# DEVELOPMENT OF FFAG

## STRAIGHT LINES AND DISPERSION SUPPRESSORS

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- Scaling straight lines
- Dispersion suppressor
- Application: PRISM case

# MOTIVATIONS

- Scaling FFAG transport lines.
- Insertions in scaling FFAG rings:
  - Straight sections
  - Dispersion suppressors in straight lines and bending lines.

# SCALING STRAIGHT LINES

Transverse linearized equations of motion

$$\begin{cases} \frac{d^2x}{dy^2} + \frac{1}{\rho^2}(1 - K\rho^2)x = 0 \\ \frac{d^2z}{dy^2} + \frac{1}{\rho^2}(K\rho^2)z = 0 \end{cases} \quad \text{with} \quad K = \frac{1}{B\rho}\left(\frac{\partial B_z}{\partial x}\right)_{z=0}$$

These equations have to be independent of momentum

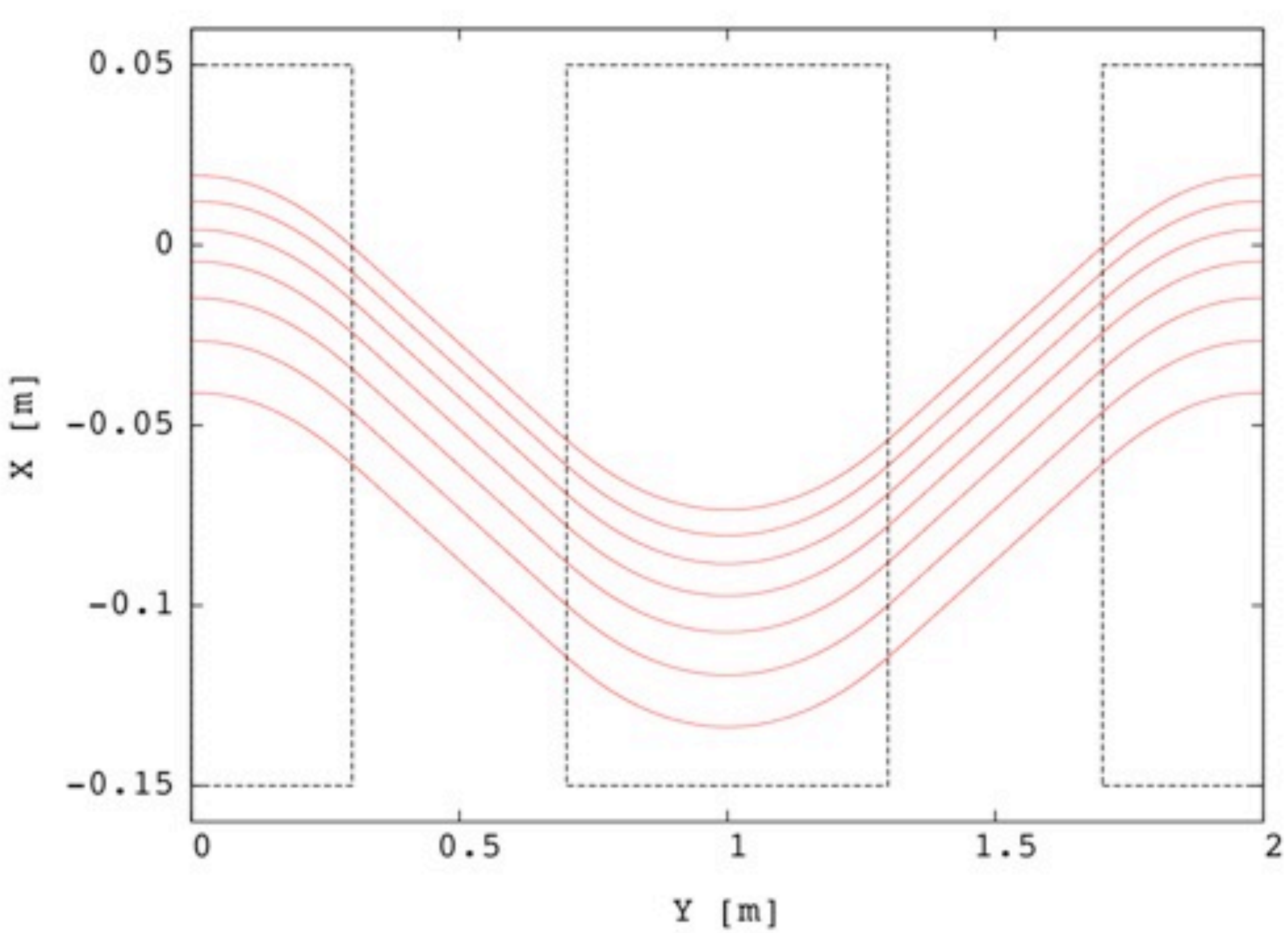
$$\begin{cases} \frac{d\left(\frac{1}{\rho^2}\right)}{dP} = 0 \\ \frac{d(K\rho^2)}{dP} = 0 \end{cases} \Leftrightarrow \begin{cases} \rho = const. \\ \frac{1}{B}\left(\frac{\partial B_z}{\partial x}\right)_{z=0} = \frac{n}{\rho} \end{cases}$$

