CP violation at low energy and leptogenesis

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Collaboration with
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How is CP violation of low energy related to CP violation required from Baryon number asymetry

A plausible scenario for $B \neq 0$ is Baryogenesis through leptogenesis $L \neq 0$.

Sphaleron ($T > 1$TeV) $\frac{d(B+L)}{dt} = Anomaly \neq 0$ washes out $(B+L)_{initial}$.

$$B_{now} = \frac{1}{2}(B-L)_{ini.} + \frac{1}{2}(B+L)_{ini.} \exp[-\frac{\Delta t}{\tau}] \rightarrow \frac{1}{2}(B-L)_{ini.}$$

Fukigita and Yanagida: "Baryogenesis without Gut". $B_{ini.} = 0$

They use Lepton number production from Heavy Majorana Neutrinos decays.

$$L_{ini} \sim \Gamma[N \rightarrow l^- \phi^+] - \Gamma[N \rightarrow l^+ \phi^-]$$

Purpose of my talk is give a specific example which shows "a direct Link" between the size and sign of Baryon number and CP violation in neutrino oscillation:

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu_\mu} \to \bar{\nu_e}) \sim J$$

$$J = Im(K_{e1}K^*_{e2})(K_{\mu 1}K^*_{\mu 2})^*.$$

K: MNS matrix

$$J^{cc}_\mu = \bar{l}_L \gamma_\mu K \nu_L$$

How CP violation in neutrino oscillation is related to CP violation for Baryon number (L) ? Seesaw model:

1) CP violation and mixing in neutrino sectors

2) CP violation for Baryon number asymmetry

Neutrino oscillations: $J \sim P - \bar{P} = \Delta P$

$$m_D = y_\nu \frac{v}{\sqrt{2}}$$

$$m_{eff} = -m_D \frac{1}{M} {m_D}^T$$

Lepton number asymmetry from Heavy Majorana decays: L

$$\begin{aligned} \epsilon_1 &= \frac{\Gamma[N_1 \to l^- \phi^+] - \Gamma[N_1 \to l^+ \phi^-]}{\Gamma[N_1 \to l^- \phi^+] + \Gamma[N_1 \to l^+ \phi^-]} \\ &= -\frac{M_1}{2M_2} \frac{Im[(m_D^\dagger m_D)_{12}^2]}{V^2 (m_D^\dagger m_D)_{11}}, \\ & \quad (V = \sqrt{4\pi} v \sim 1(TeV)) \end{aligned}$$

(In the base, Heavy Majorana neutrinos are real diagonal, also $M_2 > M_1$). **We consider "the minimal seesaw" which generates $L \neq 0, \Delta P \neq 0$.**

Minimal seesaw model:

(3, 2) model = 3 light neutrinos ν_1, ν_2, ν_3 + 2 heavy Majorana neutrionos $N_1 N_2$:

$$\bar{l^i} m_{l_i} l^i + \bar{\nu^i} m_{D_{ij}} N_{Rj} + \frac{1}{2} (\bar{N_{Rj}})^c M_j N_{Rj}$$
$$(i = 1 \sim 3, j = 1, 2).$$

Dirac mass term 3×2 matrix: (No freedom of rotating from left and right, i.e., m_l M are real diagonal base.)

$$m_D = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{pmatrix}$$

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad m_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

Property of the model:

1) one light neutrino is exactly massless:
$det[m_D \frac{1}{M} m_D^T] = 0$

2) Three CP violating phases
$3 = 6 - 3$ imaginary part in $m_D \to \exp(iPhase) m_D$, $m_l \to \exp(iPhase) m_l$

A parametrization of m_D (6 real + 3 CP phases) : (bi-unitary transformation)

$$m_D = U_L m V_R : \quad m = \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix}$$

$$\begin{aligned} U_L &= O(\theta_{23})U(\theta_{13}\delta_L)O(\theta_{12}) \times \\ & \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(-i\frac{\gamma_L}{2}) & 0 \\ 0 & 0 & \exp(i\frac{\gamma_L}{2}) \end{pmatrix} \\ V_R &= \begin{pmatrix} c_R & s_R \\ -s_R & c_R \end{pmatrix} \begin{pmatrix} \exp(-i\frac{\gamma_R}{2}) & 0 \\ 0 & \exp(i\frac{\gamma_R}{2}) \end{pmatrix} \end{aligned}$$

