Measuring and Analyzing the Transverse Emittance of Charged Particle Beams

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Abstract. The emittance is an important characteristic of propagating charged particle beams, because it allows the prediction of focus size and beam losses. After introducing the concept and various definitions of the emittance and Twiss parameters, useful applications of this representation will be explored for the transverse emittances of charged particle beams. This is followed by a presentation of different measuring methods, together with their advantages and disadvantages. The second part discusses the analysis of emittance data, and possible mitigations for effects introduced by noise, bias, and other artifacts.

INTRODUCTION

The theory and practice of charged particle beam emittances have been a topic of interest since the early days of charged particle accelerators. Initially the interest was limited to the very few laboratories with accelerators, and therefore the topic was mostly discussed in internal reports and design studies. The shocking demonstration of the awesome power of the nuclear force and the prospect of its peaceful use generated widespread interest in nuclear physics. Accordingly, a rapidly growing number of institutions acquired accelerators, and the topic of beam emittances gained broad interest, as evidenced by many publications in journals and conference proceedings starting in the late 1950's.

In 1980 Lejeune and Aubert published an in-depth, 100-page review that gave detailed discussions of the theory, applicability, and the history, as well as state of the art of measurement techniques [1]. Their paper remains the most valuable resource on the topic of emittance.

In 1989, at the (1^{st}) Accelerator Instrumentation Workshop, Sander presented a tutorial on transverse emittances. His concise 29-page paper focuses on emittance definitions, the ellipse and its propagation, updated state of the art of measurement techniques, and some analysis methods, all very valuable for newcomers and experts [2].

This paper, presented at the 12th Beam Instrumentation Workshop, is structured around a similar introduction. It contains a detailed discussion of the

emittance, the emittance ellipse with its Twiss parameters, their physical meanings, and their units that remain points of discussion. After a brief introduction to beam sampling, the measurement techniques are recounted together with their potential problems. While most emittance distribution measurement techniques remain basically unchanged, refinements continue to be implemented, partly driven by new inventions and partly driven by growing computing power.

Interesting new techniques have been introduced to measure beam diameters that can be used to determine emittance parameters without measuring the emittance distribution. Some of these techniques are so minimally invasive that they can be used without impacting standard operations. Other techniques use signals from naturally-occurring byproducts so that it is possible to monitor stored beams without shortening their lifetime.

And finally, this paper discusses all common analysis methods that are in use. Three sample distributions are used to show the strength and limits of the most common techniques. It ends with a discussion of methods that are suited to assess the tails of the distributions, which gain in importance with the implementation of high-power accelerators [3] and ramping up their power [4].

DEFINITIONS

When an electric field connects to a plasma or a very hot surface, the field accelerates properly charged particles in the direction of the field, the axial direction z, until they reach the final momentum $p_i = m \cdot v_i = (p_{ix}, p_{iy}, p_{iz})$. Transverse momentum components are normally present, with a fraction originating from the initial particle motion in the plasma (ion temperature), and with the other fraction being gained from the transverse components of the electric field. The latter fraction causes a substantial correlation between the particle's transverse position components (x_i, y_i) and the transverse momentum components (p_{xi}, p_{yi}). Alternating axial electric fields in bunchers, Radio-Frequency-Quadrupoles (RFQ), and linear accelerators (LINAC) divide the stream into many small longitudinal bunches, in which there is a substantial correlation between the axial position within the bunch, z_i , and the axial component of the momentum, p_{iz} . Particle tracking is simplified by expressing the particle position and momentum coordinates with respect to the moving center (x_0 , p_{x0} , y_0 , p_{y0} , z_0 , p_{z0}) of the particle bunch as indicated in Fig.1.



FIGURE 1. A particle bunch in the 6-dimensional phase-space of location and momentum.

As a consequence of Liouville's theorem, the hyper-volume in the sixdimensional phase space (x, p_x , y, p_y , z, p_z) occupied by any fraction of these particles is conserved when the particles drift or pass through fields of conservative forces [5]. Conservative forces include the static electro-magnetic fields that are normally used to reshape and transport charged particle bunches to the desired location. Accordingly, this conservation law can be used to calculate the behavior of particle bunches as they propagate through an accelerator and reach the desired target. This is the basis for designing charged particle accelerators and beam lines, and for predicting beam losses as well as target yields. Accordingly, measurements of this hyper-volume can be used to initially characterize a beam for beam transport and loss studies and later to verify the optimal tune of the transport elements.

Often called "emittance," the volume emittance \mathcal{V}_6 is by definition the six dimensional hyper-volume of a certain fraction of the particles in a bunch:

$$\mathcal{V}_{6} = \iiint \iint dx \cdot dy \cdot dz \cdot dp_{x} \cdot dp_{y} \cdot dp_{z}$$

Because there is often no coupling between the axial motions and the transverse motions, the 6-dimensional volume emittance can be separated into a 2-dimensional longitudinal emittance area \mathcal{A}^L and a 4-dimensional transverse emittance volume \mathcal{V}^T :

$$\mathcal{V}_{6} = \iiint dx \cdot dy \cdot dp_{x} \cdot dp_{y} \cdot \iint dz \cdot dp_{z} = \mathcal{V}_{4}^{T} \cdot \mathcal{A}^{L}$$

The longitudinal emittance is given by the particles' axial positions and their variations in time and therefore is determined from the beam current measured as a function of time at a specific location. Here we focus on the transverse emittance that is given by the particles' transverse positions and their variations as a function of time or as a function of longitudinal position z:

$$z = \int \mathbf{v} \cdot dt = \int (dz/dt) \cdot dt$$

As the components of forces act on the components of the position or the velocity, the 4-dimensional emittance volume can normally be separated into two orthogonal emittance areas \mathcal{A}^{X} and \mathcal{A}^{Y} .

$$\mathcal{V}_4^T = \iint dx \cdot dp_x \cdot \iint dy \cdot dp_y = \mathcal{A}^x \cdot \mathcal{A}^y$$

There are cases where this formula can be misapplied. For example, lowenergy beam particles travel on helical trajectories through the core of a solenoid. This causes the transverse exit coordinates to become linear combinations of the transverse input coordinates, which introduces a strong coupling between the coordinates. The emittance of a beam without axial symmetry appears to grow inside the solenoid unless the coordinate system is rotated together with the beam.

Where the particle energy remains constant along the beam axis, normally one can factor out the momentum p, yielding the transverse emittance V^T in the trace space x, y, x'=dx/dz, and y'=dy/dz, where x' and y' are the Cartesian trajectory angles:

$$\mathcal{V}_4^T = \iiint dx \cdot dy \cdot dp_x \cdot dp_y = \vec{p}^2 \cdot \iiint dx \cdot dy \cdot dx' \cdot dy' = \vec{p}^2 V_4^T$$

The emittance in the "trace space" is most convenient because it directly describes the trajectories and therefore easily allows for the determination of losses on apertures or walls. If the coordinates are not coupled, the 4-dimensional emittance volume can again be separated into 2-dimensional emittance areas in the trace space $\{x, x'\}$ and $\{y, y'\}$:

$$V_4^T = \iint dx \cdot dx' \cdot \iint dy \cdot dy' = A^x \cdot A^y$$

The acceptance of rectangular apertures and wave-guides are hyperrectangular, while the acceptance of round apertures and beam tubes are hyperelliptical. The emittance distributions of real beams, however, vary. Most ion sources feature relatively small, limiting extraction apertures that intercept particles with trajectories far from the axis. Often the results are expanding beams with nearly elliptical distributions, such as the distribution shown in Fig. 2. This density plot shows the current distribution as a function of the vertical position y and the vertical trajectory angle y' of the beam emerging from the SNS baseline H source, a cesium-enhanced, RF-driven, multicusp source.



FIGURE 2. The emittance distribution of the beam emerging from the SNS ion source.



FIGURE 3. The vertical emittance distribution of the beam emerging form the SNS LEBT.

The diverging, low-energy beams emerging from ion sources grow rapidly in size and therefore are often subjected to considerable aberrations in the limitedaperture lenses used in low energy beam transport systems (LEBT). The individual trajectories encounter focusing forces that increase more than linearly with the distance from the axis. This causes the outer trajectories to form crossovers before the central part of the beam does, resulting in S-shaped distributions.

