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THE LINEAR ACCELERATOR STRUCTURES WITH SPACE-UNIFORM QUADRUPOLE FOCUSING

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Introduction

The linear accelerators with space-uniform quadrupole focusing do not require high-voltage injector and allow to have high capture efficiency without any preliminary bunching. Wide capture region and large acceptance allow to get high values of current limits. The units with space-uniform focusing are very effective as an initial part of high-current linear accelerators for medium and high energies.

Description of operation

In linear accelerators with drift tubes the quadrupole focusing system has the strongly marked space periodicity: either the poles polarity of quadrupole lenses or the geometry of poles alternates along the axis. But with time alternating voltage there may be used the quadrupole system of the focusing electrodes which is uniform along the accelerator axis. Such system is shown in Fig.1. The magnitude $U_l = 2U_a$ is the voltage amplitude between two adjacent electrodes. As the electrodes are supplied with HF voltage $\pm U_a \cos\omega t$, so the particles are sequentially exposed to fields with alternating gradient signs while they are travelling along the axis. In the space-uniform system this effect leads to the quadrupole focusing.

If the distance between opposite electrodes of the same polarity in four-wire line varies periodically along the axis, there appears a longitudinal accelerating component of the HF field. The space period of the variation must be equal to the synchronous particle path during a period of the HF. The phases of distance changings in the mutually perpendicular planes have a half-period shift. The electric field potential at the axis under these conditions is modulated with period $\beta\lambda$, that create resonant accelerating effect. Fig.2 shows round electrodes of alternating diameter with conical transitions; there are given a section of electrodes by the plane passing through the longitudinal axis and three cross sections with the co-ordinates $z = -\frac{1}{4}\beta\lambda$; 0; $\frac{1}{4}\beta\lambda$. The longitudinal axis shows variable quantity $k_1 z$, where k_1 - is the wave number of the accelerating wave: $k_1 = 2\pi/\beta\lambda$. The function defining the electrodes diameter is odd relatively to the section with exact quadrupole symmetry. The sections of the modulated four-wire line consisting of the electrodes of "crank-shaft" type are shown in Fig.3.

HF resonators

The HF supply of the four-wire line may be fulfilled by a resonator with longitudinal magnetic field. Fig.4 shows possible types of resonators: the four-chamber resonator [1] and double H-resonator [2]. The H-resonator, due to preposition of its inventor V.A.Tepljakov, one calls a construction, the main resonating element of which is a cylinder with longitudinal gap along its wall; the electric field is mainly concentrated in the gap.

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In Fig.4 are shown the directions of longitu dinal magnetic field and electric lines of force in the region of interaction with the beam. The shown sections correspond to the planes with exact quadrupole symmetry. The magnetic fluxes connection takes place at the bottoms of the resonator. The intercham ber partitions do not reach to bottoms. The first realized units of the accelerator wit space-uniform focusing were made as double H-resonator [3]. At the first stage of constru tion it seemed to be technologically simple and more reliable then the four-chamber res nator. Nevertheless the four-chamber resona tor have some advantages in comparison with the double H-resonator. The symmetry of fou chamber resonator corresponds to the quadru pole symmetry of electric field in the regi of interaction with the beam. This simplifi the adjustment. The four-chamber resonator a little less in dimensions and has smalle HF losses than the double H-resonator. In conclusion in four-chamber resonator it is much simpler to vary the shape of the modu lated electrodes. The technological diffic ties of the four-chamber resonator may be successfully solved.

Let us define the distributed capacitan per unit of the four-wire quadrupole line by the equality

$$\frac{dJ}{dz} = -C_u \frac{dU_l}{dt}$$

where J - the full conduction current comi up to one electrode. Let us neglect the ma netic field in the region of the interacti with beam. Then the radius R of the infin tely long four-chamber resonator with thin partitions will be defined by the equation

$$\frac{J_1(kR)}{N_1(kR)} = \frac{J_0(ka) + \frac{\pi C_u}{4\varepsilon_0} ka\, J_1(ka)}{N_0(ka) + \frac{\pi C_u}{4\varepsilon_0} ka\, N_1(ka)} \quad ;$$

a- interaction region radius; $k = 2\pi/\lambda$. value of the magnetic field in each chamb only slightly depends on co-ordinates. As ming the value of the magnetic field in t chamber to be constant and $a \ll R$, one can such approximate dependence between the w length of the quadrupole mode of oscillat and the resonator radius

$$\lambda = R\sqrt{\frac{\pi^3 C_u}{2\varepsilon_0}}$$

For the double H-resonator with the radiu the resonating cylinder R_1 and the radiu the shield cylinder R_2 under the same app ximation

$$\lambda = 2R_1\sqrt{\frac{\pi^3 C_u}{\varepsilon_0}\left[1 - 2\left(\frac{R_1}{R_2}\right)^2\right]}$$

The expressions (2,3) are the more accur the bigger the distributed capacitance p unit. The resistance losses of HF power unit length of the resonator may be eval by the expression

$$P = f\, U_l^2 \sqrt{\frac{1}{\sigma}(\omega C_u)^3} \qquad \text{Wt/m},$$

where σ - the specific conductivity of r nator walls (ohm^{-1} m^{-1}).

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$$\frac{{}_o(ka) + \frac{\pi C_u}{4\varepsilon_o} ka\, J_1(ka)}{(ka) + \frac{\pi C_u}{4\varepsilon_o} ka\, N_1(ka)} \quad ; (1)$$

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$$= R\sqrt{\frac{\pi^3 C_u}{2\varepsilon_o}} \quad (2)$$

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g cylinder R_1 and the radius of
linder R_2 under the same appro-

$$\sqrt{\frac{\pi^3 C_u}{\varepsilon_o}}\left[1-2\left(\frac{R_1}{R_2}\right)^2\right] \quad (3)$$

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$$\sqrt{\frac{1}{\sigma}(\omega C_u)^3} \qquad \text{Wt/m}, \quad (4)$$

specific conductivity of reso-
ohm⁻¹ m⁻¹).

double H-resonator

$$\sqrt{2\pi}\left[1-2\left(\frac{R_1}{R_2}\right)^2\right]^{\frac{3}{2}}\left\{1+\frac{2\left(\frac{R_1}{R_2}\right)^3\left(1+2\frac{R_1}{R_2}\right)}{\left[1-2\left(\frac{R_1}{R_2}\right)^2\right]^2}\right\};$$

four-chamber resonator

$$f = \frac{4+\pi}{2\sqrt{\pi}}.$$

evaluations show that the dimensions of
resonators and resistance losses are
l; so under $C_u = 40$ pF/m the cavity radi-
s $R \approx \lambda/10$. The calculated value of resis-
e losses in four-chamber resonator under
4m and $U_L = 300$ kV are approximately
Wt/m; but as it is known from experience
Alvarez resonator, the real resistance
ses may be 2-3 times more than calculated.