1) Leptogenesis controled by a CP phase γ_R in V_R

$$\begin{aligned} \epsilon_1 & \sim -Im[(m_D^\dagger m_D)^2_{12}] \\ & \sim -({m_3}^2 - {m_2}^2)^2 {s_R}^2 {c_R}^2 \sin 2\gamma_R \end{aligned}$$

2) $P - \bar{P}$:CP violation in neutrino oscillation depends on all three CP violating phases $\delta_L, \gamma_L, \gamma_R$

$$J = Im[K_{e1}K^*_{e2}K^*_{\mu 1}K_{\mu 2}]$$

K is detemined by the diagonalization : $-K^\dagger m_D \frac{1}{M} m_D^T K^*$.

More on K (MNS) in (3,2) model:

$$K = U_L \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta\exp[-i\phi] \\ 0 & -\sin\theta\exp[i\phi] & \cos\theta \end{pmatrix} P$$

$$m_{eff} = -m_D \frac{1}{M} {m_D}^T = -U_L m V_R \frac{1}{M} V_R^T m^T U_L^T$$

$\phi, \theta(P)$ **are functions of** $(\gamma_R, s_R, M_i, m_i)$ **and are obtained by diagonalization:**

$$\phi = Arg.(Z_{22}^* Z_{23} + Z_{23}^* Z_{33}) \to 0(\gamma_R \to 0)$$

$$\tan 2\theta = \frac{2|Z_{22}^* Z_{23} + Z_{23}^* Z_{33}|}{|Z_{33}|^2 - |Z_{22}|^2} \to 0(\theta_R \to 0)$$

$$-m V_R \frac{1}{M} {V_R}^T m^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} \\ 0 & Z_{23} & Z_{33} \end{pmatrix}$$

$$\begin{aligned} Z_{22} &= -{m_2}^2 \left(\frac{{c_R}^2}{M_1 \exp[+i\gamma_R]} + \frac{{s_R}^2}{M_2 \exp[-i\gamma_R]} \right) \\ Z_{33} &= -{m_3}^2 \left(\frac{{s_R}^2}{M_1 \exp[+i\gamma_R]} + \frac{{c_R}^2}{M_2 \exp[-i\gamma_R]} \right) \\ Z_{23} &= -m_2 m_3 s_R c_R \left(-\frac{1}{M_1 \exp[+i\gamma_R]} + \frac{1}{M_2 \exp[-i\gamma_R]} \right) \end{aligned}$$

Computing J. $K = U_L(\theta_{ij}, \delta_L, \gamma_L) U(\theta, \phi) P$

$$
\begin{aligned}
J &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \times \\
& [c_{13} \cos 2\theta \sin \delta_L \sin 2\theta_{23} \\
&+ c_{12} \sin 2\theta \sin(\delta_L - \gamma_L - \phi) \cos 2\theta_{13} \\
&- \frac{1}{2} s_{12} s_{13} \sin 2\theta \sin 2\theta_{23} \sin(2\delta_L - \gamma_L - \phi)] \\
&+ \frac{1}{8} \sin(\theta_{12} - \theta_{13}) \sin 2\theta \sin 2\theta_{12} \sin 2\theta_{23} \times \\
& \sin(\gamma_L + \phi)
\end{aligned}
$$

At least three angles $s_{13}, s_{23}, s_{12}/\sin\theta$ **must be non-zero for** $J \neq 0$**.**
At least one combination of phases, δ_L **/** $\gamma_L + \phi$ **must be non-zero.**

Prospects for determing correlation.