A typical example can be seen in Fig. 3 which shows the 65 keV H⁻ beam emerging from the SNS LEBT [6]. The figure shows the core of the beam to converge as necessary for an optimal transmission through the RFQ. However, a part of the distribution's tail indicates a crossover, and the outermost part of the tail diverges past its crossover.

RFQs and LINACs have a relatively narrow acceptance because they are unable to accelerate the part of the beam that enters without the proper convergence. They act as emittance filters and the emerging beams have a rather elliptical emittance, such as the one shown in Fig. 4. The figure shows the



FIGURE 4. The horizontal emittance distribution of the beam in the SNS MEBT.

emittance distribution of the 2.5 MeV H⁻ beam in the middle-energy beam transport section (MEBT) that is downstream of the RFQ [7]. All equal density boundaries are of nearly elliptical shape. The density boundaries embracing the beam core are less tilted than the large boundary that embraces the tail, which is a signature of non-linear space charge forces.

Due to the predominately elliptical nature of emittances, most authors, but not all [8], proceed to define the "emittance" as the half-axis product of an ellipse with an area equal to the emittance area occupied by a certain fraction of the particle distribution:

$$E_4^T = \frac{V_4^T}{\pi^2}, \ E^x = \frac{A^x}{\pi}, \ \text{and} \ E^y = \frac{A^y}{\pi}$$

The inconsistent definition and the subtle distinction between the "emittance" and the "volume or area emittances" continue to cause significant confusion. Rather than dealing with this confusion, many authors circumvent the definition of emittance, area- and volume emittance. It is therefore important that every emittance value is characterized either as representing a half-axis product or an area or a volume. The half-axis products are used throughout this paper.

When the charged particles are accelerated, the axial velocity component increases dramatically, while any transverse field components at the entrance and the exit of the acceleration field can be factored out as the conservative force field of an emittance-conserving lens. As shown in Fig. 5, the increase of the axial velocity components decreases the trajectory angles, and hence the emittance in the trace space. For example, the 6-fold velocity increase in the SNS RFQ causes the ~100 mrad divergence seen in Fig. 2 to shrink to ~20 mrad in Fig. 4.

The shrinkage of the trajectory angles can be factored out by multiplying the emittance with the relativistic velocity of the charged particles. The normalized emittance is then defined as

$$E_{norm} = \beta \cdot \gamma \cdot E$$
 with $\beta = \nu/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

where β is the ratio between the particles' velocity *v* and the speed of light *c*, and γ is the relativistic correction factor.



FIGURE 5. Trajectory angles shrink as the axial velocity increases.

In low-energy beam transport systems, which often simultaneously transport different charge states, β can be conveniently calculated from the ion source potential (energy/charge) and the charge state:

$$\beta = \sqrt{|\text{charge}[e]| \cdot \frac{\text{energy}}{\text{charge}} [kV] / \text{mass}[amu] / 685}$$

Further acceleration isolates the desired charge state and β is simply calculated from the ratio of the energy divided by the mass. An older normalization using the square root of the energy per nucleon in MeV became unfashionable when the main interest moved to accelerators capable of relativistic energies.

The normalized emittance remains roughly the same throughout the entire accelerator. It can decrease through collimation, but more typically it increases gradually throughout the accelerator, caused mostly by slightly mismatched, coupled systems and by non-conservative force fields, such as stripper foils.

The area or volume emittance of 100% of a Gaussian distribution is infinite because the distribution extends to infinity. The area or volume emittance of a finite, real beam is finite, but dominated by particles that are often very far from the central trajectory and encounter highly non-linear fields, and therefore are easily lost. For this reason the emittance is normally given for a fraction of the beam, with 90% being most common. The emittance changes rapidly with the fraction of the enclosed beam and therefore it is important to characterize emittances with the % level of the included beam fraction.

This limitation was eliminated through the introduction of the rms emittance by Chasman [9] and refined by Lapostolle [10]. For the (x,x') it is defined as:

$$E_{rms}^{x} = \sqrt{\left\langle x^{2} \right\rangle \cdot \left\langle x^{\prime 2} \right\rangle} - \left\langle x \cdot x^{\prime} \right\rangle^{2}, \text{ with } \left\langle x^{2} \right\rangle = \frac{\sum_{adl} (x - \bar{x})^{2} \cdot c(x, x^{\prime})}{\sum_{adl} c(x, x^{\prime})}, \ \bar{x} = \frac{\sum_{adl} x \cdot c(x, x^{\prime})}{\sum_{adl} c(x, x^{\prime})},$$
$$\left\langle x^{\prime 2} \right\rangle = \frac{\sum_{adl} (x^{\prime} - \bar{x})^{2} \cdot c(x, x^{\prime})}{\sum_{adl} c(x, x^{\prime})}, \ \bar{x}' = \frac{\sum_{adl} x^{\prime} \cdot c(x, x^{\prime})}{\sum_{adl} c(x, x^{\prime})}, \text{ and } \left\langle x \cdot x^{\prime} \right\rangle = \frac{\sum_{adl} (x - \bar{x}) \cdot (x^{\prime} - \bar{x}^{\prime}) \cdot c(x, x^{\prime})}{\sum_{adl} c(x, x^{\prime})},$$

The rms emittance is finite for the entire beam, as long as the total beam current is finite. Despite its finite nature, rms emittances are often given for a fraction of the beam, which should be clearly stated.

The rms emittance is defined as a semi-axis product. Since it always was consistently defined, there seems to be a trend to use the rms emittance definition, which avoids the confusion regarding other emittance definitions.

The rms emittance is conserved for conservative, linear forces. It is not conserved for non-linear forces that distort the emittance ellipse into an S-shape. Not being conserved, the rms emittance becomes a valuable quality parameter because it uses "average distances from the center of the particle distribution in the position versus trajectory-angle phase space".

Each transverse emittance area has the unit of a length times an angle. The length is normally given in m, cm, or mm, while the angle is given in rad(ians), or mrad. Since the angle is actually a ratio some people omit this unit and write μ m rather then mm·mrad [11].

When the emittance is a half-axis product, it is common to write the symbol π in front of the units, often separated with a symbol for multiplication. This custom follows a suggestion that tried to clarify which emittance definition was used in the evaluation [1]. The symbol π was supposed to show that π was factored out, which is technically incorrect for rms-emittances, which are always defined as half-axis products. In addition, the symbol π is often called a factor, which is incorrect and can lead to the misinterpretation that rms-emittances should be divided by π . A well-intended suggestion to omit the " π " is of limited benefit, because it replaces one confusing convention with another one [11]. The confusion originates in the fact that two different definitions have been established, and that the name emittance is often used as a shorthand for area-emittances and volume emittance value represents a half-axis-product or an area or a volume.

THE EMITTANCE ELLIPSE AND ITS TRANSFORMATION

The emittance ellipse of a drifting beam rotates when the beam gradually changes from a convergent to a divergent beam. The correlation between positionand angle coordinates briefly disappears in the upright ellipse found at the beam waist. Inside a focusing element the ellipse flips over before the beam emerges, possibly with a different convergence as shown in Fig. 6.



FIGURE 6. The beam envelope and the emittance ellipse of a drifting beam that is refocused.

At each location along the axis z, the rotating ellipse can be described with the squares and cross product of x and x', and the Twiss parameters α , β , γ , and ϵ :

$$\gamma \cdot x^2 + 2 \cdot \alpha \cdot x \cdot x' + \beta \cdot x'^2 = A/\pi = \varepsilon$$

The Twiss parameters give a simple description of the ellipse. The dimensionless α , for example, relates to the x,x' correlation, and is negative for diverging beams, zero in beam waists or antinodes, and positive for converging beams. The parameter β is by definition positive (β >0) and is measured as length per angle (such as m/rad).

