The effects of matter density uncertainties on neutrino oscillations in the Earth

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Outline

- Introduction

- Matter density profiles: Models

  1. A single matter density perturbation

  2. Random matter density fluctuations

  3. Mean density as measured quantity

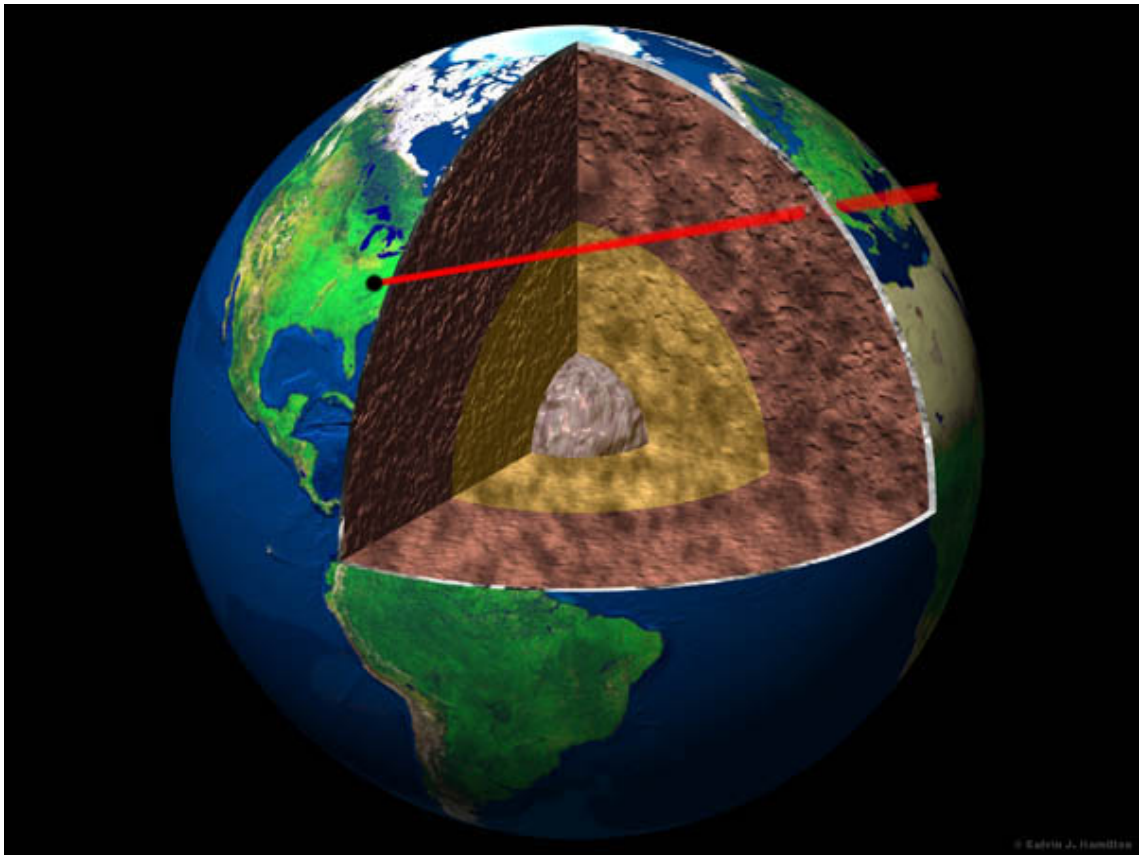
- Summary: Comparison of models

- Conclusions

Introduction

Neutrino oscillations are assumed to be influenced by coherent forward scattering in matter (MSW effect)

Earth matter effects are important especially for LBL experiments:



Key questions:

1. How precisely does one know the matter density profile?
2. How precisely does one need to know the matter density profile?

The evolution operator method

Propagation in a layer of length x_j and constant density ρ_j described by *evolution operator*

$$\mathcal{U}(x_j, \rho_j) = e^{-i\,\mathcal{H}(\rho_j)\,x_j},$$

where $\mathcal{H}(\rho_j)$ is the Hamiltonian in constant matter density:

$$\mathcal{H}(\rho_j) \equiv U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} A(\rho_j) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

For an arbitrary matter density (step) profile:

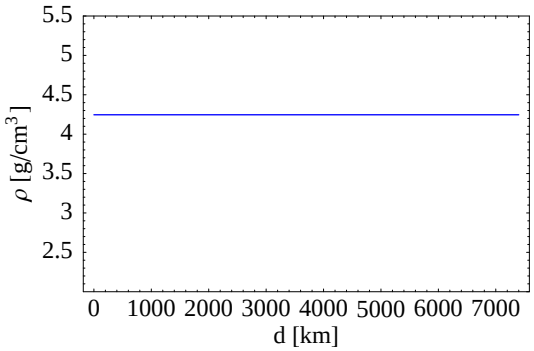
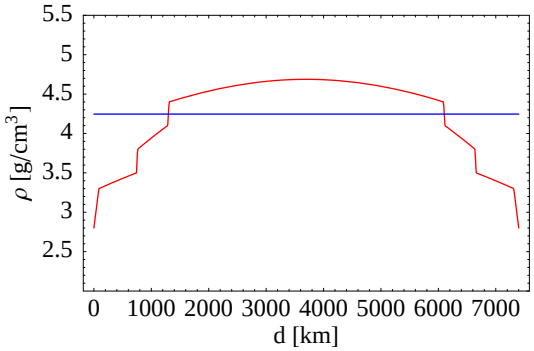
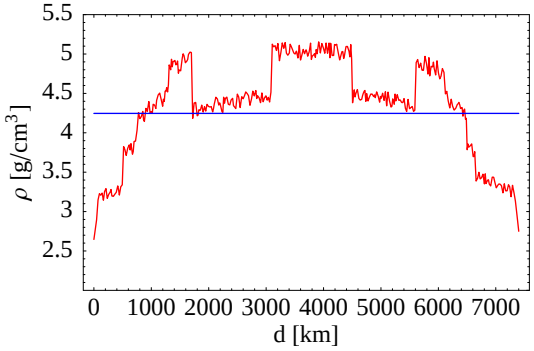
$$P_{\alpha\beta} = \left| \langle \nu_\beta | \mathcal{U}(x_n, \rho_n) \ldots \mathcal{U}(x_1, \rho_1) | \nu_\alpha \rangle \right|^2.$$

Note that in general $[\mathcal{U}(x_i, \rho_i), \mathcal{U}(x_j, \rho_j)] \neq 0$ for $i \neq j$

$\rightarrow$ Interference effects

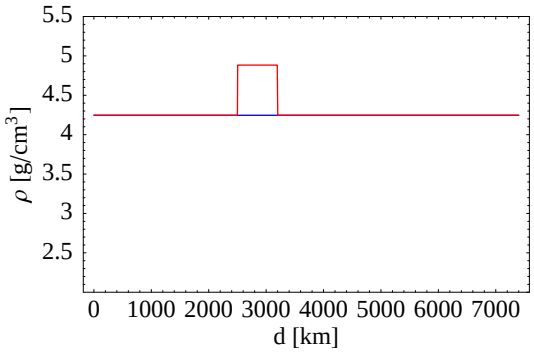
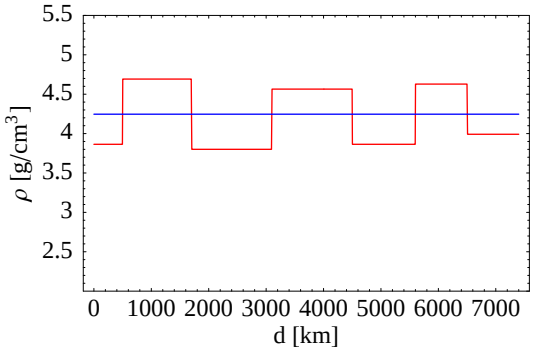
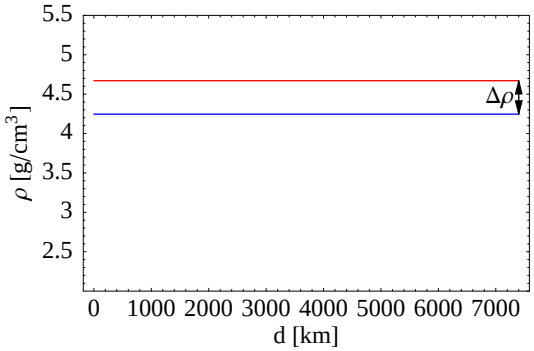
$\rightarrow$ Taking the mean density is not necessarily a good approximation

Density profiles for long baseline experiments

Density profile	Description	Problem
Mean density: 	Average density along baseline	Only 0th order approximation
PREM profile: 	Preliminary Reference Earth Model	Fluctuations up to 5% without agreement in geophysics
"Realistic" profile: 	Actual profile along a specific baseline to be built	Yet unknown

→ Models necessary!