Accelerating and focusing electrodes

Let us consider the four-wire quadrupole
e. At quasi-stationary approximation the
ctric potential in the region of the axis
re the beam interacts with the field may
presented as

$$U(r,\psi,z,t) = U_o(r,\psi,z)\cos\omega t.$$

function of the amplitude distribution in
mon case is

$$(r,\psi,z) = -\frac{U_L}{2}\left[F_o(r,\psi) + \sum_{n=1}^{\infty} F_n(r,\psi)\sin(2n-1)k_1 z\right] \quad (5)$$

function

$$F_o(r,\psi) = \sum_{s=0}^{\infty} A_{os}\, r^{2(2s+1)} \cos 2(2s+1)\psi \quad (6)$$

es the law of potential distribution in
section with co-ordinates $k_1 z = \nu\pi$
$= 0,1,2,\ldots$), where the field has accurate
drupole symmetry. The coefficients F_n
the harmonics of the space modulation of
potential are defined by the series

$$F_n(r,\psi) = \sum_{s=0}^{\infty} A_{ns} I_{4s}\left[(2n-1)k_1 r\right]\cos 4s\psi \quad (7)$$

are the modified Bessel function. The
metry of the field is taken into account
expressions (5 - 7). The first term of the
ies (7) describes the axial symmetrical
ponent of the potential; the rest of the
ies gives the components with the symmet-
s of higher order.
Near of the axis the electric field is of
π mode wave. The portions of particle
rgy are being gained along each $1/2\cdot\beta\lambda$ of
. Let φ to be the field phase at the
ent when the particle is in the plane with
t quadrupole symmetry. Then with the
t degree of approximation for any particle
$k_1 z + \varphi$. Let us assume that during acce-
tion period the transverse co-ordinates
particles remain approximately constant.
increase of the particle energy along the
leration period $L = \frac{1}{2}\beta\lambda$ in this case is
ned by expression

$$= \frac{ekU_L}{2}\sum_{n=1}^{\infty}(2n-1)F_n(r,\psi)\int_0^L \cos(2n-1)k_1 z\cos(k_1 z+\varphi)\,dz,$$

follows

$$\Delta W = \frac{\pi}{4} eU_L F_1(r,\psi)\cos\varphi. \quad (8)$$

can see that only the first harmonic of
space modulation of potential gives the
energy increase. The value

$$T = \frac{\pi}{4}F_1(0) = \frac{\pi}{4}A_{10}$$

is the analogue of the transit time factor of the particle moving along the axis and defines the acceleration efficiency. For the particle moving along the axis

$$\Delta W = e\,U_L\,T\cos\varphi. \quad (9)$$

The transversal oscillations of the particles in nonrelativistic approximation are described by the equation

$$\frac{d^2x}{dt^2} = \frac{eU_L}{2m_o}\left[\frac{\partial F_o}{\partial x}\cos\omega t + \sum_{n=1}^{\infty}\frac{\partial F_n}{\partial x}\sin(2n-1)k_1 z\cos(k_1 z+\varphi)\right].$$

Assuming that the half of the period of the space electrode modulation is much shorter than the transversal oscillations wavelength, it is possible to change the second term in square brackets by the value averaged for the half of the period L. Then the last equation may be simplified

$$\frac{d^2x}{dt^2} = \frac{eU_L}{2m_o}\left[\frac{\partial F_o}{\partial x}\cos\omega t - \frac{1}{2}\frac{\partial F_1}{\partial x}\sin\varphi\right] \quad (10)$$

The main quadrupole focusing effect is defined by the quadratic term of the series (6). The other components give rise to the beginning of the various nonlinear effects. Let us confine ourselves with the linear approximation to the quadrupole component of the electric field assuming

$$F_o(r,\psi) = \varkappa\left(\frac{r}{a}\right)^2\cos 2\psi. \quad (11)$$

Later on the value a we will regard as minimum distance from the axis to the electrode; this distance define the aperture of the channel and accodingly the acceptance of the channel. The coefficient $\varkappa$ depends upon the depth of modulation of the electrodes. As it follows from the equations (8,10) the acceleration and defocusing of the particles for a first approximation depend only on the function $F_1(r,\psi)$. The paraxial particles are forced mainly by the cylindrically symmetrical components of the function F_1; so we can assume

$$F_1(r,\psi) = \frac{4T}{\pi}I_o(k_1 r) \quad (12)$$

The modified Bessel function of the zero order with small values of the argument does not differ much from unity. Usually it is possible to assume $I_o(k_1 r) = 1$, neglecting thus the longitudinal movement dependence on the transversal oscillations. But the accelerator with space-uniform focusing allows to use the particles injection with rather low energy. The wave number $k_1 = 2\pi/\beta\lambda$ under these conditions may turned out not very small and the dependence between longitudinal and transverse oscillations will play its part[4].

The accurate calculation of the coefficients T and $\varkappa$ requires the numerical solution of the electrodynamics equations for the electrodes of concrete shape. But for these coefficients it is easy to get approximate expressions, suitable to choose the main parameters of the accelerator.