 ϵ is also positive, but measured as the product of a length and angle, such as m·rad, as previously discussed. The parameter γ is also positive, measured in angle per length (e.g. rad/m) and dependent on α and β :

$$0 < \gamma = (1 + \alpha^2) \beta$$

For the rms emittance ellipse $\varepsilon_{\rm rms}$, α , β , and γ can be calculated as:

$$\alpha = -\langle x \cdot x' \rangle / \varepsilon_{rms}$$
, $\beta = -\langle x^2 \rangle / \varepsilon_{rms}$, and $\gamma = -\langle x'^2 \rangle / \varepsilon_{rms}$

When a beam drifts through a field-free region, the Twiss parameter γ remains constant because the related maximum divergence x'_{max} remains constant:

$$x'_{\max} = \sqrt{\gamma \cdot \varepsilon}$$

The Twiss parameter β , however, changes because the related maximum radius x_{max} changes throughout a drift region.



FIGURE 7. The emittance ellipse and its geometrical properties

More relations between the geometry of the emittance ellipse and the Twiss parameters can be found in Fig. 7. The orientation angle θ and the aspect ratio a/b shown in Fig. 7 are given by:

$$\tan(2\theta) = \frac{(2\alpha)}{(\gamma - \beta)}$$
$$\frac{a}{b} = \frac{\alpha}{\beta} + \frac{(\beta + \gamma)}{2} + \frac{\sqrt{(\beta + \gamma)^2 - 4}}{2}$$

The formula for the beam radius is used to calculate the beam envelope, as is the one shown in Fig. 6. The effect of drifts on charged particle trajectories through electric and magnetic fields, as well as through field-free regions, is described with a transfer matrix R that acts on the initial transverse-position and trajectory-angle vector *i* to calculate the final vector f[12]:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{f} = \begin{bmatrix} R \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}_{i}$$

As simple examples, the transfer matrix for a drift region of length L and a lens with focal length f are:

$$\begin{bmatrix} R \end{bmatrix}_{L} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} R \end{bmatrix}_{f} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

The transfer matrix of a sequence of ion optical elements is the product of the individual transfer matrixes. The example shown in Fig. 6, which has drift spaces a and b next to the lens f, is described by:

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$$\begin{bmatrix} x_f \\ x'_f \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x'_i \end{bmatrix}$$

In the presence of coupling that may be introduced by dipole magnets, skew quadrupoles, and RF deflectors, the equations need to be written for the entire four or six dimensional vectors and matrixes to include all off-diagonal elements.

By introducing the $\boldsymbol{\sigma}$ matrix, the ellipse equation can be written in matrix form:

$$\begin{bmatrix} x \\ x' \end{bmatrix}^T \cdot \begin{bmatrix} \sigma \end{bmatrix}^{-1} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = 1 = \begin{bmatrix} x \\ x' \end{bmatrix}_f^T \cdot \begin{bmatrix} \sigma \end{bmatrix}_f^{-1} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}_f = \begin{bmatrix} x \\ x' \end{bmatrix} \cdot \begin{bmatrix} R \cdot \sigma \cdot R^T \end{bmatrix}_f \cdot \begin{bmatrix} x \\ x' \end{bmatrix}^T$$

with
$$\left[\sigma\right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \mathcal{E} \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

Using the well-known transfer matrixes of ion optical elements allows for propagating the sigma ellipse. For example, for the drifting beam shown in Fig. 6 the ellipse rotates with position z as

$$\begin{bmatrix} \sigma \end{bmatrix}_{Z} = \mathcal{E} \cdot \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \beta_{0} & -\alpha_{0} \\ -\alpha_{0} & \gamma_{0} \end{bmatrix}_{I} \cdot \begin{bmatrix} 1 & 0 \\ z & 1 \end{bmatrix} = \mathcal{E} \cdot \begin{bmatrix} \beta_{0} - 2 \cdot \alpha_{0} \cdot z + \gamma_{0} \cdot z^{2} & \gamma_{0} \cdot z - \alpha_{0} \\ \gamma_{0} \cdot z - \alpha_{0} & \gamma_{0} \end{bmatrix}$$

Rather than calculating numerous individual trajectories, propagating the emittance ellipse through the system allows for conveniently calculating the beam radius throughout the accelerator or the beam line:

$$x_{\max}(z) = \sqrt{\beta(z) \cdot \varepsilon} = \sqrt{\sigma_{11}(z)}$$

BEAM SAMPLING FOR EMITTANCE MEASUREMENTS

Most beams contain an overwhelming number of ions, ranging from 10^5 in beams of very highly stripped ions from EBIS or ECR ion sources to the 10^{16} protons per second that will be delivered to the SNS target at full power. No technique exists to make all those individual trajectories visible or to capture the copious amount of information from such short-lived phenomena.

Rather than individual ions, emittance measurements sample a small fraction of the beam, called a beamlet, which contains thousand to billions of ions. These beamlets are then analyzed to obtain their trajectory angle distribution. To obtain an adequate resolution, the beamlets must be small, which means that practically the entire beam has to be stopped to determine the emittance contribution from each of the many beamlets. Since the samples must be small, their contributions to the emittance are small, and therefore it is important that the samples are clean. Obtaining small samples that are free from signatures caused by stopping ~99% of the beam is an intrinsic challenge.

At the 11th BIW, Plum gave an in-depth tutorial on interceptive beam diagnostics, which is an excellent introduction to stopping beams [13]. This paper restricts itself to phenomena relevant for stopping 99% of the beam with slits. Briefly, ions penetrating a solid rapidly decelerate through numerous, predominately small-angle, collisions. The projected range for protons, shown in Fig. 8, was calculated with PSTAR [14] for several possible slit materials. If parts of the slits are thinner than the projected ion range, ions with reduced energy and a broadened trajectory angle distribution will exit from the back of the slits. They contaminate the sampled beamlet with large trajectory angles, and possibly with altered charge states. In addition to the ions, many electrons will exit, mostly with



FIGURE 8: The range of protons in various slit material as a function of proton energy.

very low energies and none of them exceeding twice the ion velocity. Due to their low energy, the electrons can be easily deflected to prevent contamination of the sampled beamlet.

If the beam hits a surface grazingly, the small-angle scattering returns a fraction of the beam back into the vacuum space with nearly the initial energy, slightly altered trajectory angles, and possibly altered charge states. Since the number of near-surface collisions increases with the inverse of the sine of the grazing angle, the fraction of reflected beam grows rapidly with decreasing impact angle and thus easily becomes a source of contamination. This problem can be avoided by tapering the edge-defining surfaces. As shown in Fig. 9 the downstream taper angle θ_D should always exceed the maximum divergence of the beam to keep the downstream edge surface in the shadow of the beam.

Figure 9a shows a slit design recommended for low-power, low-energy (<<1MeV/amu) beams. At these energies, the ion range is shorter than the typical



FIGURE 9: Slits for a) low-energy, b) high-energy, and c) high-power, high-energy beams.

width of the rough edge found on carefully-machined slits [15]. The fraction of the edge that is too thin to stop the ions is therefore given by the roughness of the edge rather than by its geometry, which means that the beam can be stopped with a simple, flat surface.

Figure 9b shows a slit design recommended for high-energy beams. Here the upstream, beam-facing side of the edge should be tapered with an angle θ_U that matches or exceeds the acceptance of the downstream element. This assures that surface-scattered particles are not included in the analysis. The width w of the edge that is too thin to stop the beam is then given by

$$w = r_i \cdot tan\theta_U \cdot tan\theta_D / (tan\theta_U + tan\theta_D)$$

where r_i is the ion range. In well designed systems, the downstream acceptance matches the maximum divergence of the beam, which makes

$$w \cong (r_i \cdot tan\theta)/2$$

Accordingly, slit scattering is minimized with small taper angles θ within the recommended limits.

Tapered surfaces reduce the surface density of the deposited beam power proportional to $\sin(\theta)$, favoring a small taper angle θ . If the beam power density requires a taper, the design must assure that the beam core is always intercepted on the tapered surface. This can require bulky slits for small tapers. Fig 9c shows a typical compromise angle θ_P selected for a high-power, high-energy beam. Slit designs for high-power beams normally require thermal modeling.

A beam hitting a surface generates a large number of secondary electrons. Most secondary electrons have a rather low energy (<20 eV) and are emitted close to normal to the surface. As a consequence of the escape probability, the electron intensity decreases proportional to the cosine of the exit angle with respect to the surface normal. Because of the large original number, even a small fraction of the secondary electrons can significantly contaminate the beam sample if it is allowed to reach the current probe that samples the beamlet.