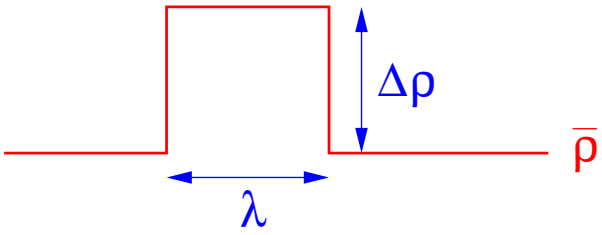
Our models for density profiles

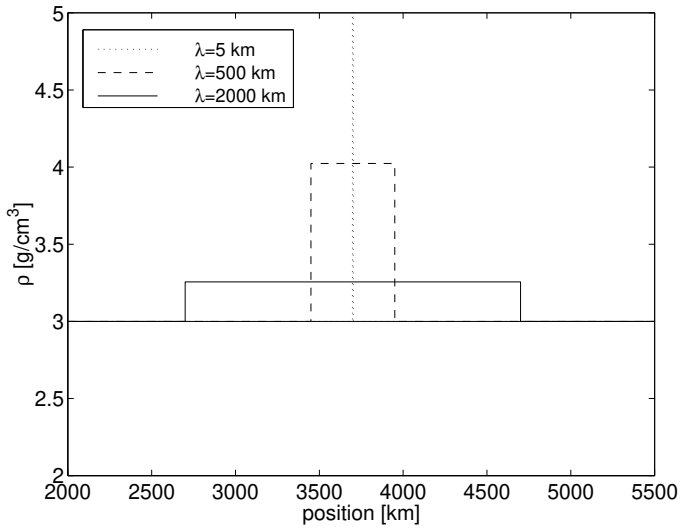
Model	Evaluation	Problems
1. Single perturbation (mine, mountain, lake) 	Analytical for probabilities	Only simple applications
2. Random fluctuations 	Numerical for probabilities	High computational effort; only average statements
3. Mean density as measured quantity 	Numerical for full statistical analysis	Not as realistic as 2.

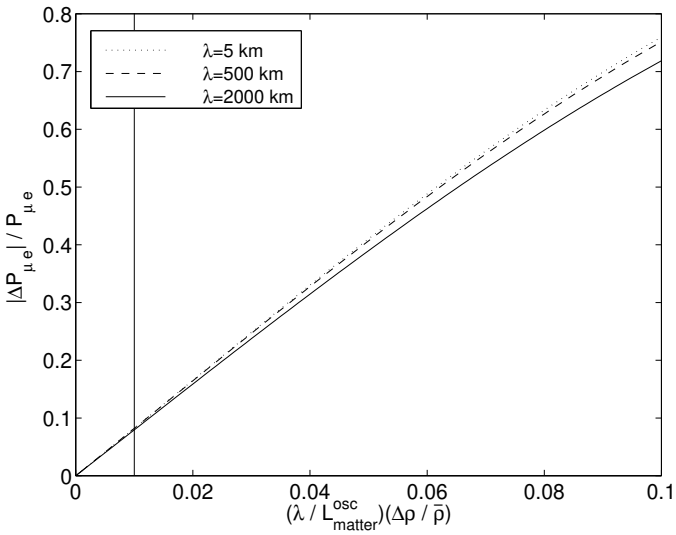
1. A single perturbation

Modeling in terms of

- Amplitude $\Delta\rho$
- Length scale λ







(LMA, $L^{\rm osc}_{\rm matter} \simeq 17000\,{\rm km}$, $L = 7400\,{\rm km}$, $E = 30\,{\rm GeV}$)

For coal mine, etc.:

$\lambda \sim 10\,{\rm km}$, $\Delta\rho/\bar{\rho} \sim 5 - 100\%$: error $\lesssim 1\%$.

Analytical (perturbation theory):

Change in probabilities is proportional to the product

$$\lambda \cdot \Delta\rho$$

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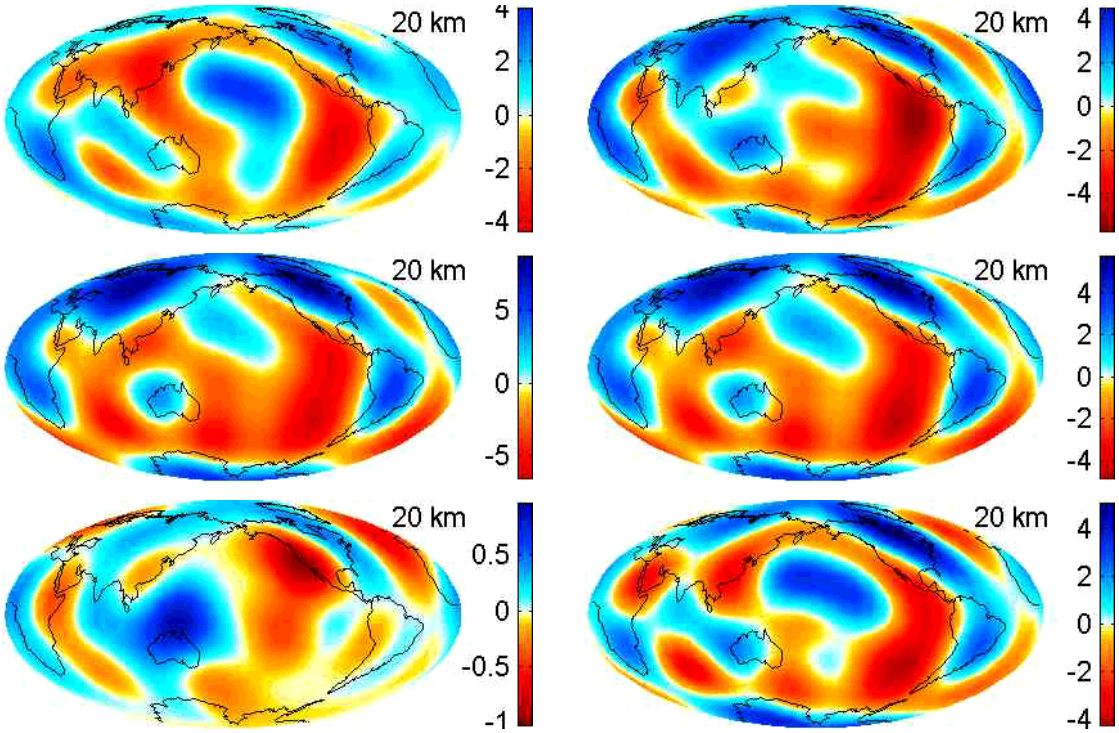
2. Random density fluctuations