The pieces of electrodes with constant section in Fig.2,3 correspond to the drift tubes and transitions between the adjacent pieces of constant section to the accelerating gaps. The exact solution of the boundary-value problem for the electrodes of constant section may be achieved if the electrodes sections are approximated by the field equipotentials of four linear wires with quadrupole symmetry of charge. Let us define the depth of the electrodes modulation m as a ratio of the maximum distance from the axis to the electrode to the minimum distance. The equipotentials coinciding with the electrodes surface are defined by the three parameters: the aperture radius a, the depth of modulation m and the formfactor V. The curvature radius of the electrodes at the nearest to the axis points are correspondingly (Fig.2,3)

$$R_x = \frac{a}{1+\frac{8}{1+m^2}sh^2V}\,;\quad R_y = \frac{ma}{1+\frac{8m^2}{1+m^2}sh^2V} \qquad (13)$$

where a is the distance from the axis to the electrode at the plane XOZ. Under $V \lesssim 0.1$ the sections of the electrodes are near hyperbolic and under $V \gtrsim 0.25$ are near circles with the radii R_x, R_y. The distributed capacitance of the four-wire quadrupole line is $C_u = \frac{\pi\varepsilon_0}{2V}$. Under $a \ll \beta\lambda$ one can assume the longitudinal component of the field at the channel axis to be equal zero at the electrodes pieces with constant section. Then the expression for the acceleration efficiency may be obtained

$$T = \frac{1}{2V}\,\frac{\sin\pi\alpha}{\pi\alpha}\,\ell n\,\frac{1+p}{1-p}\,, \qquad (14)$$

where $\alpha = \mathcal{g}/\beta\lambda$; $p = \frac{m^2-1}{m^2+1}\,th V$;

$\mathcal{g}$ is the transition length between the pieces of electrodes with constant section.

The efficiency of focusing in modulated four-wire line consisting of interchanging pieces of constant section electrodes are approximately defined by the expression

$$\varkappa \approx \frac{1}{J_0(k_1a_1)}\cdot\frac{sh\,2V}{V}\cdot\frac{m^2+1}{m^4+2m^2ch\,2V+1} \qquad (15)$$

where $a_1 = a(m^2-1)/\sqrt{2(m^2+1)}$. In a uniform line $m=1$ and $T=0$; the acceleration is absent but the focusing effect is maximum. With growth of the modulation depth the efficiency of acceleration is increasing, but the efficiency of focusing is decreasing.

From the expressions (13) it follows that under

$$sh\,V > \sqrt{\frac{1+m^2}{8m}} \qquad (16)$$

we have $R_x > R_y$; under $m < 3/2$ the expression (16) roughly represents the cylindrical electrodes with conic turnings (Fig.2); but it is nevertheless highly rough as under this approximation the unequality $a+R_x < ma+R_y$ always takes place.

If the unequality

$$sh V < \sqrt{\frac{1+m^2}{8m}} \qquad (17)$$

is true, then $R_x < R_y$. This case corresponds to the electrodes of the "crankshaft" type; the line electrodes radii are approximately proportional to the distances from the electrodes to the axis (Fig.3). In the case (17) usually is $p \ll 1$, so the expression (14) may be simplified:

$$T \approx \frac{thV}{V}\cdot\frac{\sin\pi\alpha}{\pi\alpha}\cdot\frac{m^2-1}{m^2+1}\,.$$

In contrast to the case (16) in the case (17 the acceleration efficiency has weak dependance of the parameter V and the focusing quadrupole field is more linear. Calculation of the boundary effect in the last case give the approximate equality

$$T \approx \frac{1}{I_0(k_1a_2)}\cdot\frac{thV}{V}\cdot\frac{\sin\pi\alpha}{\pi\alpha}\cdot\frac{m^2-1}{m^2+1}\,,$$

where $a_2 = a\sqrt{2m^2/1+m^2}$. The condition C_u = const (V = const) must be met along the whole resonator. Under the changing modu lation depth this condition defines, with ac cordance to the equalities (13), the ratios R_x/a and R_y/ma. In the parts of the accelerato with constant modulation depth the parameter V may be choosed under the condition $sh^2V = \frac{1+m^2}{8m}$; it gives $R_x = R_y = \frac{ma}{m+1}$.

Beam dynamics

It is more comfortable to study the longi tudinal oscillations of the particles in the device with space-uniform focusing in canoni cally conjugated variables $\zeta = z - z_s$, $\rho = v - v_s$. The phase difference of the equilibrium and nonequilibrium particles is $\psi = -k_1\zeta$. The replacement of the equations in finite diffe rences by differential equations does not ad appreciable error in spite of the low injection energy as it is advantageous to choose the partial increase of the energy (9) at th beginning of the accelerator rather small, a it will be shown later. The longitudinal oscillations equations may be drawn from the equality (8) directly. The Hamiltonian describing the longitudinal motion of any particle relatively the synchronous one is

$$H(\zeta,\rho) = \frac{1}{2}\rho^2 + \frac{eU_L T}{\pi m_0}\left[k_1\zeta\cos\varphi_s - I_0(k_1 r)\sin(k_1\zeta - \varphi_s \right. \qquad (1$$

The equation of the small longitudinal oscil lation is

$$\frac{d^2\zeta}{dt^2} + \Omega^2 I_0(k_1 r)\zeta = \frac{\Omega^2}{k_1|tg\varphi_s|}\left[I_0(k_1 r) - 1\right], \qquad (1$$

where $\Omega^2 = \omega^2 eU_L T\sin|\varphi_s|/\pi m_0 v_s^2$

The particle which is moving along the accelerator axis performs small longitudinal oscillations with frequency Ω . Let now the particle to have finite amplitude of transvers oscillations. The averaged motion of this particle may be presented as $r = R\cos\Omega_r t$

From the equation (19) it follows that the rather strong coupling of transversal and longitudinal oscillations gives rise to the periodical modulation of the small longitudi nal oscillation frequency and to appearance of an external force. Let us at first consider the equation (19) without its lefthand part. The coefficient attached to ζ is an even periodical function of time with frequency $2\Omega_r$ and may be represented as a Fourie series

$$I_0(k_1R\cos\Omega_r t) = I_0^2\left(\frac{k_1R}{2}\right) + 2\sum_{n=1}^{\infty} I_n^2\left(\frac{k_1R}{2}\right)\cos 2n\Omega_r t$$

$V \cdot \frac{\sin\pi\alpha}{\pi\alpha} \cdot \frac{m^2-1}{m^2+1}$.

