NUCLEAR EXPERIMENTAL TECHNIQUES

LINEAR ION ACCELERATOR WITH SPATIALLY HOMOGENEOUS STRONG FOCUSING

I. M. Kapchinskii and V. A. Teplyakov

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A linear ion accelerator is suggested, in which focusing and acceleration is accomplished by means of a h-f electric field generated by a long line of four conducting wires. The accelerator contains no drift tubes. The advantages of this accelerator are its relative simple design, the small transverse dimensions, and the possibility of an essential reduction of injection energy without influencing the high intensity of the accelerated beam.

In linear accelerators with drift tubes the strong-focusing system displays spatial periodicity: either the polarity of the quadrupole-lens poles or the geometry of these poles is alternated along the axis. When operating with alternating voltage a quadrupole system of focusing electrodes can be used which is homogeneous along the accelerator axis (Fig. 1). A high-frequency voltage $(\pm V/2)\cos(\omega t + \varphi)$ is applied to the electrodes, and the particles moving along the axis are exposed successively to the action of fields with alternating signs of the gradient. This results in the appearance of a strong-focusing effect in the spatially homogeneous quadrupole system.

In the high-frequency focusing field a longitudinal accelerating component can be produced if the distance between the opposite electrodes of like polarity is changed periodically along the axis. The spatial period of variation of electrode distance must be equal to the path traveled by the particle within a high-frequency period, while the phase of distance variation in the vertical and horizontal planes must be shifted by half a period. The potential of the axial electric field is then modulated with the period $\beta\lambda$, which gives rise to a resonance acceleration effect. Figure 2 shows schematically a cross section of the electrodes at y = 0 and x = 0; z is the longitudinal coordinate and k is the wave number: $k = 2\pi/\beta\lambda$.

Let us consider a four-conductor quadrupole line. In a quasisteady approximation the potential distribution of the electric field generally has the form

$$u(r, \psi, z, t) = -{}^1\!/_2 V \left[F_0(r, \psi) + \sum_{n=1}^{\infty} F_n(r, \psi)\sin knz\right]\cos(\omega t + \varphi). \quad (1)$$

The function

$$F_0(r, \psi) = \sum_{s=0}^{\infty} A_{0s} r^{2(2s+1)} \cos 2(2s+1)\psi$$

determines the potential in cross sections with the coordinate $kz = l\pi$ ($l = 0, 1, 2, \ldots$), where the distribution has an exact quadrupole symmetry. With good accuracy this function describes the form of the electrodes in these cross sections. In the case of harmonic spatial modulation of the electrode distances, the coefficients F_n are given by

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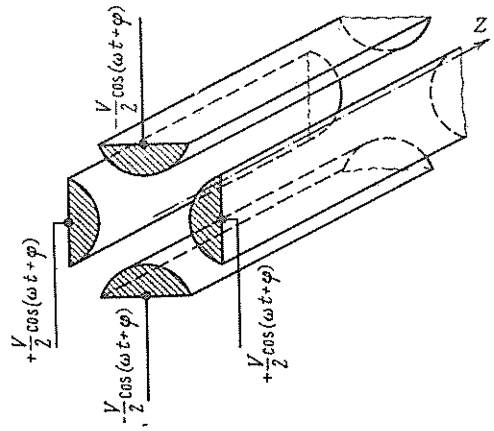

Fig. 1. Four-conductor long line with quadrupolar symmetry.