MEASURING 4-DIMENSIONAL EMITTANCE DISTRIBUTIONS

Measuring the full 4-dimensional transverse emittance distribution requires a 2-dimensional sampling of the beam, with each sampled beamlet being sampled by the second slit to determine its transverse velocity distribution, as shown in Fig.10.

The first set of 4-jaw slits is positioned to $x-\Delta x/2$, $x+\Delta x/2$, $y-\Delta y/2$, $y+\Delta y/2$ to sample a $\Delta x \cdot \Delta y$ large beamlet at position (x,y). The second set, at a distance L downstream of the first set, is positioned to $x_2-\Delta x_2/2$, $x_2+\Delta x_2/2$, $y_2-\Delta y_2/2$, $y_2+\Delta y_2/2$ to sample a $(\Delta x+\Delta x_2)\cdot(\Delta y+\cdot\Delta y_2)/L^2$ large angular range around the



FIGURE 10. Four dimensional emittance distributions are measured with 2 sets of 4-jaw slits

trajectory angle $((x_2-x)/L,(y_2-y)/L)$. The sub-beamlet that passes through both sets of 4-jaw slits enters a Faraday-cup, which measures the very small fraction of the beam current. If one is only interested in the beam core it can be sufficient to sample 20 positions with each set of opposite 2-jaw slits. 100 positions are more appropriate if the interest extends to the tails of the distribution. The required 10^5 to 10^8 samples take an excessively long acquisition time even at the highest repetition rates allowed with the required stepping motors.

The pepper pot scanner, however, circumvents the impossible scanning requirements entirely: it uses a viewing screen to see the trajectory-angle distribution of the beamlets sampled with a pepper-pot plate. The plate features a 2-dimensional, equidistant array of sub-mm holes, which are separated by several mm. As shown in Fig. 11 the plate lets pass several tens of small, circular position samples that expand or contract behind the plate according to the trajectory angle distribution of each sampled beamlet. Optimal accuracy requires an adjustable distance L between the pepper-pot plate and the viewing screen for allowing the individual beamlets to expand without overlapping on the screen.



FIGURE 11. Four dimensional emittance distributions measured with a pepper pot scanner

When introduced over 30 years ago, it was mostly used for instantaneous qualitative assessments using fluorescent screens. Quantitative assessments required photographic screens that had to be removed and analyzed offline. The analysis needs to evaluate all beamlets for their mean angle with respect to the beam axis and for the contours of different percentage levels of the most intense beamlet, a time consuming and cumbersome task.

To simplify the emittance evaluation for axially symmetric beams the following approximation was derived for the radial emittance ε_r :

$$\varepsilon_{\rm r} = ({\rm R} \cdot {\rm D}/(2 \cdot {\rm L})) \cdot ({\rm D}_{\rm L}/{\rm d} - {\rm S}_{\rm L}/{\rm S}) \cdot (1 - ({\rm S}/{\rm R})^2)^{-1/2}$$

where D and D_L are the diameters of the holes and the images, S and S_L are the respective distances from the beam centers, L the distance between the pepper-pot plate and the screen, and R the full beam radius at the pepper-pot plate [16].

The advent of small, high-resolution digital cameras with high-speed electronic readouts has dramatically improved the convenience of the method. Quickly recording the screen image, and comparing the distribution with a prerecorded calibration net, a computer can evaluate and display the results in a split-second. Using a screen with sufficient sensitivity drastically reduces the data acquisition time, and actually allows for measuring shot-to-shot variations with pulsed beams.

The minimal acquisition time can enable emittance measurements where conventional slits fail due to thermal loading. For example, GSI replaced a slitharp scanner with a pepper-pot scanner to measure the emittance in the new prestripper of the UNILAC, where heavy ions with several 100 kV/amu energy kept damaging the slits. Their calibration grid is established with screen shots obtained with an aligned laser illuminating the pepper pot plate [17]. GSI has also developed an amazing emittance analysis system that uses the pepper pot emittance data to reconstruct the beam in many interesting projections [18]. GSI has successfully demonstrated emittance measurements with up to 15 mA Ar beams [17].

Several Labs are developing pepper pot emittance scanners for low-energy ion beams over a wide range of beam currents [19].

The short acquisition time of pepper pot emittance scanners enable emittance measurements on low and medium energy beams almost without a serious interruption of normal operation. This is a significant advantage for user facilities.

A significant number of beamlets need to be sampled to obtain the desired accuracy, which limits the pepper pot scanner to beams that are relatively large at the pepper pot plate as well as on the screen.

Potential drawbacks are the accuracy of the alignment grid, as well as the linearity of the beam intensity versus the light output of the screen, the screen resolution, and the camera response. Alumina screens are currently used by GSI, where phosphor and ruby screens are tested for the low energy beams.

MEASURING 2-DIMENSIONAL EMITTANCE DISTRIBUTIONS

If there is no coupling between two transverse, orthogonal directions, it is sufficient to measure the 2-dimensional emittance distributions in each of those directions, x and y. Because the 2-dimensional emittance distribution is independent of the orthogonal coordinate, the current should be integrated over this coordinate to enhance the signal without changing the measured distribution. Compared to a four-dimensional emittance measurement, the integration increases the measured current signal 1-2 orders of magnitude and makes the results potentially less sensitive to bias, noise, and artifacts.

As shown in Fig. 12, the transverse integration is experimentally accomplished with 2-jaw slits that are set to $x-\Delta x/2$ and $x+\Delta x/2$ to sample Δx large beamlets at position x, while a second set, at a distance L downstream of the first set, is positioned to $x_2-\Delta x_2/2$ and $x_2+\Delta x_2/2$ to sample a $(\Delta x+\Delta x_2)/L$ large angular range around the trajectory angle $(x_2-x)/L$. The sub-beamlet that passes through both sets of 2-jaw slits enters a Faraday cup, which measures the current as a function of slit position x and trajectory angle x'. In some emittance scanners, the second slit and Faraday cup are replaced with a single wire with a diameter that corresponds to the slit width Δx_2 . In either case, a relative alignment within tight tolerances is required to accurately measure the emittance of beams with small angular divergence.

Artifacts generated by the stopped beam can also limit the accuracy of measured emittance distributions. For example, Fig. 13a shows the background of an emittance distribution of a 35 kV H- beam measured with a 2-slit and a Faraday cup scanner. The plot shows increasing signal strength with darkening gray shades, with black representing the beam core. The background appears to be independent of the position of the second slit, but undulates and slowly increases with increasing position of the first slit. The observed dependence suggests that some of the secondary electrons emitted from the first slit reach the Faraday cup,



FIGURE 12. Two dimensional emittance distributions are measured with 2 sets of 2-jaw slits



FIGURE 13. a) A 2-slits and Faraday cup emittance plot with an undulating, tilted background.b) A slit and a wire emittance plot shows a background with several artifacts.

possibly through multiple scattering. Such stray currents can be intercepted with deflecting fields or with well-placed shields, including a shield around the back of the Faraday cup.

Figure 13b shows the emittance data from a slit and wire measurement probing a low-energy proton beam. The pure white represents negative signals. Apparently electrons are responsible for the clustered white areas around the -5 mm slit position where the largest fraction of beam passes the slit. The asymmetric dependence on the wire position can be explained with a tilted or offset surface, which stops the fraction of the beam that passes the slits. The triangular flat socket found for negative slit positions and positive trajectory angles is unexplained, but likely an artifact because beams normally rapidly fade with trajectory angles. Due to the open geometry, wires are normally more prone to artifacts.