Density uncertainties in 20 km depth (geophysics):



Summary: `http://cfauvcs5.harvard.edu/lana/rem/index.htm`

- Fluctuations up to 5% documented, increasing with depth
- No general agreement among seismic wave groups

$\rightarrow$ Averaging over large ensembles random matter density profiles with realistic characteristics

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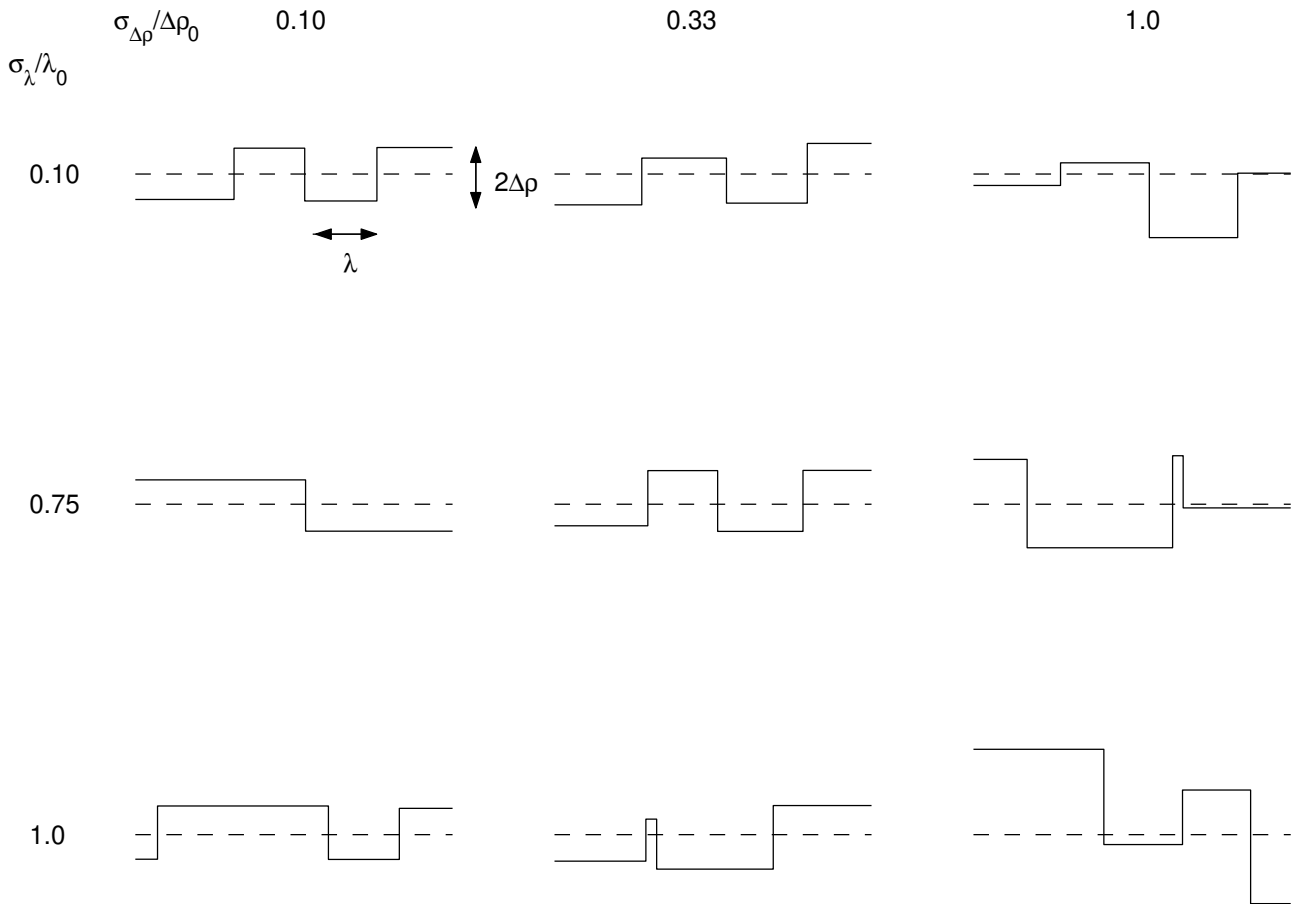
Random density fluctuations: The model

