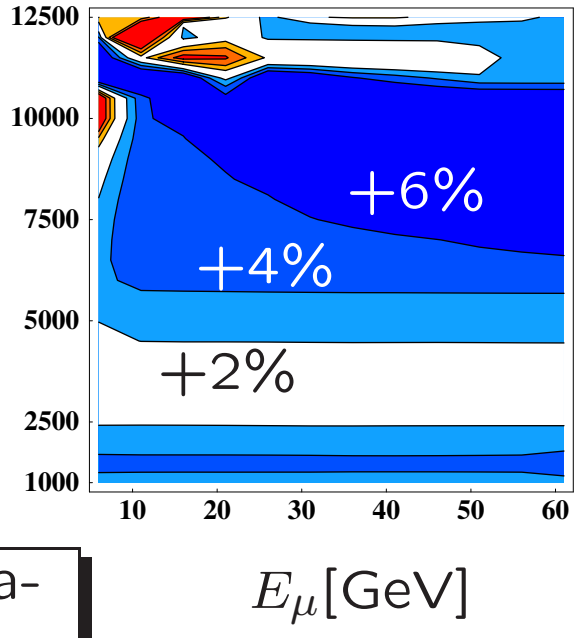
+

The difference can be compensated by the shift of a_0.

▼

In this region, the correlation exists.