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$\frac{thV}{V} \cdot \frac{\sin\pi\alpha}{\pi\alpha} \cdot \frac{m^2-1}{m^2+1}$,

$\sqrt{1+m^2}$. The condition = const) must be met along tor. Under the changing modu- s condition defines, with ac- equalities (13), the ratios

the parts of the accelerator

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$$\ldots k_1\zeta\cos\varphi_s - I_0(k_1 r)\sin(k_1\zeta - \varphi_s)\Big] \qquad (18)$$

the small longitudinal oscil-

$$= \frac{\Omega^2}{k_1|tg\varphi_s|}\left[I_0(k_1 r) - 1\right], \qquad (19)$$

$$\omega^2 eU_L T\sin|\varphi_s| / \pi m_0 v_s^2$$

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$$\ldots\left(\frac{k_1R}{2}\right) + 2\sum_{n=1}^{\infty} I_n^2\left(\frac{k_1R}{2}\right)\cos 2n\Omega_r t$$

he coefficients of the series are decreasing fast and this allows to confine ourselves with Mathieu equation. The expressions

$$1)\quad \left(\frac{\Omega_r}{\Omega}\right)^2 > I_0^2\left(\frac{k_1R}{2}\right) + I_1^2\left(\frac{k_1R}{2}\right)$$

$$2)\quad \left(\frac{\Omega_r}{\Omega}\right)^2 < I_0^2\left(\frac{k_1R}{2}\right) - I_1^2\left(\frac{k_1R}{2}\right)$$

correspond to the two first stability regions of the Mathieu equation solution. The first condition will be completed for all possible amplitudes of the transversal oscillations, if it is completed for the maximum amplitude. The second condition is true if it is correct for the particle moving along the axis. Let us assume the maximum amplitude of the transversal oscillations to be equal to the aperture radius of the channel a . Then the parametric stability criterion may come to one of two expressions

$$\left(\frac{\Omega_r}{\Omega}\right)^2 > I_0^2\left(\frac{k_1a}{2}\right) + I_1^2\left(\frac{k_1a}{2}\right); \quad \left(\frac{\Omega_r}{\Omega}\right)^2 < 1.$$

It is possible to neglect the frequency modulation outside of parametric resonance regions. The equation of the small longitudinal oscillations under these simplifications is

$$\frac{d^2\zeta}{dt^2} + \Omega^2 I_0^2\left(\frac{k_1R}{2}\right)\zeta =$$

$$= \frac{\Omega^2}{k_1|tg\varphi_s|}\left[I_0^2\left(\frac{k_1R}{2}\right) - 1 + 2I_1^2\left(\frac{k_1R}{2}\right)\cos 2\Omega_r t\right].$$

There is no any external resonances under $2\Omega_r \neq \Omega\, I_0\left(\frac{k_1R}{2}\right)$. This condition for any possible transversal amplitude leads to one of inequalities: $\frac{\Omega}{\Omega_r} > 2$ or $\frac{\Omega}{\Omega_r} < 2/I_0\left(\frac{k_1a}{2}\right)$. Then the terms of simultaneous absence of the external and parametric resonances are

$$\frac{\Omega}{\Omega_r} < \frac{1}{I_0\left(\frac{k_1a}{2}\right)}; \; 1 < \frac{\Omega}{\Omega_r} < \frac{2}{I_0\left(\frac{k_1a}{2}\right)}; \; \frac{\Omega}{\Omega_r} > 2 \qquad (20)$$

The frequencies ratio Ω/Ω_r corresponding to the stable longitudinal oscillations must be in one of three regions confined by the expressions (20). Usually one can succeed in satisfying to the first of these expressions.

The parametric coupling may lead to the resonant rise of the longitudinal oscillations not only in the accelerator with space-uniform focusing systems, but in any other accelerating system allowing the low injection energy, in phase variable focusing system for example. The conditions of the longitudinal oscillations stability (20) are correct to all the systems, where it is necessary to take into account the degrees of freedom coupling.

The Hamiltonian of the longitudinal oscillations for particle with the amplitude of the transversal oscillations R may be represented outside of the resonant regions as

$$H(\zeta,\rho) = \frac{1}{2}\rho^2 + \frac{eU_L T}{\pi m_0}\left[k_1\zeta\cos\varphi_s - I_0^2\left(\frac{k_1R}{2}\right)\sin(k_1\zeta - \varphi_s)\right].$$

The small longitudinal oscillations frequency is $\Omega \cdot I_0\left(\frac{k_1R}{2}\right)$. As the small longitudinal oscillation frequency depends on R , the oscillations of particle groups with different amplitudes of the transversal oscillations are noncoherent. The two effects - the nonlinearity of the self-focusing forces and the longitudinal oscillations dependence on the transversal ones lead to a relatively fast filling of effective phase volume on the longitudinal co-ordinates plane with the representing points of the beam, which had at first the zero volume.

The center and the saddle co-ordinates may be defined from the equation

$$\cos(\varphi_s + \psi) = \cos\varphi_s / I_0^2\left(\frac{k_1R}{2}\right)$$

For the center co-ordinate ψ_0 we have

$$\psi_0 = \left[1 - \frac{1}{I_0^2\left(\frac{k_1R}{2}\right)}\right] ctg\,\varphi_s$$

The center co-ordinate remains a small value for particles with any possible amplitude of the transversal oscillations. Really the value of the equilibrium phase at the injection in the accelerator with the space-uniform focusing is usually chosen near to 90° and $|ctg\,\varphi_s| \ll 1$; later on the difference $I_0^2 - 1$ rapidly decreases as with energy growth of the particles the argument of the Bessel function decreases. The detailed estimations [4] show that the phase stability region of the outlying particles becomes a little wider and slightly moved to the positive phases side in comparison with the particles moving along the axis. The difference of the movement invariables for axial and outlying particles with low energies and high absolute values of the equilibrium phase is unessential. As far as the particle energy increases the movement invariables draw nearer even under decreasing of the absolute value of the equilibrium phase. Later on while analysing the bunches movement let us consider all the particles to be axial and $I_0(k_1 r) \approx 1$.