In J, 4 angles and 2 combination of phases δ_L and $\phi+\gamma_L$ appears. The phase ϕ which has correlation with leptogenesis phase (γ_R) appears only with the combination $\phi+\gamma_L$. To proceed further,

A) Taking a model assumption on m_D (Gut, quark-lepton similarity...)
B) Without using any model asuumption, just using the experimental data including Baryon number (1) and neutrino mixing angles (3) + ΔP (1) + double β decay. (1) etc.

A Model : J determined by leptogenesis phase.

Suppose U_L is a real matrix ($\delta_L, \gamma_L = 0$) and $\theta_{13} = 0$.

$$J \to \quad \frac{1}{8} \sin\theta_{12} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta \sin \phi$$

$$m_D = O_{23} O_{12} \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} V_R(\gamma_R, \theta_R)$$

In this case, MNS matrix K has the form of Correlation

$$\gamma_R \to -\gamma_R$$
$$\phi \to -\phi$$
$$J \to -J \qquad L \to -L$$

Model independent approach

1) Sufficient Baryon number can be produced ? $\frac{n_B}{s} \sim 10^{-10}$.

2) neutrino mixing can be explained ?

1) Lepton number asymmetry in terms of physical quantities:

(heavy neutrino mass and (tree) decay width and light-neutrino mass: $M_1, M_2, \Gamma_1^0, \Gamma_2^0,$ n_2, n_3**)**

$$\epsilon_1 = -\frac{M_1}{2V} \times \frac{\sqrt{\left((n_3 - n_2)^2 - (x-y)^2\right)\left((x+y)^2 - (n_3+n_2)^2\right)}}{2xV}$$

$$\begin{aligned} x &= \frac{({m_D}^\dagger m_D)_{11}}{M_1} = \Gamma_1 \left(\frac{V}{M_1}\right)^2 \\ y &= \frac{({m_D}^\dagger m_D)_{22}}{M_2} = \Gamma_2 \left(\frac{V}{M_2}\right)^2 \end{aligned}$$

We can extract the possible size of CP violation for leptogenesis from the neutrino mass eigenvalue equation:

$$\begin{aligned}\epsilon_1 &= \frac{\Gamma_1 - \bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} \\ &= -\frac{M_1}{2V} \frac{Im[({m_D}^\dagger m_D)^2_{12}]}{V M_1 M_2} / \frac{({m_D}^\dagger m_D)_{11}}{M_1}\end{aligned}$$

The neutrino mass squared n^2 satisfies the following eigenvalue equation: $det[m_{eff} m^\dagger_{eff} - n^2] = 0$.

$$\begin{aligned}{n_2}^2 + {n_3}^2 &= Tr[m_D \frac{1}{M} {m_D}^T {m_D}^* \frac{1}{M} {m_D}^\dagger] \\ &= x^2 + y^2 + 2\frac{Re[({m_D}^\dagger m_D)^2_{12}]}{M_1 M_2} \\ n_2 n_3 &= \frac{det[{m_D}^\dagger m_D]}{M_1 M_2} = xy - \frac{|({m_D}^\dagger m_D)_{12}|^2}{M_1 M_2}\end{aligned}$$

CP violating part can be extracted as:

$$\frac{Im[({m_D}^\dagger m_D)^2_{12}]}{M_1 M_2} = \pm\sqrt{\left(\frac{|({m_D}^\dagger m_D)^2_{12}|}{M_1 M_2}\right)^2 - \left(\frac{Re[({m_D}^\dagger m_D)^2_{12}]}{M_1 M_2}\right)^2}$$