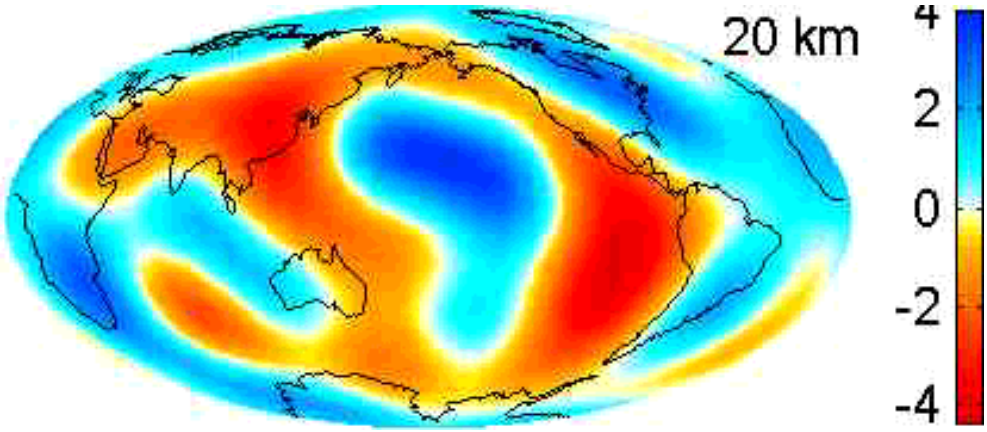


Modeling in terms of

- Length scale λ
  Fluctuating around mean λ_0 with (truncated) Gaussian distribution with standard deviation σ_λ

- Amplitude $\Delta\rho$
  Fluctuating around mean $\Delta\rho_0$ with (truncated) Gaussian distribution with standard deviation $\sigma_{\Delta\rho}$

Random density fluctuations: Standard parameter set

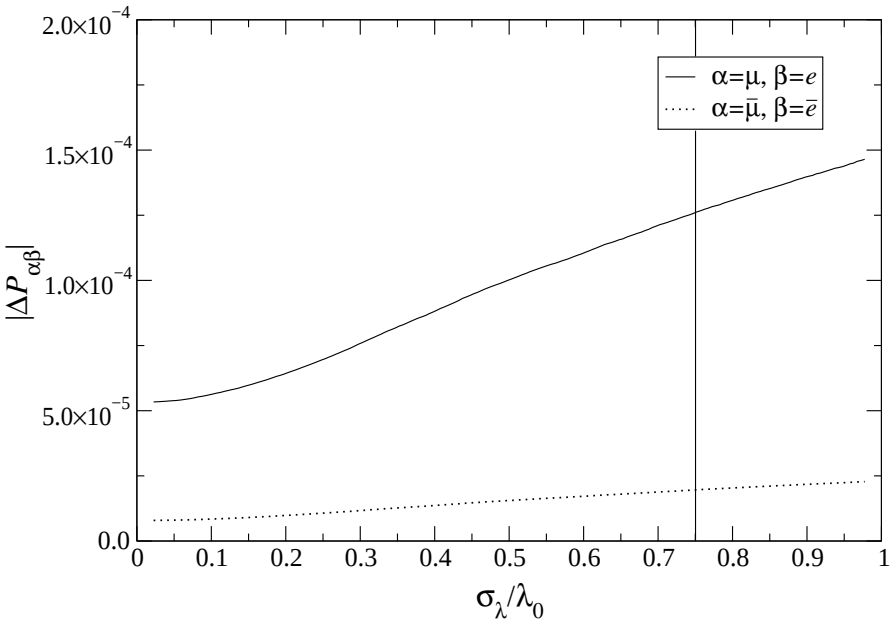


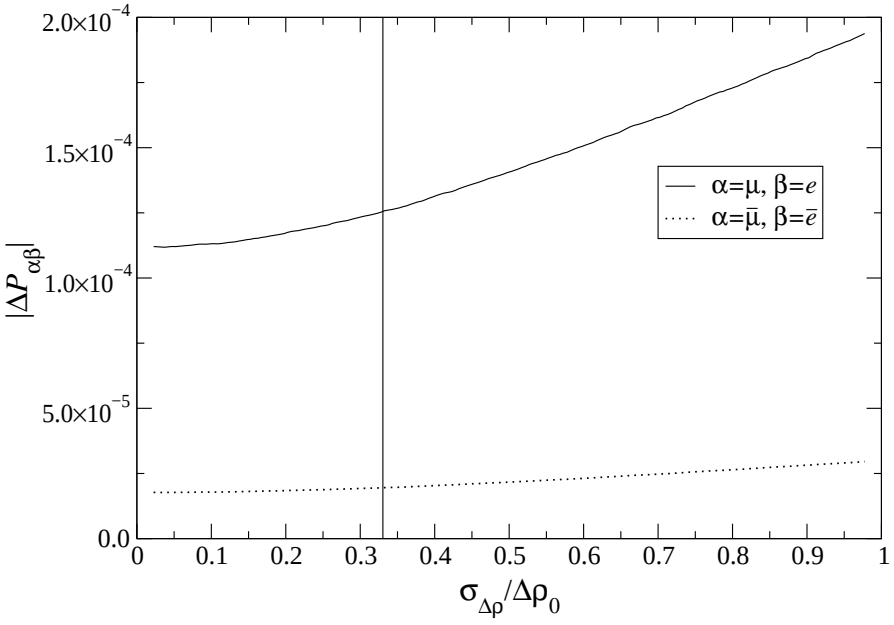
Based on these fluctuations, we choose the following standard parameter set and investigate the individual parameters:

- $\lambda_0 \sim 2000\,\mathrm{km}$
- $\sigma_\lambda \sim 1500\,\mathrm{km} = 0.75\,\lambda_0$
- $\Delta\rho_0 \sim 3\%\,\bar{\rho}$
- $\sigma_{\Delta\rho} \sim 1\%\,\bar{\rho} = 1/3\,\Delta\rho$

In addition: LMA solution, baseline 7400 km, energy 30 GeV, averaging over 20000 profiles

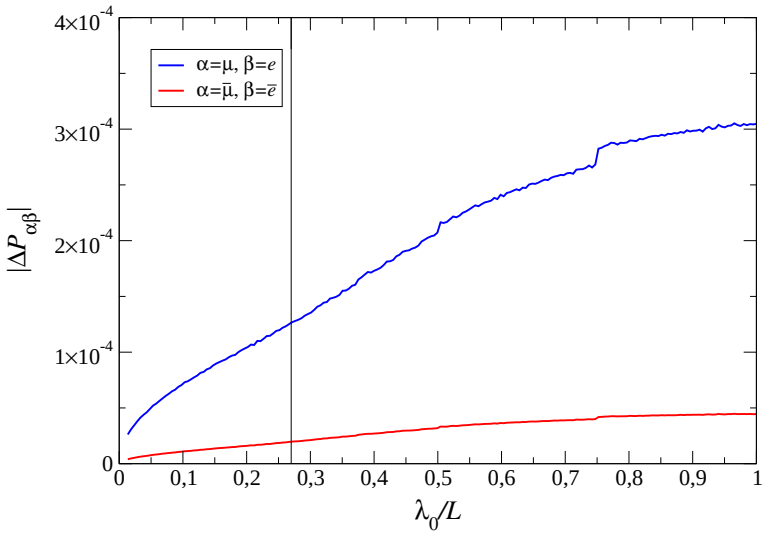
Random fluctuations:
The influence of the standard deviations σ_λ and $\sigma_{\Delta\rho}$

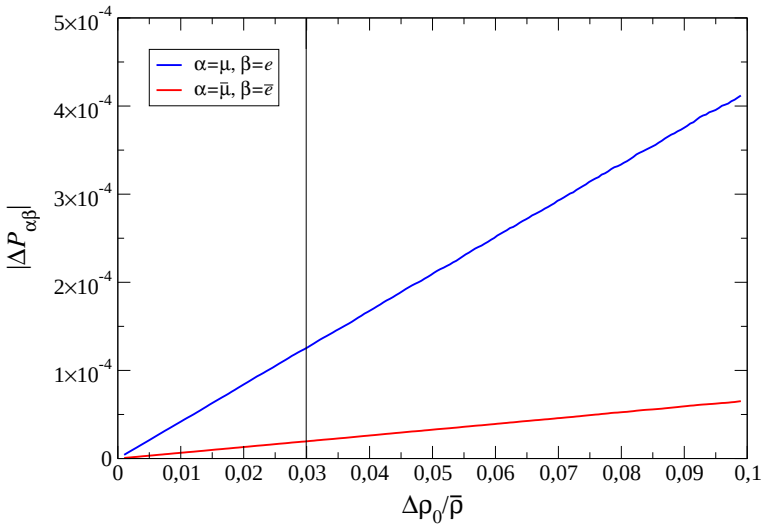




→ Quite stable around standard values (vertical lines)