The field law is given by ➡

$$\boxed{B_z = B_0 e^{\frac{n}{\rho}(X - X_0)}}$$

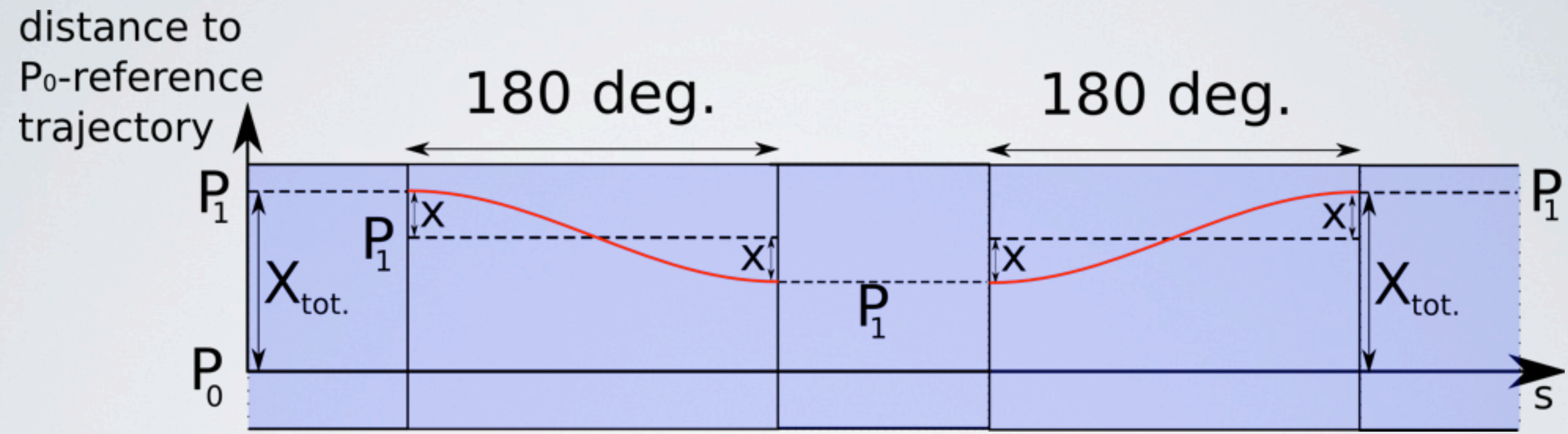
# SCALING STRAIGHT LINES

Tracking in Runge Kutta with linear fringe field for an example with protons




| | |
|---|---|
| Length of the magnets | 60 cm |
| Drift | 40 cm |
| Kinetic energy range | 80 to 200 MeV |
| Field index | 17 |
| Local curvature radius | 2.1 m |
| Step size | 1 mm |
| Phase advances: | |
| horizontal $\mu_x$ | 104.8 deg. |
| vertical $\mu_z$ | 112.5 deg. |

