Muon decay:

Is it really

non-neutrino physics ?

W. Fetscher

Institute for Particle Physics

ETH Zürich,

CH-8093 Zürich, Switzerland

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1. Introduction

$$\boxed{\mu^+ \to \quad \overline{\nu}_\mu \, (e^+ \nu_e)}$$

One of the fundaments of the standard model:

G_F : Strength of the charged weak interaction

$V - A$: Chiral structure of the charged weak interaction

Both are to be determined by experiment. Measurements exist with

$\mu^\pm$: Lifetime, decay asymmetry

$e^\pm$: Spectrum, polarization, decay asymmetry

ν_μ : Inverse muon decay

ν_e : spectrum (KARMEN)

THEORY OF WEAK INTERACTIONS IN PARTICLE PHYSICS

ROBERT E. MARSHAK
Distinguished University Professor of Physics
University of Rochester, Rochester, N.Y.

RIAZUDDIN
Director, Institute of Physics
University of Islamabad, Rawalpindi

CIARAN P. RYAN
Lecturer, Mathematical Physics Department
University College, Dublin

$$
\begin{aligned}
& d\Gamma(\mu^- \to e^- + \bar{\nu}_1 + \nu_2) = \frac{d\mathbf{p}_2}{(2\pi)^4}\frac{1}{8E_\mu E_e} \\
1\quad & \times [(|C_S|^2 + |C_S'|^2 + |C_P|^2 + |C_P'|^2)K^2\{(p_1 \cdot p_2) \overset{-}{\cancel{+}} m_\mu m_e(n_1 \cdot n_2)\} \\
2\quad & + (|C_S|^2 + |C_S'|^2 - |C_P|^2 - |C_P'|^2)K^2\{(p_1 \cdot p_2)(n_1 \cdot n_2) \\
3\quad & - (p_1 \cdot n_2)(p_2 \cdot n_1) - m_\mu m_e\} - (C_S C_P'^* + C_S' C_P^* + C_S^* C_P' + C_S'^* C_P) \\
4\quad & \times K^2\{m_\mu(p_2 \cdot n_1) - m_e(p_1 \cdot n_2)\} + (C_S C_P'^* + C_S' C_P^* - C_S^* C_P' - C_S'^* C_P) \\
5\quad & \times K^2\{\varepsilon_{\alpha\beta\gamma\delta} p_{1\alpha} n_{1\beta} p_{2\gamma} n_{2\delta}\} + 2(|C_V|^2 + |C_V'|^2 + |C_A|^2 + |C_A'|^2) \\
& \times \{\tfrac{2}{3}(K \cdot p_1)(K \cdot p_2) + \tfrac{1}{3}(p_1 \cdot p_2)K^2 + m_\mu m_e[\tfrac{2}{3}(K \cdot n_1)(K \cdot n_2) \\
6\quad & + \tfrac{1}{3}(n_1 \cdot n_2)K^2]\} + 2(|C_V|^2 + |C_V'|^2 - |C_A|^2 - |C_A'|^2)\{\tfrac{2}{3}K^2[(p_1 \cdot p_2) \\
& \times (n_1 \cdot n_2) - (p_1 \cdot n_2)(p_2 \cdot n_1) + \tfrac{3}{2} m_\mu m_e] + \tfrac{2}{3}(K \cdot n_1)[(p_1 \cdot p_2)(K \cdot n_2) \qquad \text{falsch} \\
& - (p_1 \cdot \overset{n_2}{\cancel{p_2}})(K \cdot p_2)] - \tfrac{2}{3}(K \cdot p_1)[(n_1 \cdot p_2)(K \cdot n_2) - (n_1 \cdot n_2)(K \cdot p_2)]\} \qquad (?) \\
7\quad & + 2(C_V C_A'^* + C_V' C_A^* + C_V^* C_A' + C_V'^* C_A)\{m_e[\tfrac{2}{3}(p_1 \cdot K)(n_2 \cdot K) \\
& + \tfrac{1}{3}K^2(p_1 \cdot n_2)] + m_\mu[\tfrac{2}{3}(n_1 \cdot K)(K \cdot p_2) + \tfrac{1}{3}K^2(n_1 \cdot p_2)]\} \\
8\quad & + 2(C_V C_A'^* + C_V' C_A^* - C_V^* C_A' - C_V'^* C_A)\tfrac{1}{3}(-m_\mu^2 + m_e^2)\varepsilon_{\alpha\beta\gamma\delta} p_{1\alpha} n_{1\beta} p_{2\gamma} n_{2\delta} \\
9\quad & + 2(|C_T|^2 + |C_T'|^2)\{\tfrac{4}{3}(K \cdot p_1)(K \cdot p_2) - \tfrac{1}{3}K^2(p_1 \cdot p_2) \\
& - m_\mu m_e[\tfrac{4}{3}(K \cdot n_1)(K \cdot n_2) - \tfrac{1}{3}K^2(n_1 \cdot n_2)]\} \\
10\quad & - 2(C_T C_T'^* + C_T^* C_T')\{\tfrac{4}{3}m_\mu(K \cdot n_1)(K \cdot p_2) - \tfrac{1}{3}m_\mu(n_1 \cdot p_2)K^2 \\
& - \tfrac{4}{3}m_e(K \cdot p_1)(K \cdot n_2) + \tfrac{1}{3}m_e(n_2 \cdot p_1)K^2\}] \qquad (3.10)
\end{aligned}
$$