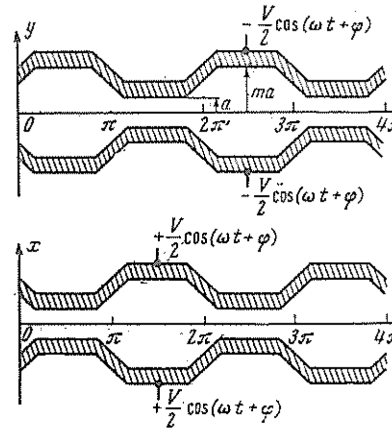

Fig. 2. Electrode cross sections of orthogonal planes through the longitudinal axis of the accelerator.

$$F_n(r, \psi) = \sum_{s=0}^{\infty} A_{ns} I_{2s}(knr) \cos 2s\psi.$$

Along a path equal to the period of spatial modulation $\Delta z = \beta\lambda$ the particle energy grows by

$$\Delta W = {}^1/_2\, ekV \sum_{n=1}^{\infty} n \int_0^{\beta\lambda} F_n(r, \psi) \cos knz \quad \cos(kz + \varphi)\, dz.$$

For particles moving along the accelerator axis

$$\Delta W = 2eV\theta \cos\varphi,$$

where φ is the phase of the field (referred to the maximum) at the moment of time when the particles are at a point corresponding to a cross section with exact quadrupolar symmetry; θ is the efficiency of acceleration

$$\theta = {}^1/_4 \pi A_{10}. \tag{2}$$

In a first approximation only the first harmonic of the spatial modulation of the electrodes contributes to the acceleration.

Let us introduce a nondimensional longitudinal coordinate

$$\tau = (\omega t + \varphi)/2\pi.$$

Transverse vibrations of the ions in the nonrelativistic case can then be described by

$$\frac{d^2x}{d\tau^2} = \left(\frac{2\pi^2 eV}{m\omega^2}\right)\left[\frac{\partial F_0}{\partial x}\cos 2\pi\tau + \sum_{n=1}^{\infty}\frac{\partial F_n}{\partial x}\sin n(2\pi\tau - \varphi)\cos 2\pi\tau\right]. \tag{3}$$

Considering the linear approximation we restrict ourselves to the quadrupolar component of the field and the first harmonic of the spatial modulation of the electrodes. Bearing in mind that the main contribution to acceleration is made by the term with the coefficient A_{10}, we obtain

$$\left.\begin{aligned} F_0(r, \psi) &= A_{00} r^2 \cos 2\psi \\ F_1(r, \psi) &= A_{10} I_0(kr) \\ F_n(r, \psi) &\equiv 0;\ n \geqslant 2. \end{aligned}\right\} \tag{4}$$

Let us denote by a the minimum distance from the electrode to the axis, which determines the acceptance of the channel. When the distance between the electrodes is not modulated, the radius of the channel will be equal to a at any point of the axis, while with perfect hyperbolic poles the gradient of the electric field is

$$G_0 = V/a^2.$$

In the presence of modulation the electric field gradient on the channel axis in cross sections with exact quadrupolar symmetry, is given by

$$G = \partial E_x/\partial x = V A_{00}.$$

We call G/G_0 the focusing efficiency ($\varkappa$). It is easy to see that

$$\varkappa = A_{00}a^2. \tag{5}$$

The equation of transverse oscillation (3) can then be written in the form

$$\frac{d^2x}{d\tau^2} = \left[\frac{\varkappa V}{E_0}\left(\frac{\lambda}{a}\right)^2 \cos 2\pi\tau - \frac{2\pi V}{\beta^2 E_0}\theta \sin\varphi\right]x, \tag{6}$$

where ε_0, in eV, is the rest energy of an ion. The advance of the phase of transverse oscillations in the period $\beta\lambda$ (or $\Delta\tau = 1$), in a continuous approximation, is given by the equation

$$\mu^2 = \frac{1}{8}\left(\frac{\varkappa V\lambda^2}{\pi E_0 a^2}\right)^2 + \frac{2\pi V}{\beta^2 E_0}\theta\sin\varphi. \tag{7}$$

The first term on the right-hand side of Eq. (7) is determined by the effect of strong focusing. The second term is nonzero if the electrode distance is modulated ($\theta \neq 0$); this term is connected with the defocusing action of the field which accompanies the effect of acceleration.