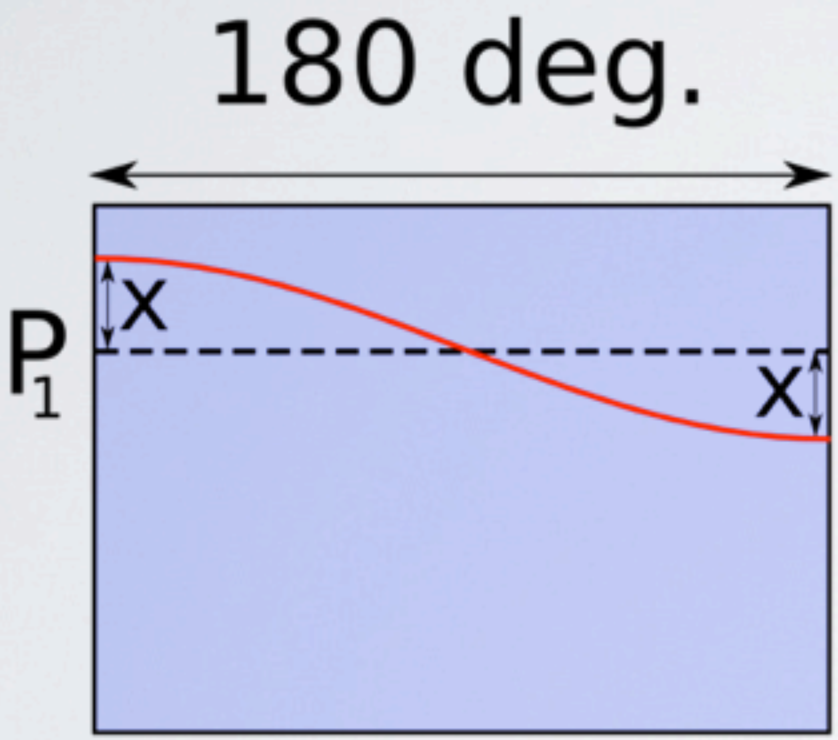
# DISPERSION SUPPRESSOR




| | | | | | |
|---|---|---|---|---|---|
| Bending: | $k_1$ | $k_2$ | $k_3$ | $k_2$ | $k_1$ |
| Straight: | $\left(\frac{n}{\rho}\right)_1$ | $\left(\frac{n}{\rho}\right)_2$ | $\left(\frac{n}{\rho}\right)_3$ | $\left(\frac{n}{\rho}\right)_2$ | $\left(\frac{n}{\rho}\right)_1$ |