MEASURING TRAJECTORY ANGLE DISTRIBUTIONS WITH MULTI-COLLECTORS

The 10^3 to 10^4 stepper motor movements and measurements required for 2-slit and slit and wire scanners consume a considerable amount of time. A multicollector that measures the trajectory angle distribution simultaneously can speed up the acquisition time by 1 to 2 orders of magnitude. Multi-collectors come in two basic forms: multi-collectors and wire harps. As shown in Fig. 14, the trajectory angle distribution can be deduced from an alternating stack of metal and insulator plates that is mounted in place of the second slit. The current from each collector plate needs to be amplified, converted and recorded by a computer. The angular acceptance of each collector is given by $(\Delta x+d)/L$, where d is the thickness of the collector plates. The angular resolution is about the same because the insulators should be rather thin to minimize charging problems. The angular acceptance of the collector is limited by the number of collector plates, which is typically limited to ~20 by the economy of the electronic hardware required for each plate.

Adjustable bias voltages for the collectors are a significant benefit. A positive bias voltage prevents secondary electrons from escaping and should give the most reliable measurements. However, as shown in Fig. 14b, it can lead to cross talk between the plates, which inflates the measured emittance. In addition, a positive bias attracts electrons that are generated elsewhere, which can cause a nonuniform background.

A negative bias pushes the secondary electrons away, which alters the signal strength to $I = (1\pm\gamma) \cdot I_{\pm}$ where the sign matches the sign of the intercepted ion beam current I_{\pm} and γ is the secondary electron emission coefficient. γ depends on the ions' species, their charge state, their energy, and their impact angle as well as on the composition and condition of the intercepted surface. Because γ depends on the surface condition, it can be expected to change with beam exposure. γ ranges from much less than 1 to several tens, and therefore can reverse the signal polarity when intercepting negative ion beams. For example, Fig. 15 shows the positive signals measured with a negatively biased multi-collector intercepting a low-energy H⁻ beam.



FIGURE 14. a) Multi-collector emittance scanners use a multiple collector to measure the trajectory angle distribution. The collectors can be run with a positive (b) or a negative (c) bias.



FIGURE 15) Emittance distribution measured with a negative biased multi-collector.

In addition, neutral and photon beams I₀ will also generate a signal I with

 $I = \gamma \cdot I_0$

Such signals can be seen as a diagonal strip in Fig. 15 because the collectors that intercept the flux change linearly with the changing position of the slit.

Normally, the bias is tuned for most-convincing or best results. However, measuring the emittance distribution for different bias voltages allows for assessing the effect of those problems and for putting an upper limit on the accuracy of the measurements.

The variations in the background of Fig 15 are caused by the variations in the zero-offset of the different amplifiers. The individual zero-offsets can be easily subtracted for each amplifier after being determined from the many data that show no signals or artifacts.

Replacing the second slit with a wire harp increases the design flexibility. The angular resolution of an individual wire is given by $(\Delta x+d)/L$, where d is the wire diameter, while the angular acceptance of the harp is $(\Delta x+d+(n-1)\cdot s)/L$, where n is the number of wires and s is the spacing between the centers of two adjacent wires. This is especially interesting when the wire harp position is controlled independently of the slit [20]. For example, as shown in Fig. 16, two measuring positions can be combined to double the angular resolution (16a) or double the angular acceptance (16b). Most interesting is the combination shown in Fig. 16c because it gives a better assessment of the tails and an increased resolution of the beam core where the signals change rapidly.



FIGURE 16) Two position measurements with a wire harp can double the angular resolution (a), or double the angular acceptance (b), or do a combination of both.



FIGURE 17) Electric Sweep Scanners display the emittance distribution on X-Y oscilloscopes.

ELECTRICAL SWEEP SCANNERS

Electrical scans can be much faster than mechanical scans and shorten the acquisitions time. This technique allowed for introducing an instantaneous emittance display over 30 years ago, long before the dawn of high-speed computing. As shown in Fig. 17, a pair of adjacent electric deflectors with reversed polarity is powered with saw-tooth-profiled voltages. This sweeps the beam over the first set of slits, by varying the dogleg offset of the beam without changing its trajectory angles. A third set of electric deflector plates is powered with significantly higher frequency voltage sources, which sweep the selected beamlet over the second slit to determine the trajectory angle distribution. The actual beam emittance distribution can be displayed on a X-Y oscilloscope with the measured and amplified current signal being swept as a function of the two, properly delayed, sweep voltages [21].

The lack of moving parts gives the electric sweep scanner a high reliability and availability and allows for installing highly effective shields to block secondary electrons from the first set of slits. As discussed in the next section, the electric sweep scanners can only be used at low beam energies. An additional drawback is the insertion length required for the three electric deflectors. Accordingly electric sweep scanners were often installed in a dedicated side-beam line to support the tuning of the upstream LEBT [21].

ALLISON SCANNERS

The required insertion length can be reduced by $\sim 2/3$ if the position is probed with a mechanical scan. Accordingly Allison introduced a hybrid scanner [22], which is sometimes called, less-precisely, an "electric sweep scanner". As shown in Fig. 18, a mechanical scan probes the position distribution, while an electric scan probes the trajectory angle distribution by measuring the current passing a second slit and entering a suppressed Faraday cup. The scanner has several significant design advantages: being mounted on a single block allows for alignment of the two slits within tight tolerances. Having a single mechanical position control allows for surrounding the entire trajectory angle analysis area



FIGURE 18) Allison scanners scan the position mechanically and the angles electrically.

and the Faraday cup with a light-tight shield that intercepts all charged particles generated by the beam impacting on the scanner entrance. In addition, the internal electric field sweeps away all particles entering the scanner with low energies, such as secondary, convoy, and binary electrons.

The voltage to angle conversion was derived as

$$x' = V \cdot L_{eff}/(2 \cdot g \cdot U)$$
 or $V = 2 \cdot U \cdot x' \cdot (g/L_{eff})$

where g is the gap between the deflection plates, V is the voltage of opposite polarity that is applied to the electrical deflection plates, L_{eff} the effective length of the electric deflection field [23], and U is the kinetic ion energy per charge [24]. In addition, the gap g limits the angular acceptance to

$$x'_{max} = 2 \cdot g/L_{eff}$$



FIGURE 19) a) The clusters of inverted signals shown in black are caused by ions backscattered from the deflector plates. **b)** A background with a random pattern of positive and negative signals is found with stair-cased deflector plates.



FIGURE 20) Stair-cased deflector plates eliminate forward scattering into the Faraday cup.

On the other hand, the angular acceptance is also limited by the maximum voltage V_0 that can be applied to the deflector plates. An optimal design matches these to limits, from which one gets the lesser-known design equation:

$$V_0 \cong x'_{max}^2 \cdot U_0$$

where U_0 is the highest ion energy per charge that needs to be measured [24]. The equation shows that the application of electrical sweep scanners is limited to small ion energies per charge, typically substantially less than 1 MV.

Recent measurements identified ions, backscattered from deflector plates, as the source of the ghost signals observed with an Allison scanner [25], as shown in Fig. 19a. The inverted signals are significantly less than 1% of the measured peak current and can only be observed in low-noise measurements [24]. The small intensity and the inverted nature make this measurement an interesting test case that is used to test the robustness of emittance analysis methods for this paper.

The problem can be mitigated with stair-cased deflector plates as shown in Fig 20. They produce random pattern background data as shown in Fig. 19b. As shown in Fig. 20, it is important that all ions impact on the faces of the stairs, which can be achieved with a staircase angle of x'_{sc} of

$$x'_{sc} = (8)^{1/2} \cdot g/L_{eff}$$

A rejection ratio in excess of 99% has been achieved and optimization allows for even higher ratios [24]. With this modification, Allison scanners are expected to measure the most accurate emittance distributions for low-energy charged particle beams.

Charge reversed beams (e.g. protons in an H⁻ beam) are deflected in the opposite direction, with reversed α , forming a mirror image [26]. Insufficient suppressor voltages allow the escape of secondary electrons, which makes energetic neutral beams visible [27].





THE EMITTANCE-MASS SCANNER

Ion sources normally output a mixture of ions with different masses and charge states, especially positive ion sources. Because all ions have the same energy per charge, they follow the same trajectories in electric fields. However, different ion temperatures and stray magnetic fields often introduce slight variations in the emittance of the different species and charge states. Emittances measured at the output of the ion source or in electrical LEBTs are a linear combination of the emittance of the different species. Of interest, however, is only the emittance of the specie that will be selected in an analyzing magnet or be accelerated by the RFQ.

