High Precision Measurement of Muon Beam Emittance Reduction in MICE

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Abstract

Muon ionization cooling, an essential ingredient of a Neutrino Factory, will be demonstrated for the first time by the MICE experiment. The central part of MICE consists of a short section of a Neutrino Factory cooling channel. The emittance reduction achieved in this experiment is quite modest, 10% to 15%. In order to extrapolate the performance of a full cooling channel from these values, it is desirable for MICE to achieve an emittance measurement accuracy of 10^{-3}, absolute. So far, beam emittance has never been measured with such precision. We present the resulting constraints on the spectrometers and a method to correct the bias on the emittance due to known measurement errors.

EMITTANCE RESOLUTION OF MICE

The Muon Ionisation Cooling Experiment (MICE) is designed to demonstrate that one can design and build a section of cooling channel and measure the amount of cooling achieved on a muon beam in a variety of optical configurations [1]. The results will be used to calculate and optimize the cooling effect of a full cooling channel for a Neutrino Factory.

Muons will pass through MICE one at a time, their position in phase space being measured upstream and downstream of the cooling cell. Bunches will be constructed from individual measurements of up to 10^6 particles and the emittance calculated [2].

While the cooling effect varies for input beams of different emittance, for typical Neutrino Factory beams the cooling effect in MICE varies between 5% and 20% of the input normalised emittance. Different Neutrino Factory designs have different amounts of cooling, ranging from factors of $2-16$ [3].

For optimal cooling, the optics will be gradually tapered with decreasing β function, so as to minimise the cooling channel equilibrium emittance. In this case, for a cooling channel with n cells, the cooling performance of a full channel is given to first approximation by

$$\frac{\epsilon_{out}}{\epsilon_{in}} = (1 - \frac{\delta\epsilon}{\epsilon})^n \qquad (1)$$

where ϵ_{out} is the emittance of the cooled beam, ϵ_{in} is the emittance of the input beam and $\delta\epsilon$ is the change in emittance due to cooling in one cell. Then to predict the change in emittance for a typical Neutrino Factory cooling channel to of order 10%, it is necessary to measure the change in emittance for a single cell to 1% and the absolute emittance to approximately 0.1%.

In §2 we examine how this constrains the MICE design spectrometer resolution. In §3 we undertake a detailed study of a method to correct the bias in emittance introduced by finite tracker resolution. In §4 we show using Monte Carlo simulation that the MICE scintillating fibre tracker resolution lies within the tolerances set out in §2 and go on to calculate a corrected emittance using the methodology set out in §3.

Definition of Emittance

In this paper the normalised root mean square (RMS) emittance in $2N$ dimensions is defined by [4]

$$\epsilon_{rms} = A\sqrt{|\mathbf{V}|}. \qquad (2)$$

Here A is a normalisation factor and $|\mathbf{V}|$ is the determinant of the $2N \times 2N$ covariance matrix. We will use the phase space with position vector

$$\vec{U} = (\vec{x}, \vec{p}). \qquad (3)$$

Here $\vec{x} = (x, y, t)$, $\vec{p} = p_x, p_y, E$, $N = 3$, and the normalisation factor A is $\frac{1}{m_\mu}$.

REQUIRED TRACKER RESOLUTION

Using (2) the emittance of a particle bunch with no correlations is given by

$$\epsilon_{rms} = A\sqrt{\prod_i \sigma^2(u_i)} = A\prod_i \sigma(u_i) \qquad (4)$$

where $\sigma^2(u_i)$ is the variance of the phase space coordinate u_i and the product is taken over all u_i. To first approximation it is assumed that the errors on the phase space variables are independent. In this case, the variance of the measured phase space variable, m_i, is given by the addition in quadrature of the true phase space variable, u_i, and the error, δ_i;

$$\sigma^2(m_i) = \sigma^2(u_i) + \sigma^2(\delta_i). \qquad (5)$$

Hence, assuming $\sigma^2(\delta_i) \ll \sigma^2(u_i)$

$$\sigma(m_i) = \sigma(u_i)(1 + \frac{\sigma^2(\delta_i)}{2\sigma^2(u_i)}). \qquad (6)$$

The measurement errors on the phase space variables introduce a bias in the emittance measurement. We will show in the following that the effect of known errors, such as multiple scattering or detector granularity, can be corrected for precisely.

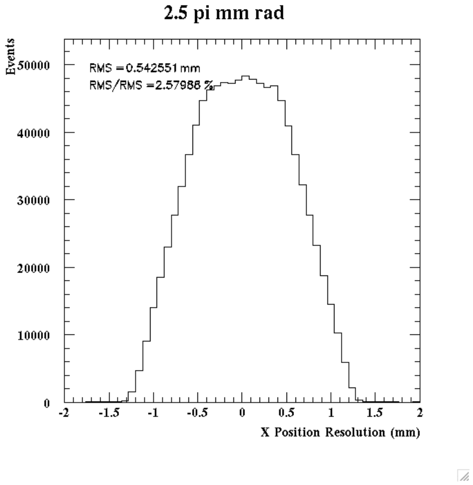

Figure 1: The simulated distribution of δx.