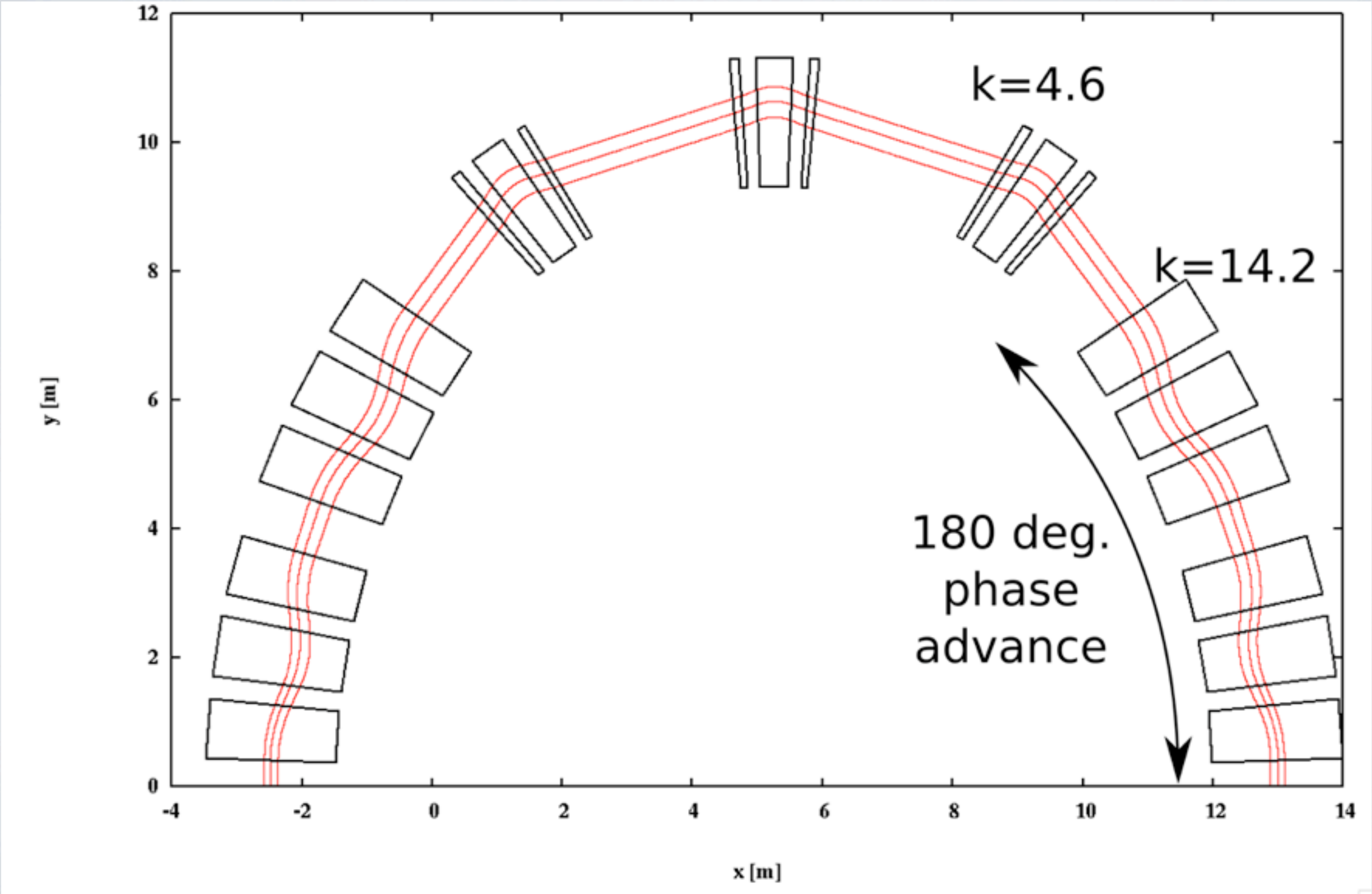
# DISPERSION SUPPRESSOR

Problem of phase advance: still scaling?




Limitation comes from amplitude dependance of the phase advance.

# APPLICATION: PRISM



# THANK YOU FOR YOUR ATTENTION

# SCALING STRAIGHT LINES

Straight section = Bending section with infinite radius

$$\lim_{r_0\to\infty}\left(\frac{r}{r_0}\right)^k = \lim_{r_0\to\infty}\left[\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{x}{r_0}k} = \left[\lim_{r_0\to\infty}\left(1+\frac{x}{r_0}\right)^{\frac{r_0}{x}}\right]^{\frac{n}{\rho}x} = e^{\frac{n}{\rho}x}$$