where $K = p_1 - p_2$.

It is instructive to examine expression (3.10) in some detail since it tells us very directly what it is possible to learn from muon decay when the neutral particles are not observed. In the first place, it is evident that we cannot fix the interaction uniquely because in (3.10) we have only ten real functions of C_i and C_i' and we need nineteen relations to

$$a = |C_S|^2 + |C_S'|^2 + |C_P|^2 + |C_P'|^2$$

F. Scheck, Muon physics

$$\alpha = |C_S|^2 + |C_S'|^2 - |C_P|^2 - |C_P'|^2$$

$$b = |C_V|^2 + |C_V'|^2 + |C_A|^2 + |C_A'|^2$$

$$\beta = |C_V|^2 + |C_V'|^2 - |C_A|^2 - |C_A'|^2$$

$$c = |C_T|^2 + |C_T'|^2$$

$$a' = 2\,\mathrm{Re}(C_S C_P'^* + C_S' C_P^*)$$

$$b' = 2\,\mathrm{Re}(C_V C_A'^* + C_V' C_A^*)$$

$$c' = -2\,\mathrm{Re}(C_T C_T'^*)$$

$$\alpha' = 2\,\mathrm{Im}(C_S C_P'^* + C_S' C_P^*)$$

$$\beta' = 2\,\mathrm{Im}(C_V C_A'^* + C_V' C_A^*)$$

$$A = a + 4b + 6c$$

$$\rho = \frac{1}{A}(3b + 6c)$$

$$\eta = \frac{1}{A}(\alpha - 2\beta)$$

$$\zeta = -\frac{1}{A}(3a' + 4b' - 14c')$$

$$\delta = \frac{1}{A\xi}(-3b' + 6c')$$

$$\xi' = -\frac{1}{A}(a' + 4b' + 6c')$$

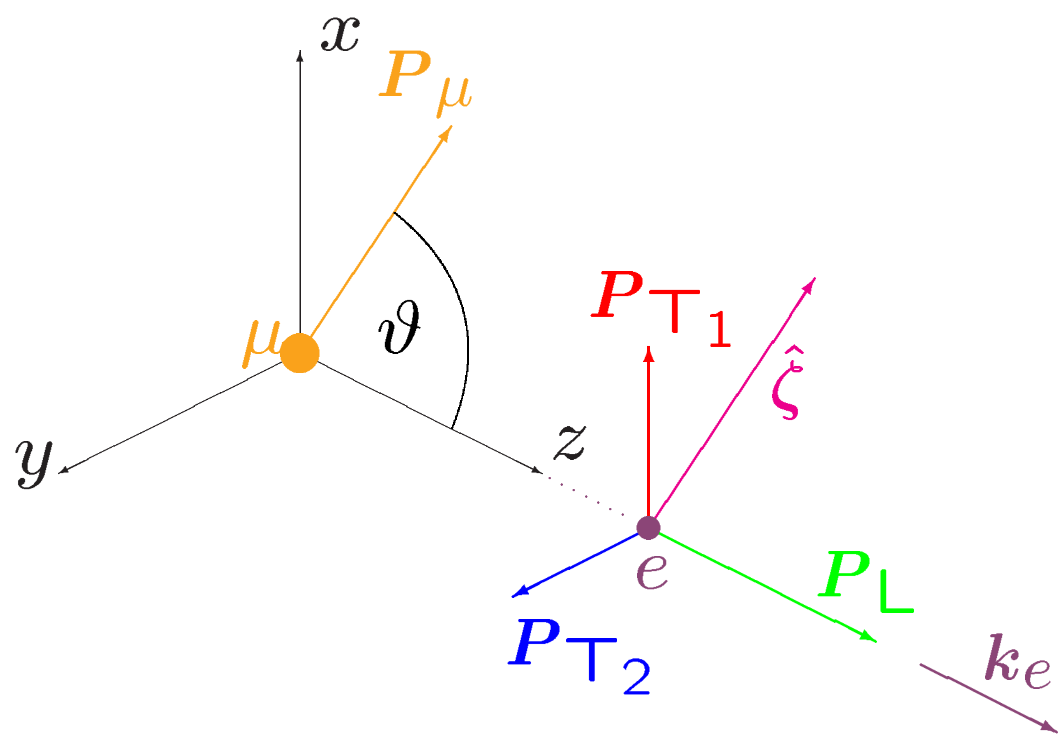
$$\xi' = -\frac{1}{A\xi'}(b' - 2c')$$

$$\xi'' = \frac{1}{A}(3a + 4b - 14c)$$

$$\rho' = \frac{1}{A\xi''}(3b - 6c)$$

$$\eta' = \frac{1}{A\xi''}(3\alpha + 2\beta).$$

2. μ- and e- Observables



$$\frac{d^2\Gamma}{dx\, d\cos\vartheta} = \frac{1}{4} m_\mu W_{\mu e}^4 G_F^2 \sqrt{x^2 - x_0^2} \cdot \left(F_{IS}(x) \pm P_\mu \cos\vartheta F_{AS}(x)\right) \cdot \left(1 + P_e \cdot \hat{\zeta}\right)$$

$\hat{\zeta}$ = Direction of greatest sensitivity of a polarization dependent detector.

$$W_{\mu e} = \frac{m_\mu^2 + m_e^2}{2\,m_\mu} \approx \frac{m_\mu}{2}$$