The longitudinal movement of the particles in the accelerator with space-uniform focusing under constant equilibrium phase does not differ of the longitudinal movement in other systems with paralleled accelerating gaps. Energy increase of the equilibrium particle along the acceleration period $L = \frac{1}{2}\beta\lambda$ is constant:

$$\Delta W_s = eU_L T\cos\varphi_s$$

The rate of acceleration decreases adiabatically as the particles energy rises

$$\frac{dW_s}{dz} \approx \frac{2\Delta W_s}{\beta\lambda} = \left(\frac{dW_s}{dz}\right)_0 \left(\frac{W_s}{W_0}\right)^{-\frac{1}{2}}$$

The phase oscillations are stable under the such index. The amplitude of the small phase oscillations decreases slower than in an Alvarez type accelerator and the amplitude of the particles momentum faster: $\Phi \sim W_s^{-1/4}$; $\frac{\Delta p}{p} \sim W_s^{-\frac{3}{4}}$. The length of the resonator ℓ to accelerate particles from the energy W_0 to the final energy W_f may be estimated by the expression

$$\ell = \frac{\sqrt{2}}{3} \cdot \frac{\lambda\mathcal{E}_0}{\Delta W_s}\left[\left(\frac{W_f}{\mathcal{E}_0}\right)^{3/2} - \left(\frac{W_0}{\mathcal{E}_0}\right)^{3/2}\right]; \quad \mathcal{E}_0 = m_0 c^2.$$

The use of the structure with the space-uniform focusing gives possibilities to decrease the injection energy and to increase the intensity of the accelerated beam. The acceleration period $\frac{1}{2}\beta\lambda$, equal to the half of the electrodes modulation period, may be rather short. This allows to begin acceleration from low energies. From the equation (10) it follows that the quadrupole focusing in space-uniform system does not depend of the particle phase and if the specific acceleration is small enough the frequencies of the transversal oscillations have weak coupling with phases of the particles. By the fixed depth of the electrodes modulation the partial increase of the energy is constant. Then the margine of the acceleration per length unit is under the low enrgies relatively large. This allows to have the equilibrium phase near to 90° at the beginning of the accelerator. The bunches follow close to each other and the mean current is almost equal to the peak one. In this system it is possible such adiabatic changing of the parameters along the accelerator axis with the energy rise, when the distance between the bunches increases, but the geometrical dimensions of the bunches under this process remain constant and therefore the density of the space charge remains constant also. These features of the system with space-uniform focusing allow to get high currents under low injection energy[5].

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$$\frac{p^2}{P^2}+\frac{\xi^2}{Z^2}=1,$$

where $P=\Omega Z$. According to the theorem of the adiabatic invriable PZ = const. It is clear from this that the condition Ω = const or

$$\frac{T\sin\varphi_s}{\upsilon_s^2}=\text{const}$$

ensures the conservation of all the phase trajectories with linear approximation. On that score this guarantees the conservation of the charge density distribution in the linear region inside the separatrix. Let then Φ_c to be the phase longitude of the separatrix. The value Φ_c is connected with the equilibrium phase by the equation[6]

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The graph of Φ_c as a function of the equilibrium phase for the whole length of the interval from zero to $-\pi/2$ is shown in Fig.5. The geometrical length of the separatrix is

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If one decreases the equilibrium phase along the accelerator axis in such a way that the value $\upsilon_s\Phi_c$ will remain constant, the length of the bunch will not change. The half of the vertical dimension of the separatrix depends on the length of the separatrix as

$$P_c=2\Omega Z_c\cdot\mathcal{F}_1(\varphi_s).$$

The function

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is shown in Fig.5. If Ω= const and Z_c =const then the vertical dimension of the separatrix is also almost constant. In particular, if one puts $\Phi_c=3|\varphi_s|$, $tg\,\varphi_s=\varphi_s+\frac{1}{3}\varphi_s^3$ the constancy of P_c is quite strict: $\mathcal{F}_1=1/3\sqrt{3}$. At the big values of the equilibrium phase, up to $\varphi_s=-1{,}5$, the function $\mathcal{F}_1$ deviation of the pointed value does not exeed 5%. So the constant charge density distribution remains along the whole length of the bunch with practically enough accuracy. The conditions of the quasi-stationarity Ω = const, Z_c = const simple define the dependence of the acceleration effectiveness and the equilibrium phase on the current energy of an equilibrium particle W_s. These parameters are connected with the final values of T_f, φ_f, W_f by the expressions

$$\Phi_c(\varphi_s)=\Phi_c(\varphi_f)\sqrt{\frac{W_f}{W_s}};\quad T(W_s)=T(W_f)\frac{W_s\sin\varphi_f}{W_f\sin\varphi_s}.$$

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$$\frac{1}{W_s}\frac{dW_s}{dz}=\frac{2}{Z_c}\left(\frac{\Omega}{\omega}\right)^2\cdot\mathcal{F}_2(\varphi_s),\qquad(21)$$

where $\mathcal{F}_2(\varphi_s)=\Phi_c(\varphi_s)/|tg\,\varphi_s|$. The function $\mathcal{F}_2$ plot is shown in Fig.5; the function $\mathcal{F}_2$ may be represented by the series

$$\frac{3}{\mathcal{F}_2}=1+0{,}3\left(\frac{\Phi_c}{3}\right)^2+0.0964\left(\frac{\Phi_c}{3}\right)^4+0.029\left(\frac{\Phi_c}{3}\right)^6+\ldots$$

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If to take into account the expression (12), (11), the equation (10) may be represented as

$$\frac{d^2x}{dt^2}-\left(\frac{\omega}{\pi}\right)^2\left[K^2\cos\omega t-\frac{1}{2}\gamma_0\sin\varphi\right]x=0\qquad(22)$$

The first term in the square brackets defines the effect of the space-uniform quadrupole focusing. As it was noted earlier, in contrast to the HF focusing in the space-periodical structures, in this case the focusing term does not depend of the particle phase. The parameter

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determines the stiffness of the focusing channel. The defocusing factor γ_0 may be written as

$$\gamma_0=\frac{\pi eU_LT}{2W_s}\qquad(24)$$

The defocusing factor is not zero at those cases when the distance between the electrodes is modulated ($T\neq0$) and the effect of

$V \cdot \frac{\sin \pi\alpha}{\pi\alpha} \cdot \frac{m^2-1}{m^2+1}$.