with $r = x + r_0$

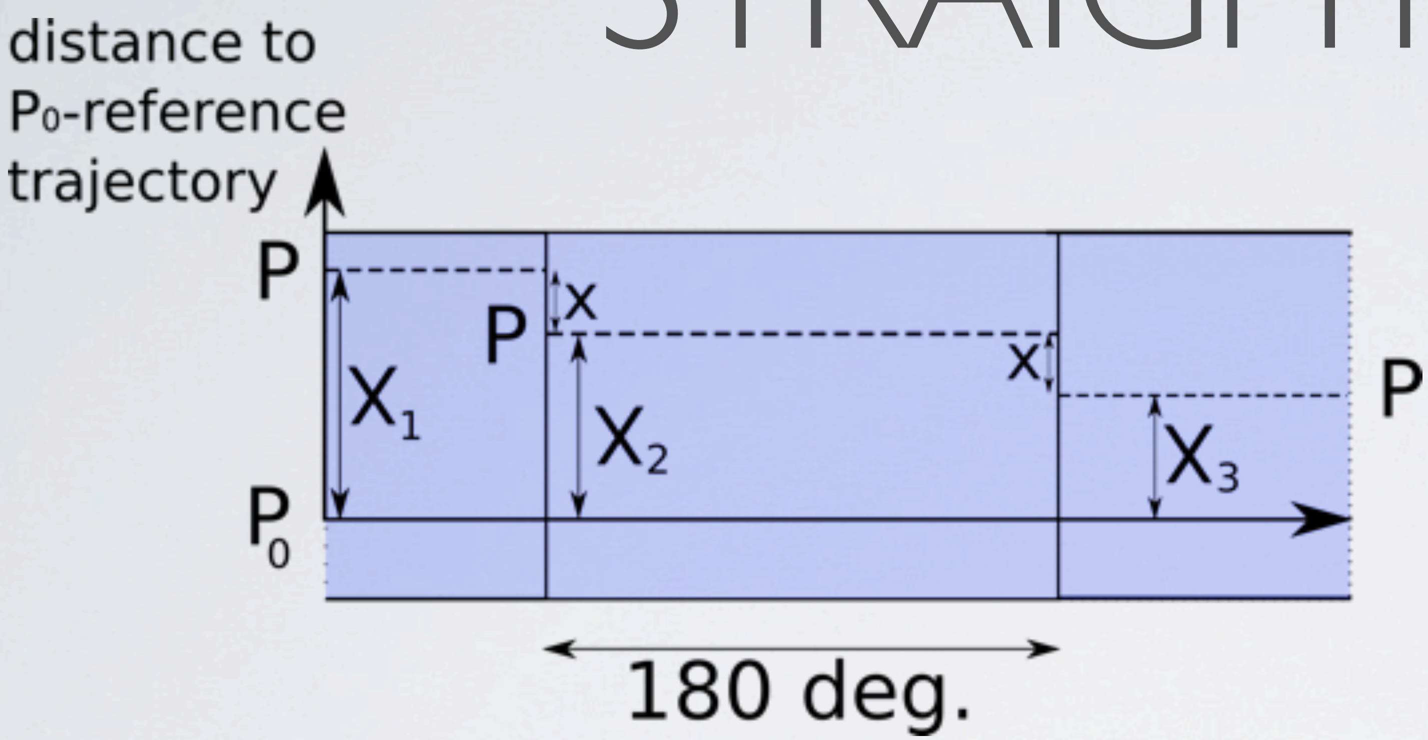
$$k = \frac{r_0}{\rho}n$$

$$n = \frac{\rho}{B}\left(\frac{dB}{dx}\right)_{z=0}$$

$\Rightarrow$

$$\boxed{B_z = B_0 e^{\frac{n}{\rho}(X-X_0)}}$$