(Maximal positron energy)

$$x = \frac{E_e}{W_{\mu e}}$$

(Reduced positron energy)

$$x_0 = \frac{m_e}{W_{\mu e}}$$

(Minimal positron energy)

$$x_0 \le x \le 1$$

(Energy range)

$\boldsymbol{P}_\mu =$ muon polarization

$\boldsymbol{P}_e =$ positron polarization

Observables:

1.	e spectrum		$\rightarrow$	ϱ	η	λ
2.	e decay asymm.		$\rightarrow$	ξ	δ	
3.	e polarization					
	longitudinal	P_L	$\rightarrow$	ξ'	ξ''	
	transverse	P_{T_1}	$\rightarrow$	η	η''	
	normal	P_{T_2}	$\rightarrow$	$\frac{\alpha'}{A}$	$\frac{\beta'}{A}$	
4.	muon life time		$\rightarrow$	τ_μ		
5.	ν_e spectrum		$\rightarrow$	ω_L		

3. Complete determination of the interaction from experiments

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MUON DECAY: COMPLETE DETERMINATION OF THE INTERACTION AND COMPARISON WITH THE STANDARD MODEL

W. FETSCHER, H.-J. GERBER and K.F. JOHNSON [1]

Institut für Mittelenergiephysik, Eidgenössische Technische Hochschule Zürich, CH-5234 Villigen, Switzerland

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Present experiments determine the most general (local, derivative-free, lepton-number conserving) leptonic four-fermion interaction hamiltonian of the normal and inverse muon decay. Numerical results are given for all ten complex coupling constants and nine T-violating amplitudes with respect to the "helicity projection form".

Assume:

- $\mu^- \quad \to \quad e^- \quad \nu_\mu \quad \overline{\nu}_e$
- point interaction
- m_ν, $m_{\overline{\nu}} \ll m_\mu$
- CPT invariance

The standard model is not assumed.

Number of parameters:

5 different interactions S, V, T, A, P	$\to$	5
Violation of P invariance	$\to$	10
Violation of T invariance	$\to$	20
Unobservable phase	$\to$	19

(L. Michel, 1950).