Lepton number asymmetry from neutrino mass

$$\epsilon_1 = -\frac{M_1}{4V}\frac{|n_3^2 - n_2^2|}{xV} \times \sqrt{\left(1 - (\frac{x-y}{n_3 - n_2})^2\right)\left((\frac{x+y}{n_3 + n_2})^2 - 1\right)}$$

$$V \sim 1(TeV)$$

$$(x, y) : x + y > n_2 + n_3; |x - y| < |n_3 - n_2|$$

$$(\rightarrow x > min.[n_2, n_3])$$

Case I

$$n_3 = \sqrt{\Delta m^2_{atom.}} \sim 5.5 \times 10^{-2} eV$$

$$n_2 = \sqrt{\Delta m^2_{solar}} = (4 \sim 5) \times 10^{-3} (LMA)$$
$$(2 \sim 3) \times 10^{-3} (SMA)$$
$$3 \times 10^{-4} (LOW)$$
$$(1 \sim 3) \times 10^{-5} (Just - so)$$

$$\frac{\epsilon_1}{10^{-8}} \sim -\frac{M_1}{V} 10^{-6} \sqrt{\frac{\Delta m^2_{atm}}{\Delta m^2_{sol}}} \times$$
$$\frac{n_2}{x} \sqrt{\left(1 - \left(\frac{x-y}{n_3 - n_2}\right)^2\right)\left(\left(\frac{x+y}{n_3+n_2}\right)^2 - 1\right)}$$

$$\epsilon_1 = -10^{-8} \to M_1 > 10^6 (TeV) \sim 10^4 (TeV).$$
$$(\sqrt{\Delta m^2_{sol}} = 4 \times 10^{-3} \sim 10^{-5}.)$$

A condition for the produced L is not destroyed by inverse decay $l\phi \to N$.

$$\Gamma_1 < H|_{T=M_1} = \frac{1}{2}\sqrt{\frac{32\pi^3}{90}}\sqrt{g^*}\frac{T^2}{M_{pl}} (@T = M_1)$$

$$x = \Gamma_1 \frac{V^2}{M_1^2} < 1.7 \times 10^{-3} (eV) \quad (g^* = 106.75)$$

Case II

$$n_3^2 - n_2^2 = \Delta m_{solar}^2$$
$$n_3 \sim n_2 = \sqrt{\Delta m_{atom}^2}$$

$$x \ > \ \sqrt{\Delta m_{atom}^2} = 0.055(eV)$$

This case contradicts with the condition, $x < 10^{-3}$ (eV).

Fig.1 Correlation between $(\sin 2\gamma_R, \sin\phi)$
with fixed $s = (x + y) \times 10^{-2}$ eV , varying $u = y - x$,
$\frac{M_1}{M_2} = 0.1$.

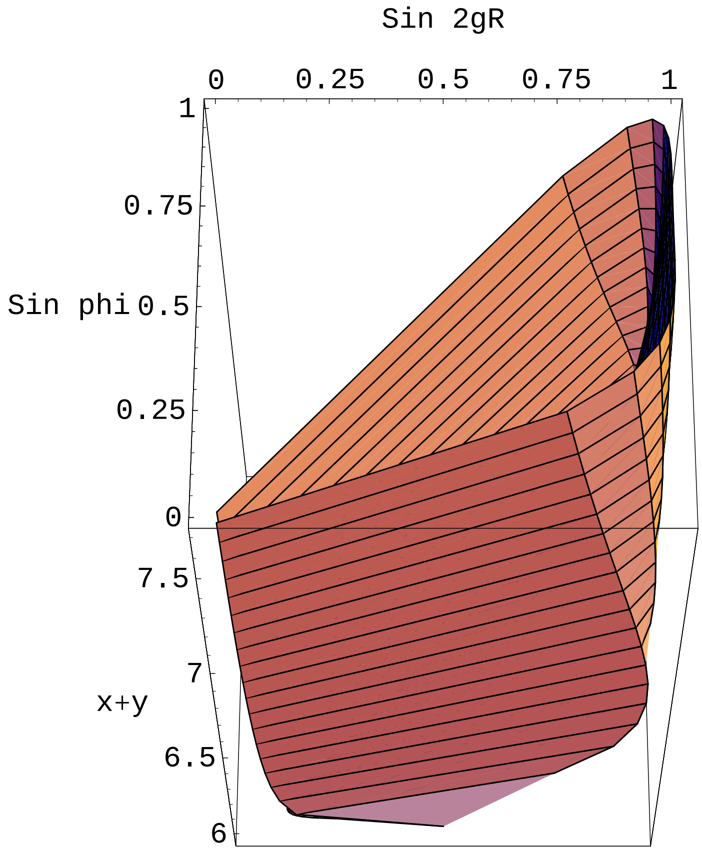


Fig.2 Correlation between $(\sin 2\gamma_R, \sin\phi)$
with fixed $s = (x + y) \times 10^{-2}$ eV varying $u = y - x$