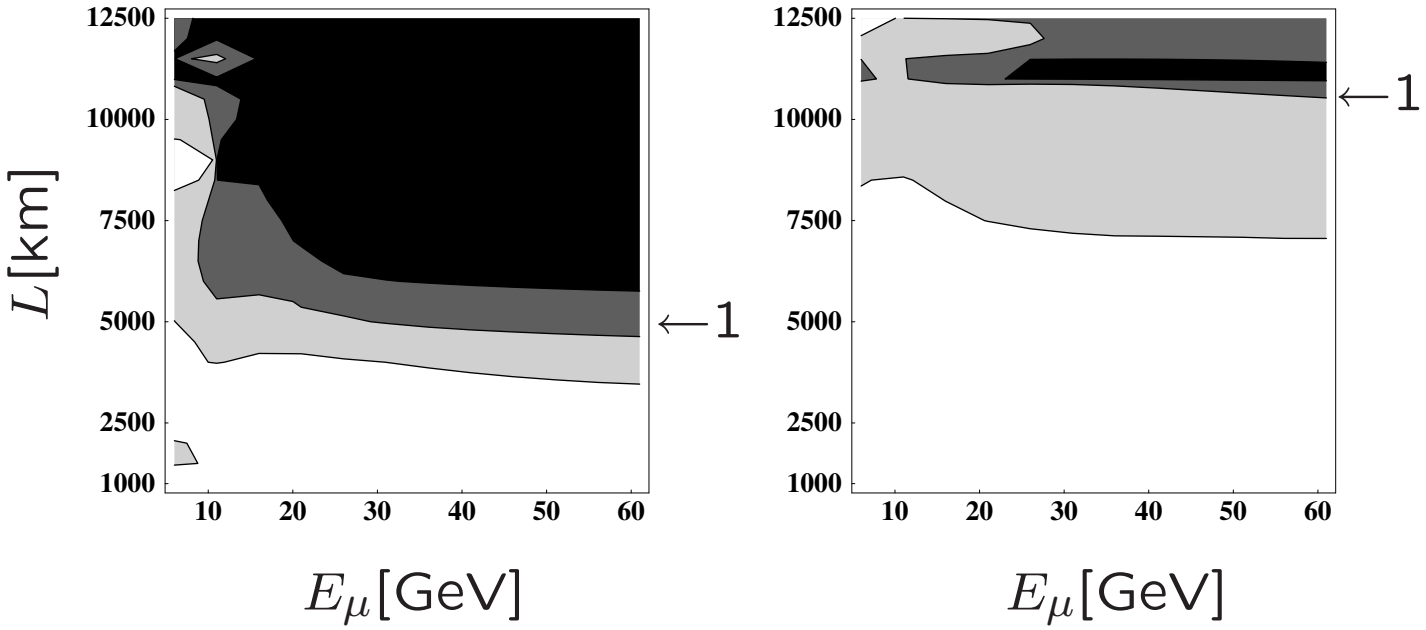
Degree of the shift of a_0



PREM vs a1: No Uncertainty

Only ν, bin resolution 5GeV

Only a_0 ⟶ a_0 and a_1

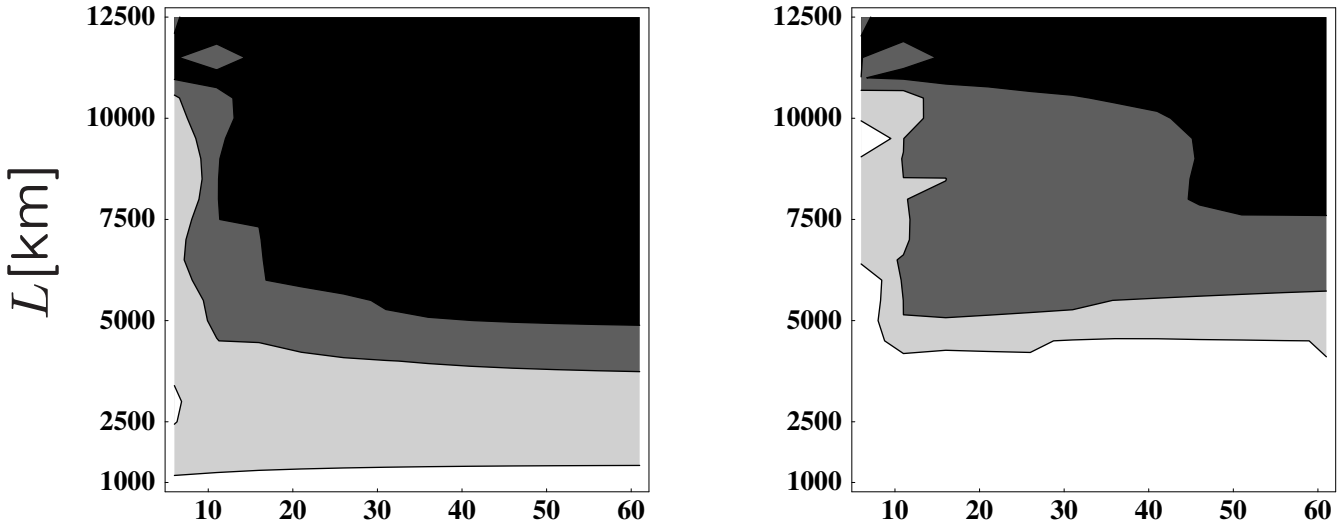
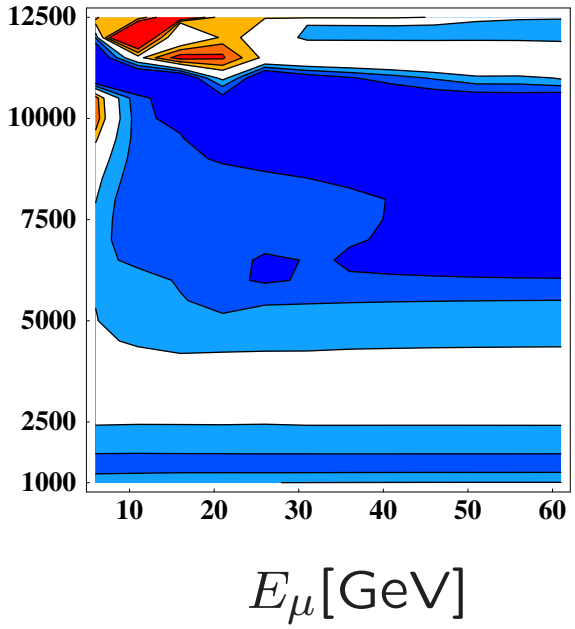


Up to 10,000km, a_1 approximation works well.

PREM vs a0: Only a_0 has 10% uncertainty

ν and $\bar{\nu}$, bin resolution 5GeV

No Uncertainty ⟶ Only a_0 has 10%



Degree of the shift of a_0