This dilemma has been addressed with a variation on the Allison scanner developed at TRIUMF [28]. They added a permanent magnetic field that is parallel to the direction of the slit gaps. The magnetic field generates a force that is parallel (or anti-parallel) to the electric force. The magnetic force is proportional to the velocity of ions, which differs for ions with different masses, but with the same energy per charge. Similarly, the ion velocity increases with charge state for ions with the same energy per charge. For ions that pass through the second slit, the electric field needs to balance the magnetic force in addition to bending the non-zero trajectory angles back onto the centerline. The mass and charge-state dependent offset separates the emittance distributions, as one can see in Fig. 21.

EMITTANCE ELLIPSE MEASUREMENTS

Emittance areas of beam percentages can be measured without the sophisticated diagnostic needed for measuring distributions. The simplest method requires two sets of adjustable slits separated by a moderate drift space with distance L. In addition one needs an upstream lens that can create a focus in the location of one of the slits, and a downstream Faraday cup with an acceptance that can measure the entire refocused beam.

The half-axis-product of the emittance rectangle that contains, for example, 90% of the beam can be found with the following procedure: First one fully opens one set of slits. Next, one alternates between closing the other set of slits and tuning the upstream focus (and steering elements if needed) until one finds the smallest opening that can pass 95% of the beam. This procedure produces a beam waist in the closed slits ($\alpha_0=0$). The measured half opening is the beam waist radius $r_0^{95\%}$, which relates as $\beta_0 \cdot \varepsilon = (r_0^{95\%})^2$.

Next, one carefully adjusts the second set of slits to find the minimum opening that passes 95% of the remaining beam. This measured half opening is the radius $r^{95\%}$ of the beam at a drift distance L with $\beta \cdot \epsilon = (r^{95\%})^2$. Using the ellipse matrix propagation equation we find for $\gamma = (\beta - \beta_0)/L^2$ and therefore

$$\epsilon^{90\%} = r_0 \cdot (r^2 - r_0^2)^{1/2} / L$$

This method is useful in marginally equipped and poorly characterized LEBTs. It is, in fact, a simplified variation of a more general method that allows for determining the emittance and Twiss parameters from a minimum of three beam radius measurements.

The 3-gradient method requires one beam profile monitor downstream of a well-characterized lens and well-characterized beam transport elements between the two. Using the standard beam model, the measured beam radius r can be expressed as a function of β at the monitor. Next, β can be expressed as a function of the Twiss parameters α_l , β_l , γ_l , and ε describing the beam entering the lens, and the corresponding transfer matrix elements R_{ij} for the beam transport between the lens entrance and the beam profile monitor. For each beam radius r_k measured with focal strength f_k , one can write the equation as a difference that should vanish.

$$r_k^2 - \varepsilon \cdot \beta_k = r_k^2 - \varepsilon \cdot \left(R_{11,k}^2 \cdot \beta_I - 2 \cdot R_{11,k} \cdot R_{12,k} \cdot \alpha_I + R_{12,k}^2 \cdot \gamma_I \right) = \Delta_k \approx 0$$

A minimum of three measurements and the identity $\beta \cdot \gamma = (1 + \alpha^2)$ are required to solve the 4 unknowns. Three measurements give exact solutions with all Δ_k vanishing. However, it is more interesting to over-determine the problem. Assuming a Gaussian distribution of the deviations Δ_k , their likelihood can be maximized by minimizing the sum of their squares.

$$\sum_{k=1}^{N} \Delta_{k}^{2} = \sum_{k=1}^{N} \left(r_{k}^{2} - \varepsilon \cdot \left(R_{11,k}^{2} \cdot \beta_{I} - 2 \cdot R_{11,k} \cdot R_{12,k} \cdot \alpha_{I} + R_{12,k}^{2} \cdot \gamma_{I} \right) \right)_{k}^{2} = S = \min$$

Differentiating the sum of the squares *S* with respect to $\varepsilon \cdot \beta_I$, $\varepsilon \cdot \alpha_I$, and $\varepsilon \cdot \gamma_I$, one obtains the following set of equations [29]:

$$\begin{bmatrix} \sum_{k=1}^{N} R_{11,k}^{4} & -2 \cdot \sum_{k=1}^{N} R_{11,k}^{3} \cdot R_{12,k} & \sum_{k=1}^{N} R_{11,k}^{2} \cdot R_{12,k}^{2} \\ -2 \cdot \sum_{k=1}^{N} R_{11,k}^{3} \cdot R_{12,k} & 4 \cdot \sum_{k=1}^{N} R_{11,k}^{2} \cdot R_{12,k}^{2} & -2 \cdot \sum_{k=1}^{N} R_{11,k} \cdot R_{12,k}^{3} \\ \sum_{k=1}^{N} R_{11,k}^{2} \cdot R_{12,k}^{2} & -2 \cdot \sum_{k=1}^{N} R_{11,k} \cdot R_{12,k}^{3} \\ \sum_{k=1}^{N} R_{12,k}^{2} \cdot R_{12,k}^{2} & -2 \cdot \sum_{k=1}^{N} R_{12,k} \\ \end{bmatrix} \cdot \begin{bmatrix} \varepsilon \cdot \beta_{I} \\ \varepsilon \cdot \alpha_{I} \\ \varepsilon \cdot \gamma_{I} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{N} R_{11,k}^{2} \cdot R_{12,k}^{2} \\ -2 \cdot \sum_{k=1}^{N} R_{11,k} \cdot R_{12,k} \cdot r_{k}^{2} \\ \sum_{k=1}^{N} R_{12,k}^{2} \cdot r_{k}^{2} \end{bmatrix}$$

The solution together with the identity $\beta \cdot \gamma = (1+\alpha^2)$ yield $\varepsilon, \beta_l, \alpha_l$, and γ_l . Being over-determined, the scatter of the deviations Δ_k is a measure of the uncertainties of either the transfer matrixes and/or the measured beam radii. Assuming well known transfer matrixes, one can determine the accuracy of the measured beam radii r_k , as well as the accuracy of the Twiss parameters α_l , β_l , γ_l , and ε . For the relative errors one gets [29]:

$$\Delta r_{rel} = \frac{1}{2 \cdot N} \cdot \sum_{k=1}^{N} \left(1 - \frac{1}{r_k^2} \cdot \varepsilon \left(R_{11,k}^2 \cdot \beta_I - 2 \cdot R_{11,k} \cdot R_{12,k} \cdot \alpha_I + R_{12,k}^2 \cdot \gamma_I \right) \right)^2 \Delta \varepsilon_{rel} = 2 \cdot \Delta r_{rel}$$

0

If the measured beam radii cover a large range compared to their uncertainties, the three-gradient method gives accurate and reliable results. The required drastic detune is incompatible with normal operations and therefore the method cannot be used for monitoring routine operations.

The 3-position method is based on beam radii measured in three or more positions. The same method and equations apply. Rather than having transfer matrix elements with different focal strengths, the transfer matrix elements include the different elements that are between the first beam profile monitor and the subsequent ones. The accuracy of the results improves with the spread of the transfer matrix elements. Since no detune is required, the measured beam radii can be used as an emittance monitor during routine operations.

Beam profiles can be measured with scintillating viewing screens. Beam losses, and lifetime issues, as well as the linearity of the response, limit their applications. One such application was the target screen that confirmed the beam profile on the Hg target during commissioning of the Hg target and early operations of the Spallation Neutron Source [3, 30].

The limited-lifetime screen allowed for calibrating the wire-harp that can be inserted upstream of the target. This wire harp features an x-, a y-, and a 45° harp to detect coupling. Intermediate wire screens suppress the cross talk between the harps [3, 31]. Heating and sputtering limit the lifetime of the wires, and therefore continuous monitoring is limited to low-power, low-intensity beams. Some wire

harps amplify their signals by operating in a low-pressure background gas [1]. The beam-induced heating and sputtering gradually changes the amplification by the secondary electron emission and, slowly, the wire diameter. These changes lead to variations in the wire sensitivity due to the increased exposure in the center.