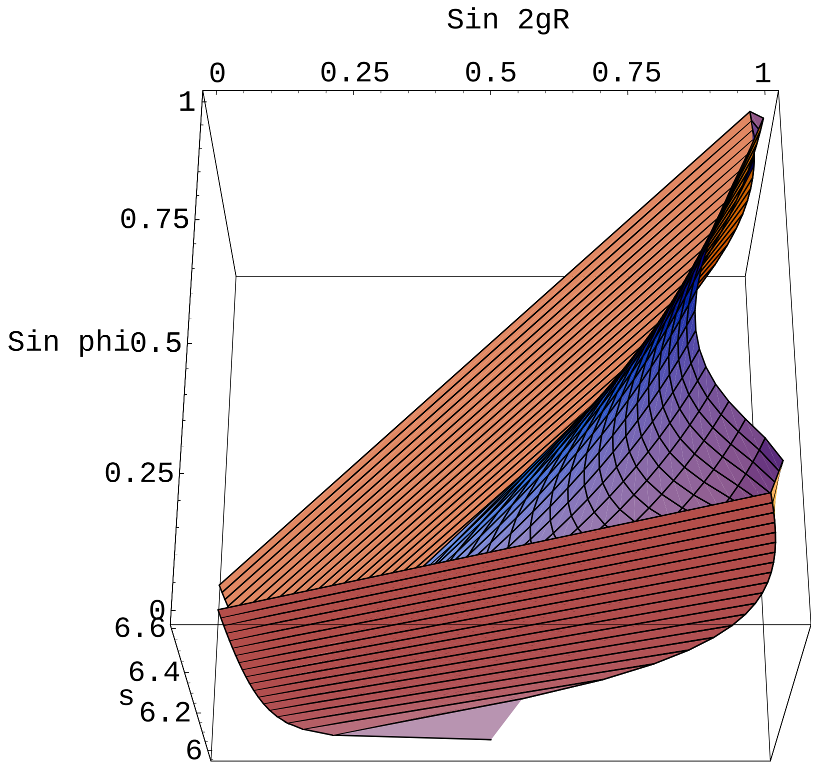
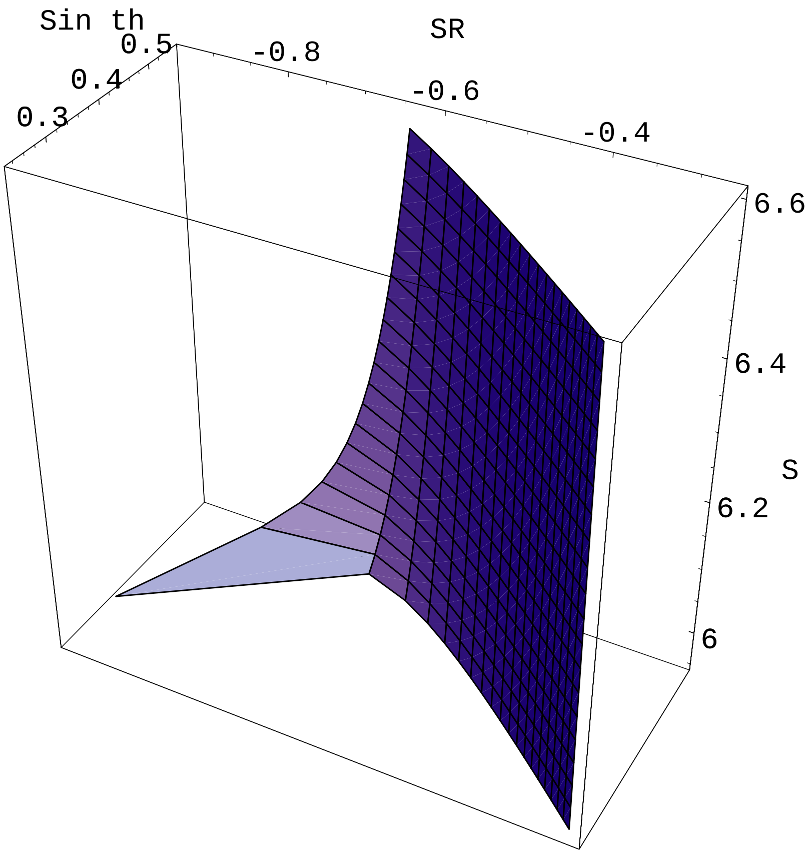