The Mathieu functions are solutions to Eq. (6). The transverse oscillations of a particle with phase φ are stable if the following conditions [1] are satisfied:

$$\left.\begin{aligned} &\frac{\varkappa V}{2\pi^2 E_0}\left(\frac{\lambda}{a}\right)^2 < 1 - \frac{2V}{\pi\beta^2 E_0}\theta\sin\varphi \\ &\frac{\varkappa^2 V}{8\pi^3 E_0}\left(\frac{\lambda}{a}\right)^4 > -\frac{20\sin\varphi}{\beta^2}. \end{aligned}\right\} \tag{8}$$

An analysis of the inequalities (8) shows that in practice the transverse oscillations are stable with any values of the synchronous phase (up to $-90°$), just as in accelerators with static lenses.

Restricting ourselves in the potential distribution function (1) to first-order terms of approximation (4), which determine essentially the effects of acceleration and focusing, we obtain

$$u(r, \psi, z, t) \approx -{}^1/_2 V\left[\varkappa\left(\frac{r}{a}\right)^2 \cos 2\psi + \frac{4\theta}{\pi} I_0(kr)\sin kz\right]\cos(\omega t + \varphi). \tag{9}$$

Solution (9) is exact if the pole surfaces satisfy the equations

$$\left.\begin{aligned} r^2\cos 2\psi &= \frac{a^2}{\varkappa}\left[1 - \frac{4\theta}{\pi} I_0(kr)\sin kz\right] \\ r^2\cos 2\psi &= -\frac{a^2}{\varkappa}\left[1 + \frac{4\theta}{\pi} I_0(kr)\sin kz\right]. \end{aligned}\right\} \tag{10}$$

The depth of spatial modulation of the electrodes can be given by means of a coefficient, m, which is equal to the ratio of maximum distance between electrode and axis to the minimum distance (Fig. 2). The efficiencies of acceleration and focusing for perfect poles are determined from Eq. (10) in terms of the channel aperture a and the electrode modulation depth m:

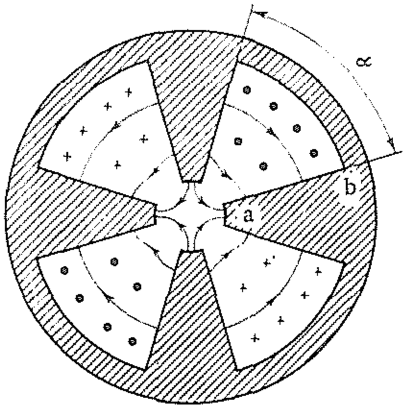

Fig. 3. Distribution of electric field in the cross section of the four-chamber resonator.

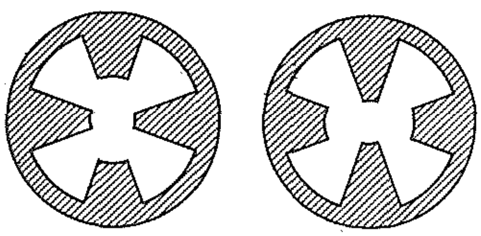

Fig. 4. Cross sections of four-chamber resonator with modulated electrodes.

$$\theta = \frac{(\pi/4)(m^2 - 1)}{[m^2 I_0(ka) + I_0(mka)]}; \quad \varkappa = \frac{[I_0(ka) + I_0(mka)]}{[m^2 I_0(ka) + I_0(mka)]}.$$

In the general case the efficiencies of acceleration and focusing must be determined from Eqs. (2) and (5).

With constant channel aperture the acceleration efficiency increases and the focusing efficiency decreases as the modulation depth is raised within wide limits; thus there exists an optimum modulation of electrode distance. Numerical estimates show that the mean increment of proton energy per unit length in the system considered is equal to about 1 MeV/m, and the normalized acceptance is about 1.5 cm mrad up to energies of 20-30 MeV, with a maximum h-f field strength of up to 250 kV/cm. Fields with such strengths are used in modern accelerators. Owing to the spatial continuity of the focusing and accelerating structures the latter can be used effectively at low injection energies of 100-200 keV, where the mean increment of energy per unit length increases essentially in the range of low energies.