# DISPERSION SUPPRESSOR IN STRAIGHT LINES



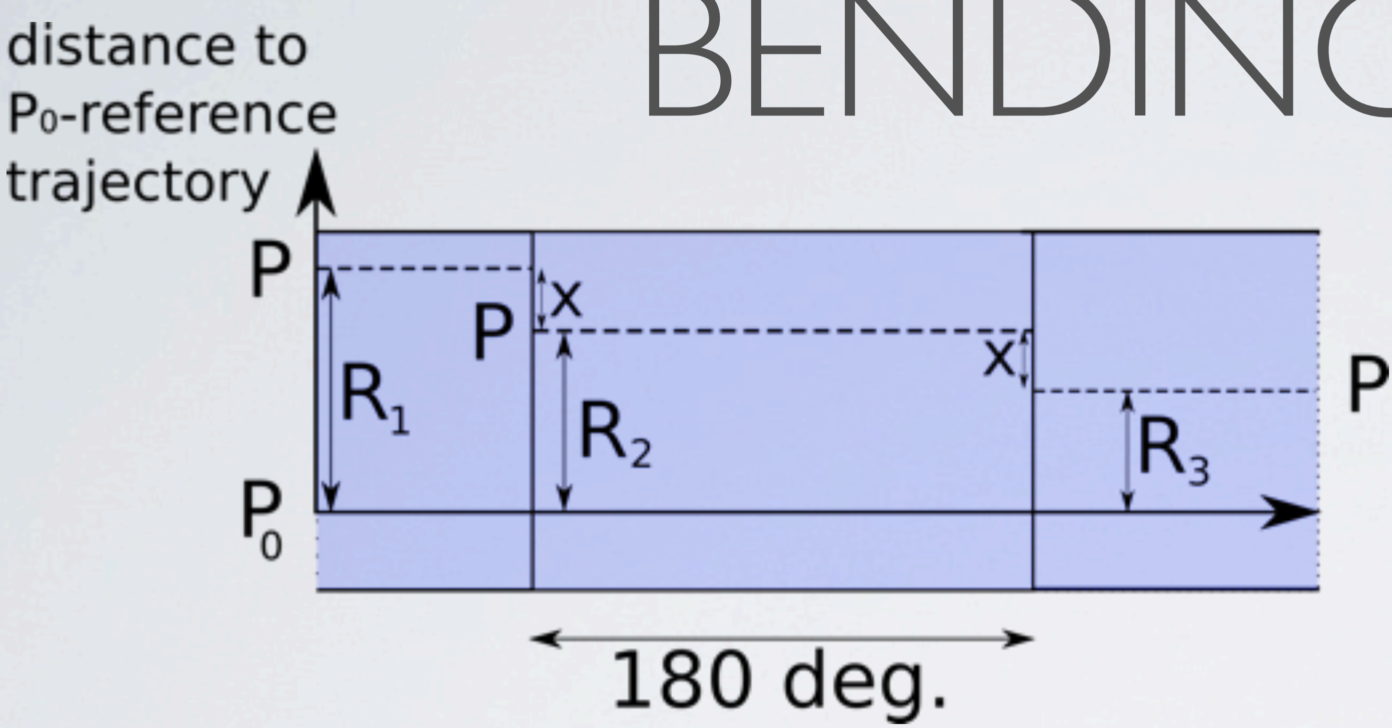

$$X_2 - (X_1 - X_2) = X_3$$

$$2X_2 = X_1 + X_3$$

$$X = \frac{\rho}{n}\ln(\frac{P}{P_0})$$

$$\Rightarrow \quad 2\frac{\rho_2}{n_2} = \frac{\rho_1}{n_1} + \frac{\rho_3}{n_3}$$