Fig.2.1 Correlation between $(S_R, \sin\theta)$
with fixed $s = (x + y) \times 10^{-2}$ eV varying $u = y - x$



Mixing Angles

This depends on Modeling of U_L.

We work on a working hypothesis:

$$m_D = O_{23} O_{12} \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} V_R(\gamma_R, \theta_R)$$

In this case, MNS matrix has the form of

$$K = O_{23}(\theta_{23}) O_{12}(\theta_{12}) U_{23}(\theta, \phi)$$

Taking $\theta_{23} = \pi/4$, $\theta_{12} = \pi/4$, we have:

$$K = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{\sin\theta \exp[-i\phi]}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\cos\theta - \sqrt{2}\sin\theta\exp[i\phi]}{2} & \frac{\sin\theta\exp[-i\phi] + \sqrt{2}\cos\theta}{2} \\ \frac{1}{2} & -\frac{\cos\theta + \sqrt{2}\sin\theta\exp[i\phi]}{2} & \frac{-\sin\theta\exp[-i\phi] + \sqrt{2}\cos\theta}{2} \end{pmatrix} P$$

$$J \to \frac{1}{8\sqrt{2}} \sin 2\theta \sin\phi$$

$$s_{13} \exp[-i\delta] = \frac{\sin\theta}{\sqrt{2}} \exp[-i\phi]$$

Fig.3 Correlation between $(J, \frac{ep_1}{10^{-8}})$ with fixed $s = (x+y) = 6, 7, 8 \times 10^{-2}$ eV , varying $u = y - x$, $\frac{M_1}{M_2} = 0.1, M_1 = 10^9$ (GeV)

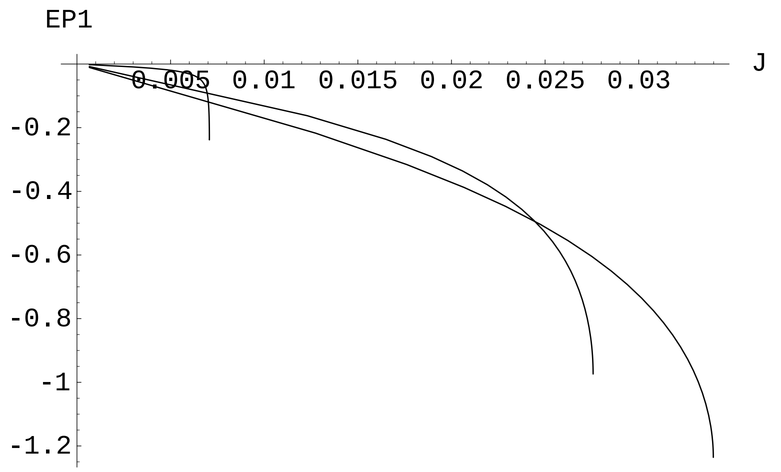


Fig.4 (s,u) regions for $\frac{1}{\sqrt{2}} < s_{13} = \frac{\sin\theta}{\sqrt{2}} < 0.1$ and $0.025 < J < 0.0255$.