The non-uniformity problem is practically eliminated with (single) wire scanners that measure the profile by scanning through the beam. Having 2 or 3 wires with different orientations allows for measuring the x-, y, and 45° profiles in a single scan [32]. Oscillating beam probes and rotating helical wires are used to measure the x- and y-profile with a single probe [33]. Continuous monitoring is again restricted to low power beams.

Beams of ions with low ionization energies can be probed with laser wires that are free of wear and related problems. The liberated electrons are guided to a multiplier, where they create a signal that is proportional to the density of the intercepted ion beam [34].

Even less invasive are beam profile monitors that use the ionization of residual gas to determine the beam profiles. Some profile monitors extract the generated ion/electron pairs to generate signals proportional to the beam density [35, 36], where others map the emitted light [37, 38].

INTRODUCTION TO EMITTANCE ANALYSIS

The analysis of emittance and Twiss parameters from measured beam radii is straightforward if the measurements are sufficiently independent [29]. The accuracy increases with the number of independent measurements and formulas have been derived to determine the accuracy of the deduced beam parameters.

The final results are normally very reliable and fairly accurate because this method is mostly used with high beam energies where the distributions are nearly elliptical.

The complexity increases for the analysis of emittance distributions, especially at low beam energy, where the distributions significantly deviate from Gaussian distribution.

Often the distribution of the beam core is nearly Gaussian and thresholding the tails gives reliable estimates for the Twiss parameters of the beam core. The emittance, however, depends strongly on the fraction of the enclosed beam, which is much more difficult to evaluate, and can cause significant uncertainties when trying to normalize the results with measurements from Faraday cups or beam current toroids that normally measure the entire beam.

Three sets of low-energy emittance data are studied to demonstrate the effects and potential problems of various analysis methods. The first set obtained with 2slits and a Faraday cup has a few % noise-variations, but no artifacts. In addition the low-noise data from Fig. 19 are analyzed to highlight the problems introduced by artifacts. The analysis was performed with Visual Basic Macro routines in an Excel spreadsheet that is available [39].

The implementation of powerful accelerators [3] and their power ramp-up [4] drive an increasing interest in the beam -tail and -halo that is found on the outside of the beam core. We will discuss some problems that can interfere with the accurate measurements needed to determine tails and halo, as well as investigative techniques. After detected problems have been sufficiently mitigated, accurate distributions can be determined and integral beam analysis becomes possible.

THE BACKGROUND'S ROLE IN THE ANALYSIS OF EMITTANCE DISTRIBUTIONS

There are many thousand data points in most measured emittance distributions, which complicate their analysis, but can make that analysis more accurate and reliable. Figure 22, for example, shows all trajectory-angle scans that were measured for 45 beam positions in an expanding, low-energy H- beam. The figure shows that only a small fraction of data contains real current signals that are significant. In addition, at the bottom of each bell curve must be real current signals that are hidden in the noise. But clearly, the dominant fraction of the data is pure background that has nothing to do with the measured beam.

Due to the dominance of the background, a bias equal to a small fraction of a percent of measured peak current can skew the normalizing sum of all measured currents by tens of percents. A bias equal to 0.01% can skew the rms emittance by 100 percent when calculated from all data. Such small biases are common and beyond the control of the experimenter and often cause arbitrary results when calculating the rms-emittance from all measured data.

Electronically zeroing a current amplifier sets the zero of the output within $\sim 0.1\%$ of the output voltage range. When calculated as a fraction of the measured peak current, the bias increases to a fraction of a percent. Measuring and digitally subtracting can reduce the net bias to $\sim 0.01\%$. This can be sufficient to give



FIGURE 22: Current versus trajectory angle measured for 45 beam positions.

reliable rms-emittance results when calculated from all data as long as the fraction of background is not excessive. However, the bias can change with time, and its measurement contains some periodic- and random-deviations. In addition it may contain charged particle currents that are related to the beam, but not related to the sampled beamlet. It is therefore most reliable to determine the bias from the measured emittance data, which is one reason to analyze the background data.

Histograms are a useful tool to study the distribution of the large number of background data. The bin width should be a multiple of the ADC's bin width that converted the original data. However, the spacing between the current values is often altered by truncations and/or digital corrections that complicate the determination of the original bin width. Sorting the current values and plotting their frequency as shown in Fig 23 circumvents the truncation problem. Figure 23a shows the data from Fig. 22, where relatively large noise amplitudes spread the background data over many channels. The two figures on the right show lownoise data in low resolution caused by a 12-bit digitizer. Figure 23b shows the data with the random background shown in Fig. 19b. The ghost-infested data of Fig. 19a were analyzed for Fig. 23c, where the inverted ghost signals are visible in the left tail.

All three distributions in Fig. 23 do not exactly peak at 0, indicating small biases that need to be subtracted for better accuracy. Bias estimates can be obtained with a frequency-weighted average of the current values that exceed 20 to 50 % of the highest frequency. High thresholds are recommended to reduce the effect of the asymmetric contributions by the real current signals as well as artifacts that may be present. The data from figure Fig 23a suggest a bias of ~0.2 % of the measured peak current while the data from Fig 23 b and 23 c suggest a 0.4% bias.

The typically published emittance distributions show uniform backgrounds like Figs 2-4. This results from assigning the same color to all measured data that yielded less than a few % of the measured peak current. Figures 13, 15, and 19 highlight background features by assigning different colors to data that yielded



FIGURE 23) Frequency of measured current values for data with a) high noise, b) low noise, and c) low noise with inverted ghosts.

slightly different values near zero, while assigning a single color to all data that yielded more than a few percent of the measured peak current. In Fig. 19 the below- and above-bias data are plotted in contrasting colors. This is a sensitive method to search for artifacts because they form uni-color clusters in the otherwise random pattern of the background.

Artifacts typically include locations far from the core of the beam and therefore can contribute excessively to rms-emittance evaluations, which is the other reason for analyzing the background.

Once artifacts have been identified it is important to understand their origin. Known origins can suggest the most effective suppression. Corrections can be applied to the data, or the hardware can be modified, which is normally more effective. However, artifact mitigation is time-consuming and sometimes of limited effectiveness. We therefore discuss analysis methods that are best suited for artifact-infested as well as artifact-free data.

THRESHOLDED EMITTANCE ANALYSIS

Applying a simple threshold can eliminate background-related problems, which explains the popularity of the method. Most commonly all data with values below the threshold are ignored, while the data exceeding the threshold are weighted with the measured value, preferably lowered by the estimated bias.

Various methods have been developed to select the most appropriate threshold. One method raises the threshold until the random contributions far from the beam core disappear in a position versus angle plot. Another method raises the threshold until all islands that are detached from the beam core are eliminated. This method completely excludes all current signals that are not significantly exceeding the noise amplitudes. Either result will depend on the plotting routine because many employ averaging or smoothing methods.

Consistency can be improved by plotting the results versus the applied threshold. Figure 24, for example, shows the rms emittance, and the rms evaluated Twiss parameters α and β as a function of the applied threshold as a percentage of the measured peak current. The figure shows the analysis of the same data that were analyzed for Fig. 23. The dashed lines show the results obtained from the original data, where the solid lines show the results obtained after subtracting the bias obtained from pseudo-histograms. As Fig 24 shows, a bias subtraction improves the accuracy of the results and allows for setting a more realistic threshold.

As seen in Fig. 24, thresholded emittance estimates appear to be dominated by the "noise peak". The near-zero peak is generated by contributions from positive noise excursions, which are not balanced by contributions from the negative noise excursions. Figure 24 shows a gradual transition from the noise peak into the contributions generated exclusively by significant current signals, at least for high-resolution measurements.



FIGURE 24) The normalized rms emittance, alpha, and beta versus thresholds applied to the data of Fig. 22 with (solid) and without (dashed) a bias subtraction.

To reduce this apparent fuzziness some groups use the often dramatic change observed in the Twiss parameters α and β when the estimates transition from being dominated by the background data to being dominated by the signals from the beam core. The improvement is of limited merit because it is well known that α and β change in the tail of the beam as we have seen in Figs. 2-4.

One would think that a threshold is the perfect method for mitigating the effects of inverted ghost signals. This is, however, not the case because the convolution with noise entangles the signals. In our case it is worse because the



FIGURE 25) Fraction of the included beam current versus thresholds for the data of Fig. 23 with (solid) and without (dashed) bias subtraction.