:he case (16) in the case (17) efficiency has weak depen- rameter V and the focusing is more linear. Calculation effect in the last case gives equality

$\frac{thV}{V} \cdot \frac{\sin\pi\alpha}{\pi\alpha} \cdot \frac{m^2-1}{m^2+1}$,

$\sqrt[3]{1+m^2}$. The condition

= const) must be met along tcr. Under the changing modu- s condition defines, with ac- equalities (13), the ratios

the parts of the accelerator

dulation depth the parameter d under the condition

it gives $R_x = R_y = \frac{ma}{m+1}$.

eam dynamics

mfortable to study the longi- ions of the particles in the e-uniform focusing in canoni- variables $\zeta = z - z_s$, $p = v - v_s$.

ence of the equilibrium and articles is $\psi = -k_1\zeta$. The he equations in finite diffe- ential equations does not add r in spite of the low injec- t is advantageous to choose ease of the energy (9) at the accelerator rather small, as later. The longitudinal os- ions may be drawn from the ectly. The Hamiltonian des- itudinal motion of any par- the synchronous one is

$\ldots k_1\zeta\cos\varphi_s - I_0(k_1 z)\sin(k_1\zeta - \varphi_s)]$ (18)

the small longitudinal oscil-

$= \frac{\Omega^2}{k_1|tg\varphi_s|}\left[I_0(k_1 z) - 1\right]$, (19)

$\omega^2 e U_L T \sin|\varphi_s| / \pi m_0 v_s^2$

:h is moving along the acce- forms small longitudinal os- frequency Ω . Let now the finite amplitude of trans- . The averaged motion of be presented as $z = R\cos\Omega_z t$.

(19) it follows that the upling of transversal and llations gives rise to the ation of the small longitudi- frequency and to appearance rce. Let us at first consi- (19) without its lefthand ient attached to ζ is an function of time with freq- be represented as a Fourier

$\ldots\left(\frac{k_1R}{2}\right) + 2\sum_{n=1}^{\infty} I_n^2\left(\frac{k_1R}{2}\right)\cos 2n\Omega_z t$

the coefficients of the series are decreasing fast and this allows to confine ourselves with Mathieu equation. The expressions

$$1)\quad \left(\frac{\Omega_z}{\Omega}\right)^2 > I_0^2\left(\frac{k_1R}{2}\right) + I_1^2\left(\frac{k_1R}{2}\right)$$

$$2)\quad \left(\frac{\Omega_z}{\Omega}\right)^2 < I_0^2\left(\frac{k_1R}{2}\right) - I_1^2\left(\frac{k_1R}{2}\right)$$

correspond to the two first stability regions of the Mathieu equation solution. The first condition will be completed for all possible amplitudes of the transversal oscillations, if it is completed for the maximum amplitude. The second condition is true if it is correct for the particle moving along the axis. Let us assume the maximum amplitude of the transversal oscillations to be equal to the aperture radius of the channel a . Then the parametric stability criterion may come to one of two expressions

$$\left(\frac{\Omega_z}{\Omega}\right)^2 > I_0^2\left(\frac{k_1a}{2}\right) + I_1^2\left(\frac{k_1a}{2}\right); \quad \left(\frac{\Omega_z}{\Omega}\right)^2 < 1.$$

It is possible to neglect the frequency modulation outside of parametric resonance regions. The equation of the small longitudinal oscillations under these simplifications is

$$\frac{d^2\zeta}{dt^2} + \Omega^2 I_0^2\left(\frac{k_1R}{2}\right)\zeta =$$

$$= \frac{\Omega^2}{k_1|tg\varphi_s|}\left[I_0^2\left(\frac{k_1R}{2}\right) - 1 + 2I_1^2\left(\frac{k_1R}{2}\right)\cos 2\Omega_z t\right].$$

There is no any external resonances under $2\Omega_z \neq \Omega\, I_0\left(\frac{k_1R}{2}\right)$. This condition for any possible transversal amplitude leads to one of inequalities: $\frac{\Omega}{\Omega_z} > 2$ or $\frac{\Omega}{\Omega_z} < 2/I_0\left(\frac{k_1a}{2}\right)$. Then the terms of simultaneous absence of the external and parametric resonances are

$$\frac{\Omega}{\Omega_z} < \frac{1}{I_0\left(\frac{k_1a}{2}\right)}; \quad 1 < \frac{\Omega}{\Omega_z} < \frac{2}{I_0\left(\frac{k_1a}{2}\right)}; \quad \frac{\Omega}{\Omega_z} > 2 \qquad (20)$$

The frequencies ratio Ω/Ω_z corresponding to the stable longitudinal oscillations must be in one of three regions confined by the expressions (20). Usually one can succeed in satisfying to the first of these expressions.

The parametric coupling may lead to the resonant rise of the longitudinal oscillations not only in the accelerator with space-uniform focusing systems, but in any other accelerating system allowing the low injection energy, in phase variable focusing system for example. The conditions of the longitudinal oscillations stability (20) are correct to all the systems, where it is necessary to take into account the degrees of freedom coupling.

The Hamiltonian of the longitudinal oscillations for particle with the amplitude of the transversal oscillations R may be represented outside of the resonant regions as

$$H(\zeta,p) = \frac{1}{2}p^2 + \frac{eU_L T}{\pi m_0}\left[k_1\zeta\cos\varphi_s - I_0^2\left(\frac{k_1R}{2}\right)\sin(k_1\zeta - \varphi_s)\right].$$

The small longitudinal oscillations frequency is $\Omega \cdot I_0\left(\frac{k_1R}{2}\right)$. As the small longitudinal oscillation frequency depends on R , the oscillations of particle groups with different amplitudes of the transversal oscillations are noncoherent. The two effects - the nonlinearity of the self-focusing forces and the longitudinal oscillations dependence on the transversal ones lead to a relatively fast filling of effective phase volume on the longitudinal co-ordinates plane with the representing points of the beam, which had at first the zero volume.