Random fluctuations: Dependence on length scale λ and amplitude $\Delta\rho$





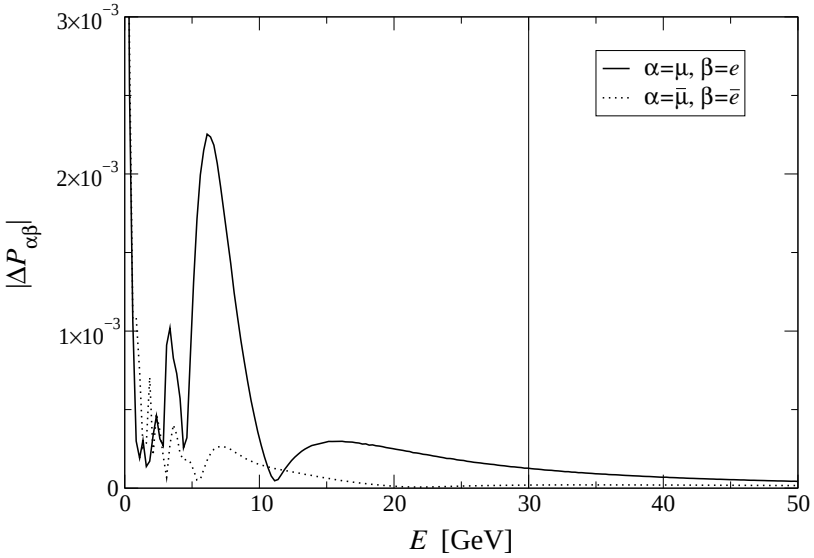
$\rightarrow$ Qualitatively similar to single perturbation
But: Suppressed by a factor of 2-3 (averaging!)

Analytical: Fast fluctuations (length scale $\lambda \ll L^{\mathrm{OSC}}$) average out

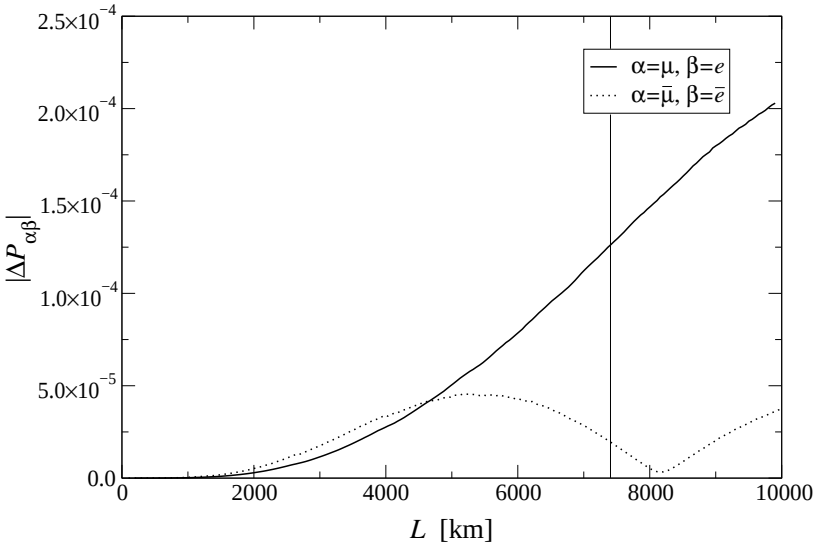
T. Ohlsson and WW, PLB 512 (2001) 357

Random fluctuations: Dependence on energy E and baseline L

(LMA solution with $\Delta m^2_{32} = 2.5 \cdot 10^{-3}\,\mathrm{eV}^2$)

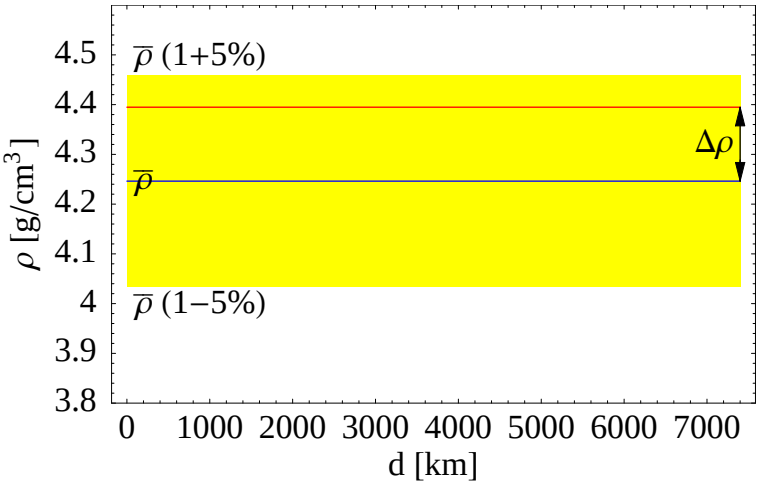


→ Very stable behavior close to neutrino factory peak energies ($20 - 50\,\mathrm{GeV}$)



→ Vanishing for short baselines, in general periodic in baseline

3. Mean density as measured quantity



Model: 5% matter density variation around the mean density allowed

Simulation: Full neutrino factory simulation including statistical, systematical, correlational, and degeneracy errors

Scenarios:

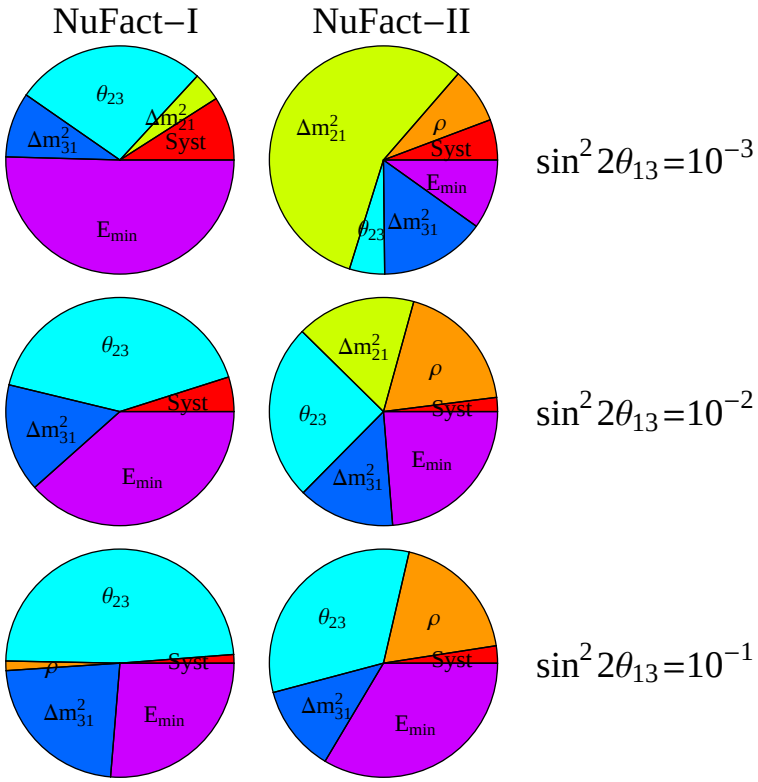
	NuFact-I	NuFact-II
Description:	Initial stage	Advanced stage
Target power:	0.75 MW	4 MW
Useful μ-decays/yr:	10^{20}	$5.3 \cdot 10^{20}$
Running time:	2.5 yr ν + 2.5 yr $\bar{\nu}$	4 yr ν + 4 yr $\bar{\nu}$
Detector mass:	10 kt	50 kt

P. Huber, M. Lindner, WW, hep-ph/0204352

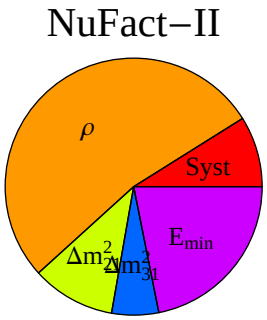
Mean density measured: Results

Impact factor: Relative influence of controllable error source on measured quantity

$\sin^2 2\theta_{13}$:



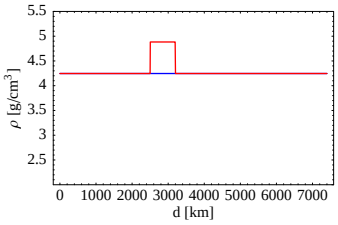
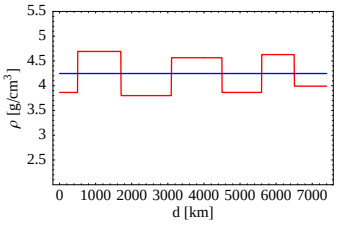
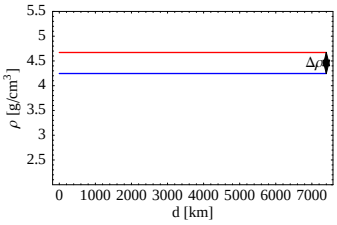
δ_{CP} (for $\sin^2 2\theta_{13} = 0.01$ and $\delta_{CP} = \pi/4$):



Advantage: Matter density uncertainty in full numerical analysis

Disadvantage: Only (conservative) approximation, no interference effects

Summary: Comparison of models

Type	1. Single perturbation	2. Random fluctuations	3. Measured mean
			
Averaging	No	Yes	No
Dep. on	$\lambda \cdot \Delta\rho$	$\lambda \otimes \Delta\rho$ (Interference effects remaining)	$\Delta\rho$
Evaluation	Analytical	Numerical	Numerical
Level	Osc. probs.	Osc. probs.	Full analysis
Impact	Small for lakes, mines, etc.	For $\lambda \ll L$: small For $\lambda > L$: 3. good approximation	Similar magnitude as other error sources
Problem	Only limited applications	High computational effort	Conservative choice

Conclusions: Key questions

1. How precisely does one know the matter density profile?

Up to 3-5 % without general agreement among geophysics groups

2. How precisely does one need to know the matter density profile?

- Quite substantial effects in a complete analysis (conservative choices)
- Random fluctuations reduce the effects by partial averaging
- The profile along a specific baseline from geophysics is probably known much better

$\rightarrow$ Realistic effects are suppressed by additional factor compared to our estimates
$\rightarrow$ Matter density uncertainties will probably not be the bottleneck of the statistical analysis

Jacobsson, Ohlsson, Snellman, WW, PLB 532 (2002) 259

P. Huber, M. Lindner, WW, hep-ph/0204352