Hamiltonian:

- Charge changing form (CCF)

- Leptons of definite handedness (Chiral fields)

1957: Gell-Mann, Feynman; chiral form for $V - A$

1983: Scheck, Mursula and Scheck; chiral form generalised to all interactions

Our matrix element (FGJ):

$$\boxed{\mathcal{M} = \frac{4\,G_{\mathsf{F}}}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T\\ \varepsilon,\mu=R,L}} g^{\gamma}_{\varepsilon\mu} \langle \overline{e}_{\varepsilon} | \Gamma^{\gamma} | (\nu_e)_n \rangle \langle \overline{\nu}_m | \Gamma_{\gamma} | (\mu)_{\mu} \rangle}$$

10 complex coupling constants $g^{\gamma}_{\varepsilon\mu}$;

19 real parameters.

The index γ labels the type of interaction:

$$\Gamma^S = \text{4-scalar}$$
$$\Gamma^V = \text{4-vector}$$
$$\Gamma^T = \text{4-tensor}$$

The indices ε, μ indicate the chiralities of the spinors of the observed (charged) leptons. The chiralities n, m of the neutrinos are uniquely determined for given γ, ε and μ.

The goal: First model independent determination of all 19 parameters using all existing experimental data.

1) μ lifetime $\Rightarrow$ Fermi coupling constant G_F.
18 parameters remain.

In the following we are only interested in the relative weights of the different couplings, therefore the $g^\gamma_{\varepsilon\mu}$ are normalized:

$$\begin{aligned}
A \equiv \quad & 4\left(|g^S_{RR}|^2 + |g^S_{LR}|^2 + |g^S_{RL}|^2 + |g^S_{LL}|^2\right) \\
& + 16\left(|g^V_{RR}|^2 + |g^V_{LR}|^2 + |g^V_{RL}|^2 + |g^V_{LL}|^2\right) \\
& + 48\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right) \\
:= \quad & 16
\end{aligned}$$

18 independent parameters left.

"$V-A$" corresponds to $g^V_{LL}=1$, with all of the remaining $g^\gamma_{\varepsilon\mu}=0$.

The normalization allows to define the four probabilities $Q_{\varepsilon\mu}$ of obtaining an electron of handedness ε and a muon of handedness μ in muon decay:

$$\boxed{\begin{aligned}
Q_{RR} &\equiv \frac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2 && = L_{RR}\\
Q_{LR} &\equiv \frac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + 3|g^T_{RL}|^2 && = L_{LR}\\
Q_{RL} &\equiv \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + 3|g^T_{LR}|^2 && = L_{RL}\\
Q_{LL} &\equiv \frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2 && = L_{LL}
\end{aligned}}$$

$\leftarrow$ THEORY $\rightarrow$ $\leftarrow$ EXP. $\rightarrow$

2) 9 measurements in normal muon decay

(without neutrino detection):

Numerical values for L_{RR}, L_{LR} and L_{RL}.

$\Longrightarrow$ Limits for the $Q_{\varepsilon\mu}$:

Q_{RR}	$<$	2×10^{-3}
Q_{LR}	$<$	4×10^{-3}
Q_{RL}	$<$	45×10^{-3}
Q_{LL}	$>$	949×10^{-3}

$\Longrightarrow$ Limits for 16 decay parameters:

$Q_{RR} \longrightarrow \lvert g^S_{RR} \rvert,\ \lvert g^V_{RR} \rvert,$
$Q_{LR} \longrightarrow \lvert g^S_{LR} \rvert,\ \lvert g^V_{LR} \rvert,\ \lvert g^T_{LR} \rvert$
$Q_{RL} \longrightarrow \lvert g^S_{RL} \rvert,\ \lvert g^V_{RL} \rvert,\ \lvert g^T_{RL} \rvert$

$$Q_{LL} > 0.949 \Longrightarrow$$

$$\boxed{\begin{array}{l} \quad \{\, g^S_{LL} = 2,\ g^V_{LL} = 0\} : S, P \\ \text{or } \{\, g^S_{LL} = 0,\ g^V_{LL} = 1\} : \text{V-A} \end{array}}$$

$\underline{\mu^-, e^-}$ lefthanded as verified by experiment.

What about the $\underline{\nu_\mu, \nu_e}$?