FIGURE 26) Fraction of the included beam current versus thresholds for a variety of measured low-energy emittance distributions compared to a Gaussian distribution (solid).

ghost signals overlap in the position-angle space with the real signals, and distort the distribution, as one can see in Fig. 24.

Figure 25 shows the sum of all signals that are included versus the applied threshold. With decreasing thresholds the curves should converge to 1 at the zero threshold, which is obviously not the case for the dashed curves showing the data before the bias correction. The bias correction improves the results, especially for the center figure. The outer two figures suggest that the bias was overestimated, which is likely caused by the small signals from the tails skewing the noise peak.

Figure 26 shows a solid line that represents the included fraction of a Gaussian distribution, for which the excluded fraction matches the threshold. In addition it shows the fraction of the beam included when evaluating thresholded emittances for four different low-energy ion beams. Most fractions are significantly smaller than the Gaussian fraction. Emittances thresholded at 10% of the measured peak current may include only 50% to 80% of the beam, which can cause substantial errors when calculating target yields or beam losses assuming Gaussian distributions that would include 90% of the beam.



FIGURE 27) Normalized rms emittances obtained with a threshold in % of the measured peak current (dashed) and the summed beam current (solid).

This dilemma can be reduced by thresholding the data as a percentage of the excluded beam current rather than the normally used percentage of the maximum peak current. It requires the mapping of the sum of the bias-free current signals versus the signal height, which was calculated to generate Fig. 25. Figure 27 shows the normalized rms emittances obtained with thresholds calculated as a percentage of the measured peak current (dashed) and as a percentage of the summed beam current (solid), again for the data that were analyzed for Fig 23. As one would expect, the beam current thresholds include more beam than peak current thresholds. For example, the left Fig. shows a 10% peak current threshold to correspond to a 15% beam current threshold. This method is very sensitive to accurate bias corrections. Figure 27 was generated with bias values determined with an exclusion analysis because the accuracy of the histogram method was insufficient. And, as the right Fig. shows, this method normally fails for data with artifacts, because the artifacts likely skew the sum of the beam current. This method is rarely used, which is not surprising considering the discussed issues.

An alternate thresholding procedure subtracts the threshold from all data and then ignores all negative data. This method has been shown to underestimate the emittance because it significantly reduces the tails of the distribution, which overproportionally contribute to the emittance [40].

In summary, bias-corrected, thresholded emittance estimates depend on the distribution of the real current signals, the distribution of the noise excursions, and the fraction of data representing pure background. The dominance transitions gradually and there are no methods that can apply thresholds delivering consistent results for different distributions. It remains, however, the most efficient and reliable way to mitigate contributions from artifacts.

EXCLUSION ANALYSIS

Rms emittance parameters are evaluated by weighing all data with the square of the distance from the beam center in the position-angle space. This is especially troublesome for large statistical excursions and artifacts that are unlikely to contain any real current signal. Accordingly, data far from the beam core should be excluded from the final analysis, which can be accomplished with the hyperrectangular exclusion that was introduced a long time ago [2].

More recently an elliptical exclusion was introduced based on the Twiss parameters and center coordinates obtained from an analysis of moderately (~10%) thresholded data [40]. Fixing most of the ellipse parameters allows for systematic studies as a function of a single parameter, the size of the ellipse. It is important to use well-fitted ellipses that can normally be found with the Twiss parameters of moderately (~10%) thresholded data.

The top of Fig. 28 shows the average current measured outside the exclusion ellipse as a function of the ellipse's half-axis-product for the same data that were previously analyzed. The dashed curves in the top figures show the average outside current to level off as soon as all real current signals are included. This



FIGURE 28) Average outside current and fraction of enclosed beam as a function of the halfaxis-product of an exclusion ellipse fitted around the core of the beam before (dashed) and after (solid) bias correction.

can yield a highly accurate determination of the bias because it can be obtained exclusively from signal-free background data. After subtracting these biases, the fraction of beam inside the exclusion ellipse reaches unity for the same small ellipse size where the average outside current starts to level off, as one can see from the solid curves in the bottom of Fig. 28.

Ghost signals normally cause undershoots or overshoots before leveling off because the ghosts normally feature a broader distribution. Biases can still be estimated if the exclusion ellipse can exclude an area of ghost-free background. Under- and especially over-shoots are sensitive indicators for the presence of ghost signals.

Figure 29 shows the rms-emittance, α , and β being evaluated with the same elliptical exclusion ellipses before (dashed) and after (solid) correcting for an average bias estimated from Fig. 28. The rms-emittance gains rapidly until all real current signals are included and then levels off if the bias was accurately subtracted. The high noise level of the signals causes the fluctuations seen in the left columns of Figs. 28 and 29.

If an exclusion boundary can separate the area with the real signals from the area with ghost signals, the method produces a plateau for the real signals before transitioning into a plateau for all signals. However, the analyzed ghost signals overlap with the real signals. The inverted polarity of the signals shrinks the rms-emittance long before it can establish a plateau, making the method useless.

After leveling out, the average outside current fluctuates due to the statistical fluctuation of the data outside the ellipse. This establishes the statistical uncertainty of the bias estimates. The self-consistent unbiased elliptical exclusion analysis (SCUBEEx) establishes the statistical uncertainty of the emittance and



FIGURE 29) Rms Twiss parameters a function of the half-axis-product of an exclusion ellipse fitted around the core of the beam before (dashed) and after (solid) bias correction.

the Twiss parameters by evaluating, for every ellipse, the average outside current, which is subtracted from all data before calculating the rms emittance parameters from the data within the ellipse. Figure 30 shows this to increase the fluctuations, which now are an integral measure for the statistical uncertainty of the method. While SCUBEEx is invaluable for estimating the threshold-free rms emittance and its accuracy for artifact-free data, SCUBEEx should not be used for data with ghosts and artifacts because the incorrect bias subtraction can be very misleading.



FIGURE 30) Rms emittance as a function of the half-axis-product of an exclusion ellipse. With SCUBEEx, for every ellipse one subtracts the average outside current from all data before calculating the rms emittance from the data within the ellipse.



FIGURE 31) Data from Fig.3 in 10 % density contour lines with the rms ellipse, the ellipse including all data exceeding 10% n peak current and ellipse enclosing 10% of beam current.

ANALYZING EMITTANCE AREAS

Most of the difficulties in evaluating rms emittance values are caused by the weight proportional to the squares of the distance from the beam center in the position-angle space. This problem does not surface with the uniformly weighted contributions to emittance areas and volumes.

Emittance areas have the advantage that they are a conserved quantity. However, when the areas are not of elliptical shape, which is typical at low energies, elliptical acceptances prevent the use of these areas for accurate prediction of losses or transmission factors. Sometimes ellipses are fitted that contain all measurements that exceeded a certain percentage of the peak current [41], for example the large ellipse in Fig. 31. These ellipses, however, contain a larger fraction of the beam than the selected % of the peak current of non-elliptical emittances. A significantly smaller ellipse can be found that contains 90% of the beam current. Their Twiss parameters are not identical and do not match the rms Twiss parameters.

It should be noted that all orientations of the ellipses differ from the elliptical beam core. This is consistent with the experience that low-energy beam transport systems yield a maximum beam transmission when tuning parameters that significantly deviate from elliptical calculations.

OUTLOOK AND ACKNOWLEDGMENTS

While the definitions and basic technologies remain the same, much progress has been accomplished in the fields of emittance measurements and analysis, partly driven by new inventions and partly driven by more powerful and smarter control systems. Emittances of high-energy beams are normally accurate and reliable and can be used for accurate predictions of beam losses and target yield. It is the non-elliptical shape that complicates low-energy emittances and limits the accuracy when used for the prediction of beam losses and transmission. More studies will be required to improve the understanding and assessment of lowenergy emittances before they can be used to accurately compare the performance of different ion sources and low-energy beam transports systems.

Using measured low-energy emittance distributions as input for trajectory calculations can improve the accuracy of model predictions. Accurate loss studies require low-noise, artifact free data to exclude the background with a very low threshold.

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