The center and the saddle co-ordinates may be defined from the equation

$$\cos(\varphi_s + \psi) = \cos\varphi_s / I_0^2\left(\frac{k_1R}{2}\right)$$

For the center co-ordinate ψ_0 we have

$$\psi_0 = \left[1 - \frac{1}{I_0^2\left(\frac{k_1R}{2}\right)}\right] ctg\,\varphi_s$$

The center co-ordinate remains a small value for particles with any possible amplitude of the transversal oscillations. Really the value of the equilibrium phase at the injection in the accelerator with the space-uniform focusing is usually chosen near to 90° and $|ctg\,\varphi_s| \ll 1$; later on the difference $I_0^2 - 1$ rapidly decreases as with energy growth of the particles the argument of the Bessel function decreases. The detailed estimations [4] show that the phase stability region of the outlying particles becomes a little wider and slightly moved to the positive phases side in comparison with the particles moving along the axis. The difference of the movement invariables for axial and outlying particles with low energies and high absolute values of the equilibrium phase is unessential. As far as the particle energy increases the movement invariables draw nearer even under decreasing of the absolute value of the equilibrium phase. Later on while analysing the bunches movement let us consider all the particles to be axial and $I_0(k_1 z) \approx 1$.

The longitudinal movement of the particles in the accelerator with space-uniform focusing under constant equilibrium phase does not differ of the longitudinal movement in other systems with paralleled accelerating gaps. Energy increase of the equilibrium particle along the acceleration period $L = \frac{1}{2}\beta\lambda$ is constant:

$$\Delta W_s = e U_L T \cos\varphi_s$$

The rate of acceleration decreases adiabatically as the particles energy rises

$$\frac{dW_s}{dz} \approx \frac{2\Delta W_s}{\beta\lambda} = \left(\frac{dW_s}{dz}\right)_0 \left(\frac{W_s}{W_0}\right)^{-\frac{1}{2}}$$

The phase oscillations are stable under the such index. The amplitude of the small phase oscillations decreases slower than in an Alvarez type accelerator and the amplitude of the particles momentum faster: $\Phi \sim W_s^{-1/4}$; $\frac{\Delta p}{p} \sim W_s^{-\frac{3}{4}}$. The length of the resonator ℓ to accelerate particles from the energy W_0 to the final energy W_f may be estimated by the expression

$$\ell = \frac{\sqrt{2}}{3} \cdot \frac{\lambda\mathcal{E}_0}{\Delta W_s}\left[\left(\frac{W_f}{\mathcal{E}_0}\right)^{3/2} - \left(\frac{W_0}{\mathcal{E}_0}\right)^{3/2}\right]; \quad \mathcal{E}_0 = m_0c^2.$$

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...cceleration takes place. The equation (22) is the Mathieu equation. The limits of first region of the stable solutions under $K^2 < \frac{\pi^2}{2}$ with enough accuracy may be defined by the expressions

$$K^4 > -\pi^2\gamma_o \sin\varphi\,; \quad K^2 < \frac{\pi^2}{2} - \gamma_o \sin\varphi \qquad (25)$$

The analysis of the expressions (25) shows that the stability of the transversal oscillations may be realised under any values of the equilibrium phase up to -90°. Practically the only first expression (25) is essential. It leads to the condition

$$\frac{e\,U_L}{W_s} > \frac{32\pi^3 T}{\text{æ}^2}\left(\frac{a}{\beta\lambda}\right)^4$$

providing the transversal stability of the particles under any phases of the longitudinal oscillations $\varphi > -\frac{\pi}{2}$.

The averaged frequency of the transversal oscillations $\mu = 2\pi\Omega_t/\omega$ under the smooth approximation is defined by

$$\mu^2 = \frac{2}{\pi^2}K^4 + 2\gamma_o \sin\varphi.$$

For the equilibrium particle

$$\mu_s^2 = \frac{2}{\pi^2}K^4 - 2\pi^2\left(\frac{\Omega}{\omega}\right)^2.$$

It is clear from this expression that in the accelerator with quasi-stationary bunches the averaged frequency of the transversal oscillations is constant if $a/\sqrt{\text{æ}} =$const.

The periodical coefficient of the equation (22) is an explicit function of the time and does not depend on the longitudinal co-ordinate. The Floquet function $\varphi(t)$ also does not depend on the longitudinal co-ordinate. This peculiarity of the solution is distinctive for an accelerator with space-uniform focusing. Really this peculiarity comes to the fact that the dimensions of the matched beam remain constant along the axis of the channel. But the beam dimensions are pulsating in time. In every of the XOZ or YOZ planes the cross sections of the beam reach their maximum dimensions at the moments when the channel is focusing in this plane and minimum dimension when the channel is defocusing in the plane.

The Floquet ellipsis in the plane $x, \frac{dx}{dt}$ is represented by the equation

$$a_o x^2 + b_o\left(\frac{dx}{dt}\right)^2 + 2c_o x\frac{dx}{dt} = \frac{c}{\gamma}V_\theta\,,$$

where

$$a_o = \frac{1}{W_1|\varphi|^2} + W_1\left(\frac{d|\varphi|}{dt}\right)^2; \quad b_o = W_1|\varphi|^2; \quad c_o = -W_1|\varphi|\frac{d|\varphi|}{dt}$$

$W_1 = 1$s; V_θ -normalized emittance of the beam. The instantenious value of the transversal oscillation frequency depends on the Floquet function modulus as

$$\omega_t = \frac{1}{W_1|\varphi(t)|^2}.$$

The matched beam envelope is proportional to the Floquet function modulus

$$R_m(t) = |\varphi(t)|\sqrt{\frac{c}{\gamma}W_1 V_\theta}$$

so the normalized acceptance of the channel will be defined by the well known expression

$$V_{ch} = \frac{\gamma}{c}\,\omega_{t\,min}\,a^2$$

The Fig.6 gives the dependance of the indimensional value $|\varphi(t)|^{-2}_{max}$ on the channel parameters K^2 and $\gamma_s = -\gamma_o \sin\varphi_s$. The coefficients of the Floquet ellipsis equation at any point of the channel including the output of the accelerator are the periodical function of time with the period $2\pi/\omega$.