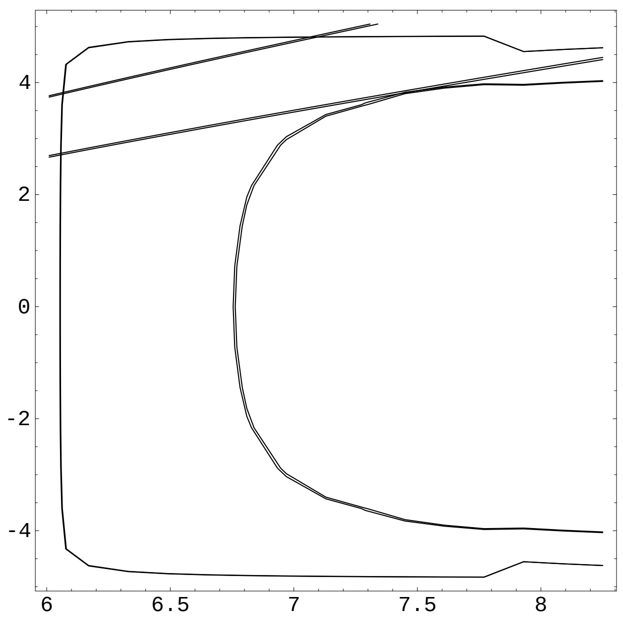
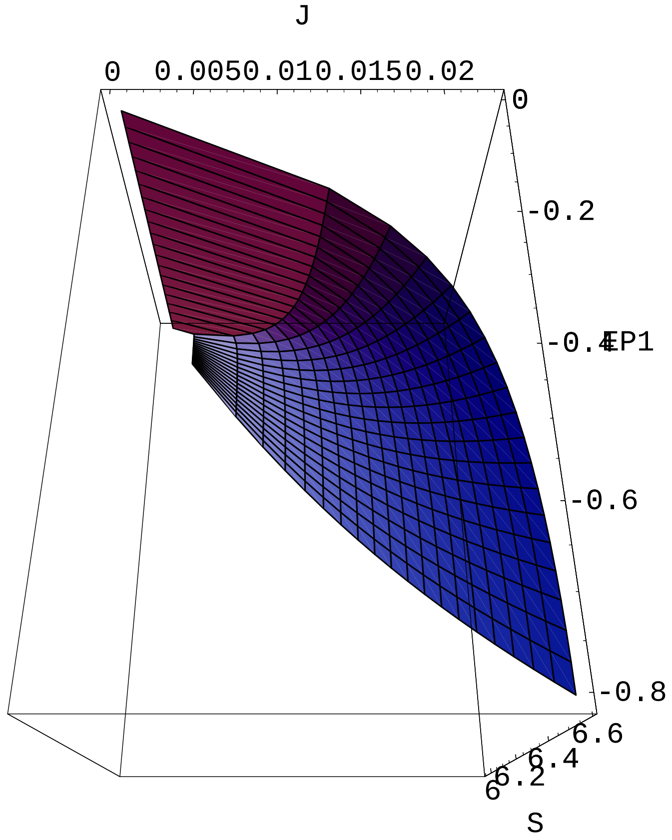


Fig.5 Correlation between $(J, \frac{ep_1}{10^{-8}})$
with fixed $s \times 10^{-2}$ eV ,
varying $u = y - x$, $\frac{M_1}{M_2} = 0.1, M_1 = 10^9$ (GeV)



Summary

- leptogenesis phase (γ_R) certainly affects the neutrino oscillation CP violation through (ϕ)

- However, in general ground, we can not distinguish the phase coming from leptogenesis (ϕ) and another phase γ_L in U_L because only a combination appear as $\phi+\gamma_L$. The isolation must be done using some other quantities, double β decay etc.

- Leptogenesis formule is written in terms of physical quantities: Heavy neutrino mass and decay width and light neutrino mass.

- Combining the condition prohibitting inverse decay (long life time of N) with the formulae, we can already discriminate the case of two nearly degenerate at atomospheric mass scale. The other case (if it is LMA solution), the condition looks marginal.

- Numerical scheme computing ϕ and θ by using the input of Γ_i, M_i, n_i are set up. We show the correlation between J and Lepton number asymmetry for a specific choice of U_L.