Poles as described by Eqs. (10) are in all cross sections close to the form of the hyperbolic poles, and the square electrode distances to the axis vary according to a sinusoidal law. In order to produce sufficiently high potential differences between neighboring poles, the latter must be cut off as in common electrostatic lenses. This results in an increase of the nonlinear field components but does not influence the acceleration efficiency. Since in all static lenses, poles with limited generatrices are used, the appearance of nonlinear effects is not specific for spatially homogeneous structures.

The behavior of a beam in a spatially homogeneous hardfocusing channel possesses characteristic peculiarities. An adjusted beam has dimensions which are constant along the channel axis but periodic functions of time. In each of the planes ($y = 0$ or $z = 0$) the maximum cross-sectional dimension is reached at the instant of time at which the channel focuses in the plane considered, and the minimum dimension is reached when the channel defocuses in this plane. The conditions for an agreement of the beam with the channel inlet vary periodically with the frequency of the electric field. It is therefore necessary to adjust the beam dynamically with the channel. This adjustment is achieved with the help of high-frequency quadrupole lenses. The envelope of the nonadjusted beam is a periodic function of time, oscillating with twice the mean frequency of the transverse oscillations of particles; these oscillations are superimposed by the high-frequency oscillations of the field. The amplitude of the "slow" oscillations runs along the beam with the velocity of the particles.

The high-frequency four-conductor line can be constructed as a four-section cylindrical resonator. All sections (chambers) of the resonator are excited to the lowest natural frequency such that the magnetic field vector is directed along the resonator; in the neighboring chambers it is in antiphase. The maximum high-frequency potential difference appears in the partition gap between the chambers, near the axis. Along the resonator the potential difference is constant at every instant of time. The field distribution in a resonator with nonmodulated electrodes is shown in Fig. 3. Figure 4 shows a cross section through a resonator with modulated electrodes in planes corresponding to the coordinates $kz = \pi/2 = 2\pi l$ and $kz = 3\pi/2 = 2\pi l$. The magnetic flux is closed at the ends of the resonator. On its ends the resonator is closed by plane bottom plates which do not touch the partition sections between the chambers.

The natural frequencies of a four-chamber resonator with nonmodulated wedge-shaped electrodes are determined by the equation

$$f(\omega b/c) = -\left(\frac{2}{\pi}\right)[p + \ln(2b/a)],$$

where c is the velocity of light, b is the radius of the resonator, and a is the radius of the interaction zone (Fig. 3); the parameter p depends on the angular dimension α of the chamber and for the quadrupole mode of the field it is equal to

$$p = 4\sin^2\alpha/\pi\alpha - \gamma;$$

while for the dipole mode it is given by

$$p = -\frac{32}{\pi\alpha}\sin\frac{\alpha}{2}\sin\frac{\alpha}{4}\left(\sin\frac{\alpha}{4} + \cos\frac{\alpha}{4}\right) - \Upsilon;$$

$\lambda = 0.5772\ldots$ is Euler's constant. The function $f(x)$ is given by

$$f(x) = N_1(x)/J_1(x) - (2/\pi)\ln x,$$

where J_1 and N_1 are first-order Bessel and Neumann functions, respectively. Estimates show that for the quadrupole mode of the field the natural frequency of the resonator depends chiefly on its radius and slightly also on the parameters a and α. For waves of $\lambda = 2$ m we obtain b $\approx$ 25 cm. The natural frequency of the dipole mode is essentially different from that of the quadrupole mode.

The quality of a four-chamber H resonator is much lower than that of cylindrical resonators excited on the E_{010} wave. Because of the small reserve of high-frequency energy in the resonator volume, however, the pulse losses of h-f power do not exceed the usual values for E resonators loaded by drift tubes. On the other hand, the low quality of the resonator permits a considerable abbreviation of the h-f pulse width and also of the mean power losses. An acceleration of long and intense beams requires a compensation of the h-f energy losses.

LITERATURE CITED

1. N. W. MacLachlan, Theory and Application of Mathieu Functions [Russian translation], Izd-vo inostr. lit. (1953).