The accelerator with space-uniform focusing does not require a high-voltage injector, gives possibility to have a high coefficient of capture of particles into acceleration conditions without increasing of phase density in transversal phase space and has wide acceptance. But such an accelerator is effective only for energies not more than 2 - 3 MeV/nucleon as there is no possibilities to get a high acceleration rate under big velocities of particles. That is why the accelerators with space-uniform focusing are effective as an initial part of the linear accelerator for high energies and big intensities. Several projects with space-uniform focusing structures as an initial part of high-current linear accelerators were proposed.[7-9]

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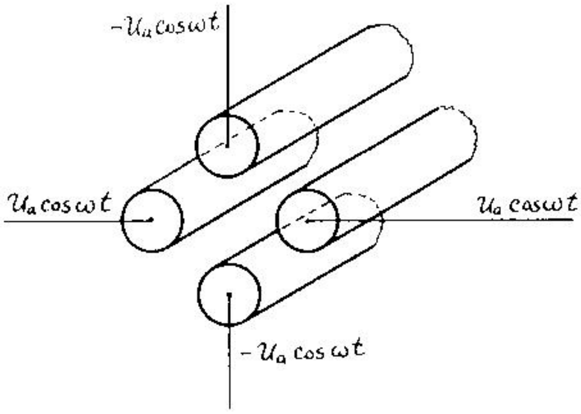


Figure 1.

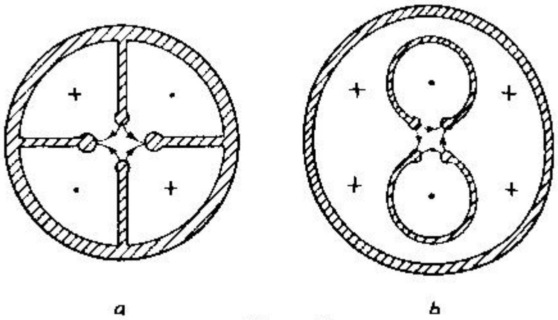


Figure 4.

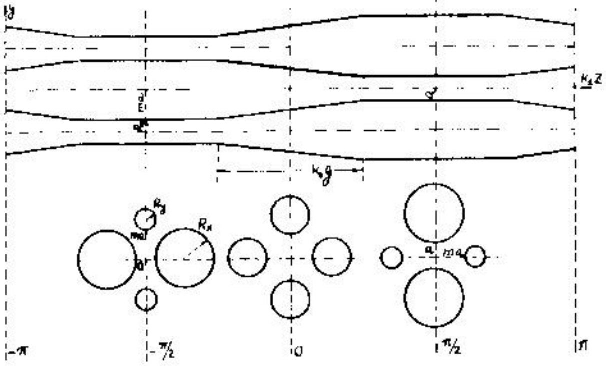
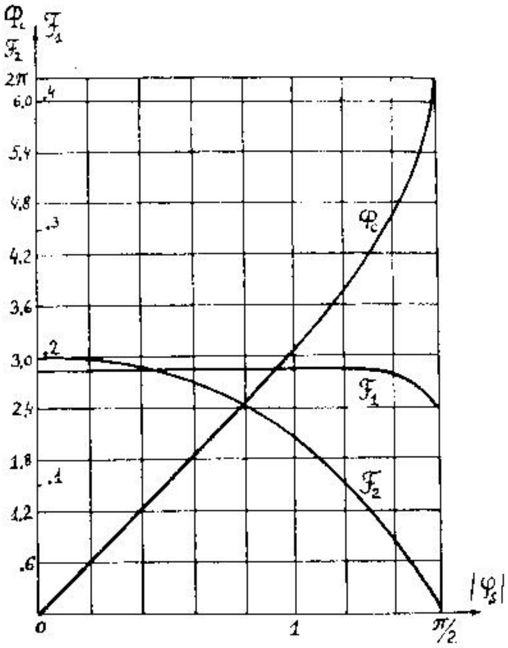


Figure 2.



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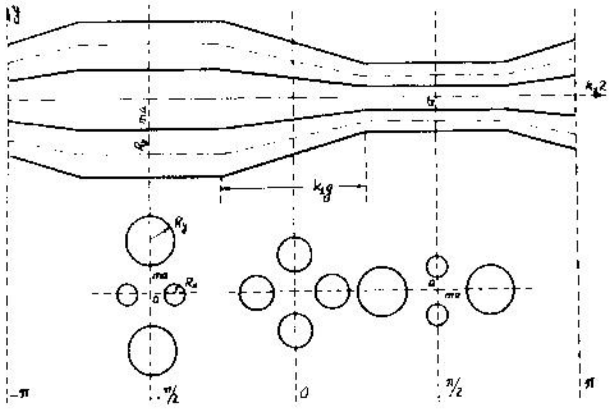

Figure 3.

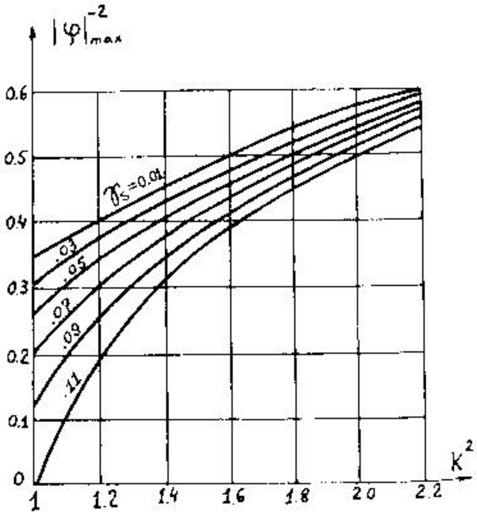

Figure 6.

period σ_Q to calculate σ_0 for the quadrupoles alone (as if $E_0T = 0$). (**b**) Calculate σ_0 when $E_0T = 1\,\text{MV/m}$ and $\phi = -30°$. Are the particles stable transversely? (**c**) For the same $-30°$ phase and the same quadrupole array, what is the maximum accelerating field E_0T for transverse stability?