Scalar interaction: Spin flip at vertex.

Vector interaction: No Spin flip at vertex.

(Jensen and Stech [1955]).

$$\Rightarrow$$

$g^S_{LL} \neq 0$	: righthanded neutrinos
$g^V_{LL} \neq 0$	: lefthanded neutrinos

Measurement of neutrinos from normal muon decay extremely difficult (C. Jarlskog [1966]).

Inverse muon decay: Data exist (CHARM [1983])!

Total production rate S , normalized to the rate predicted by $V - A$, is :

$$S = (1054 \pm 79) \times 10^{-3}$$

(CHARM II [1990]; CCFR measurement of similar precision.)

$$\boxed{S = \frac{1}{2}(1 - P_{\nu_\mu}) \cdot S_- + \frac{1}{2}(1 + P_{\nu_\mu}) \cdot S_+}$$

with

$$S_- = |g^V_{LL}|^2 + \frac{3}{8}|g^V_{RL}|^2 + \frac{3}{32}|g^S_{RR}|^2 + \frac{3}{32}|g^S_{LR} - \frac{10}{3}g^T_{LR}|^2 + \frac{4}{3}|g^T_{LR}|^2$$

and

$$S_+ = |g^V_{RR}|^2 + \frac{3}{8}|g^V_{LR}|^2 + \frac{3}{32}|g^S_{LL}|^2 + \frac{3}{32}|g^S_{RL} - \frac{10}{3}g^T_{RL}|^2 + \frac{4}{3}|g^T_{RL}|^2$$

Polarization P_{ν_μ} of the ν_μ from π decay:

a) Magnitude : From

$$\boxed{1 - P_\mu \cdot \xi \frac{\delta}{\rho} < 3.2 \times 10^{-3}}$$

(Carr et al. [1983]; Jodidio et al. [1984])

one can deduce

$$\boxed{1 - |P_{\nu_\mu}| < 3.2 \times 10^{-3}}$$

(W. F. [1984])

b) Sign :

$P_{\nu_\mu} < 0$ (Backenstoss et al. [1961])

$P_{\overline{\nu}_\mu} > 0$ (Roesch et al. [1982])

The sign is determined with the help of the electromagnetic interaction!

With $P_{\nu_\mu} \approx -1$ and, from normal muon decay,

$$\left\{g^V_{RL}, g^S_{RR}, g^S_{LR}, g^T_{LR}\right\} \approx 0$$

$$\Rightarrow \quad \boxed{S = |g^V_{LL}|^2}$$

$$\Rightarrow \boxed{|g^V_{LL}| > 0.96 \quad (90\% C.L.)}$$

$$\Rightarrow \boxed{S = |g^V_{LL}|^2}$$

$$\Rightarrow \boxed{|g^V_{LL}| > 0.96 \quad (90\% C.L.)}$$

⇒ **First experimental verification of the $V - A$ hypothesis in muon decay!**

From

$$\boxed{|g^V_{LL}|^2 + \frac{1}{4}|g^S_{LL}|^2 \leq 1}$$

follows a lower limit for $|g^S_{LL}|$:

$$\boxed{|g^S_{LL}| < 0.55 \quad (90\% C.L.)}$$

This coupling constant can only be determined indirectly!

Minimal Number of Measurements

11 measurements $\rightarrow$ 19 decay parameters!

Question: What is the minimum number of measurements necessary to determine the interaction?

Answer: Determine chirality of 3 fermions:

1) μ (Decay asymmetry)

$$\frac{1}{2}\left\{1+\frac{1}{9}(3\xi-16\xi\delta)\right\} = \frac{1}{4}|g^S_{RR}|^2 + \frac{1}{4}|g^S_{LR}|^2 + |g^V_{RR}|^2 + |g^V_{LR}|^2 + 3|g^T_{LR}|^2$$

2) e (Longitudinal polarization)

$$\frac{1}{2}\{1-\xi'\} = \frac{1}{4}|g^S_{RR}|^2 + \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RR}|^2 + |g^V_{RL}|^2 + 3|g^T_{RL}|^2$$

3) ν_μ

Measure inverse muon decay with neutrinos of a given helicity, and

4) measure lifetime for the absolute magnitude of the interaction.

$\mu^- \to e^- \bar{\nu}_e \nu_\mu$