# DISPERSION SUPPRESSOR IN BENDING LINES




$$R_2 - (R_1 - R_2) = R_3$$

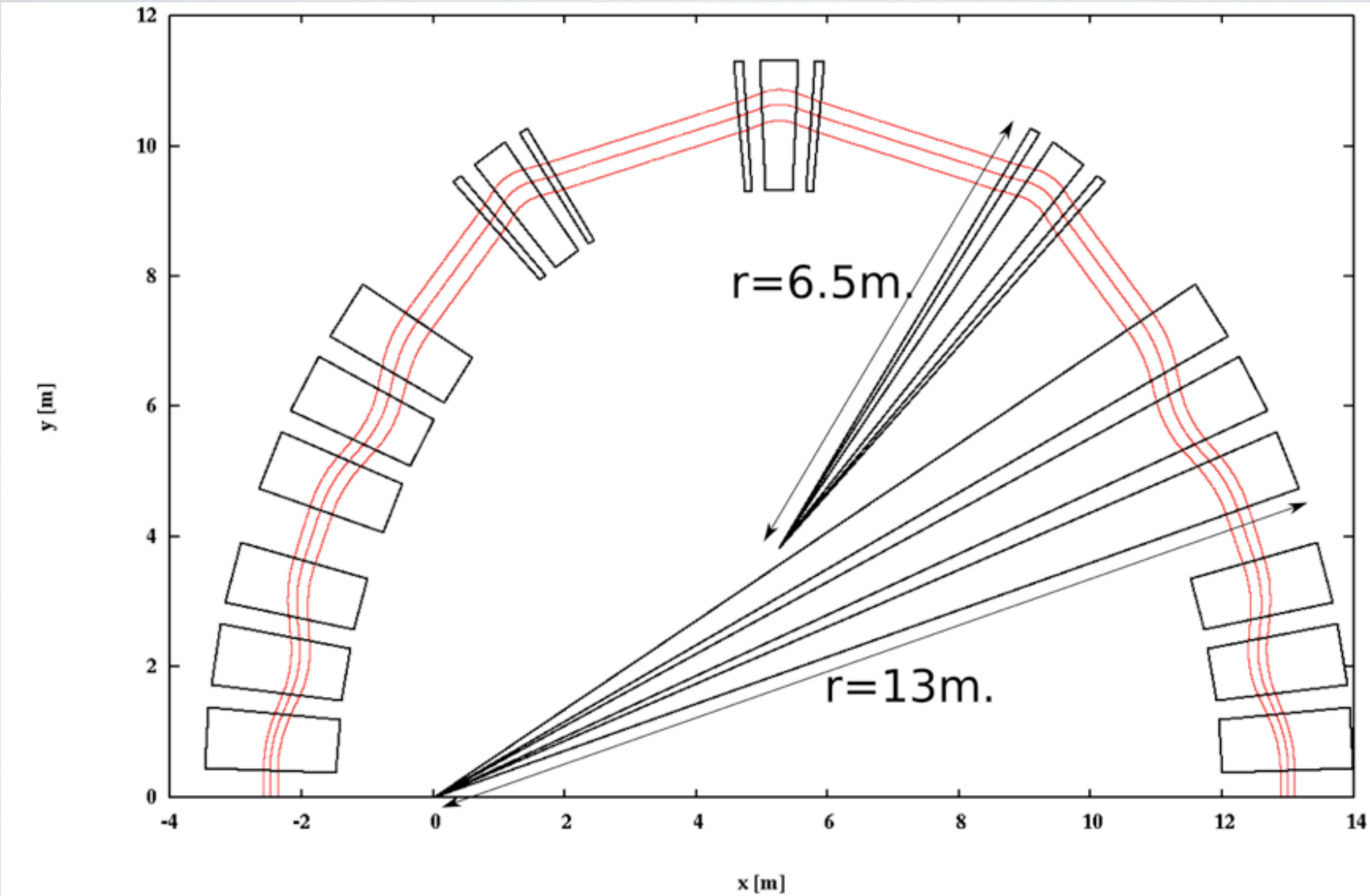
$$2R_2 = R_1 + R_3$$

$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

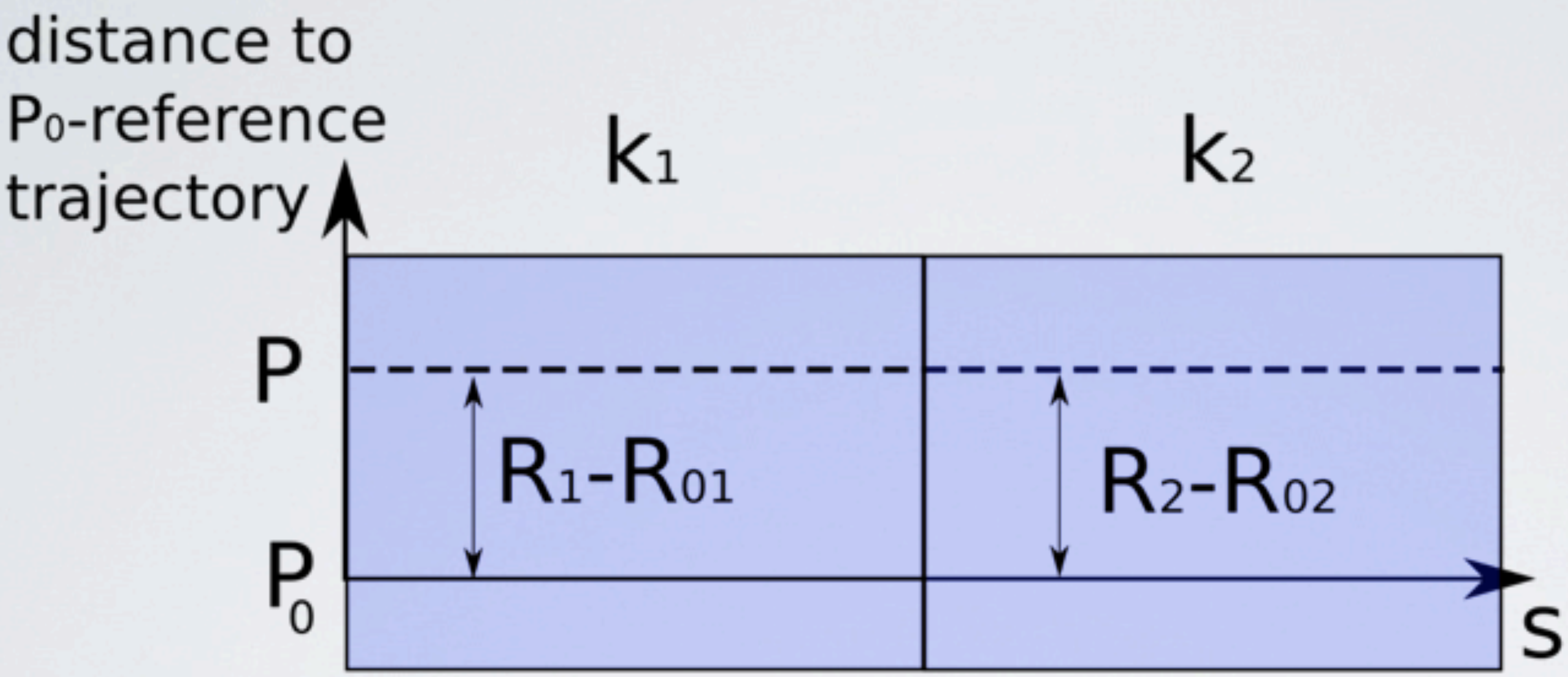
1st order

$$\frac{2}{k_2+1} = \frac{1}{k_1+1} + \frac{1}{k_3+1}$$

# APPLICATION: PRISM



# CHANGE RADIUS




$$R_1 - R_{01} = R_2 - R_{02}$$

$$R = R_0 \left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}$$

1st order ⇒

$$\frac{R_{01}}{R_{02}} = \frac{k_1 + 1}{k_2 + 1}$$

# MISMATCH BEND-STRAIGHT