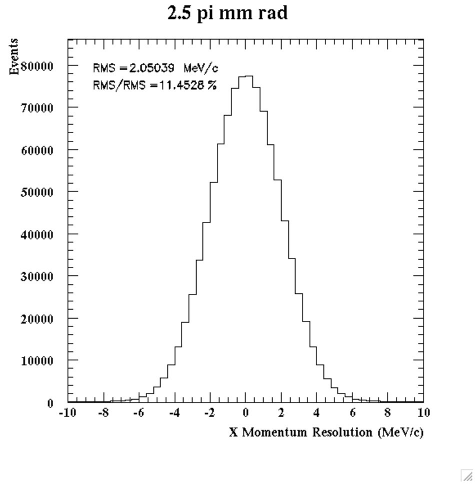

Figure 2: The simulated distribution of δp_x.

If we assume that the measurement errors can be understood with a relative accuracy of 10%, the requirement of 10^{-3} on emittance becomes a requirement of 1% on the error in each phase space variable. Using (6)

$$(1 + \frac{\sigma^2(\delta_i)}{2\sigma^2(u_i)}) < 1.01, \quad (7)$$

gives us the following requirement on the measurement accuracy

$$\frac{\sigma(\delta_i)}{\sigma(u_i)} < 14\%. \quad (8)$$

CORRECTION OF THE EMITTANCE BIAS

In the more general case, the elements of the covariance matrix are defined by

$$V_{ij} = \sigma^2(u_i, u_j) = < u_i u_j > - < u_i >< u_j > \quad (9)$$

where $< u_i >$ is the mean of u_i and n_μ is the number of muons. The measured value of u_i, m_i and the error on the measurement, δ_i, are related by;

$$m_i = u_i + \delta_i. \quad (10)$$

Substituting into (9), the true covariance is related to the measured covariance and to the error in the measurement by;

$$\begin{aligned} \sigma^2(u_i, u_j) &= \sigma^2(m_i, m_j) - \sigma^2(\delta_i, \delta_j) - \\ &\quad -\sigma^2(u_i, \delta_j) - \sigma^2(u_j, \delta_i). \end{aligned} \quad (11)$$

It can be seen that when $i = j$ and $\sigma(u_i, \delta_i) = 0$, (11) reduces to (5). In general the covariance matrix is given by

$$\mathbf{V^{true}} = \mathbf{V^{meas}} - \mathbf{R^T} - \mathbf{R} - \mathbf{C}, \quad (12)$$

where $\mathbf{R}, \mathbf{C}, \mathbf{V^{meas}}, \mathbf{V^{true}}$ are matrices with elements given by comparing terms in (11) and (12) such that:

$$\begin{aligned} V_{ij}^{true} &= \sigma^2(u_i, u_j), \\ V_{ij}^{meas} &= \sigma^2(m_i, m_j), \\ C_{ij} &= \sigma^2(\delta_i, \delta_j) \text{ and} \\ R_{ij} &= \sigma^2(u_i, \delta_j). \end{aligned} \quad (13)$$

Then, if the probability distribution of the measurement errors is known, the true value of the covariance matrix elements can be calculated. An uncertainty in the emittance will arise from the accuracy with which the error distribution is known and the statistical uncertainty introduced by using a finite number of muons. Since unforeseen sources of error can arise, it is planned in MICE to measure the actual emittance bias by placing in the muon beam the two spectrometers next to each other and comparing the values of phase space variables and emittances that they predict in the median plane situated between them.

MONTE CARLO RESULTS

The scintillating fibre spectrometer in MICE is constructed of five irregularly spaced stations, each containing three planar layers of scintillating fibres. The three layers are rotated through $120°$ with respect to each other to allow reconstruction of hit positions. Each scintillating fibre is ganged together into groups of seven fibres for readout. Momentum reconstruction is performed by measuring the helical path of muons through a 4 T field. In this section, the resolution of the spectrometer was obtained using a GEANT4-based Monte Carlo simulation of the MICE experiment [1], using as input the point resolution, efficiency, light yieldand percentage of dead channels from a tracker prototype; and background predictions from a detailed simulation of dark current emission in the RF cavities [5].

The probability distribution of δx and δp_x for the MICE spectrometer in horizontal transverse phase space is shown

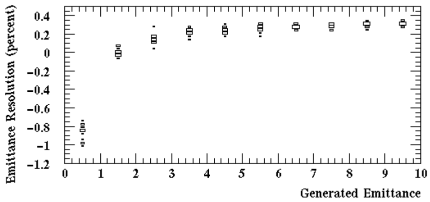

Figure 3: Simulated emittance measurement in the upstream MICE spectrometer, compared with true emittance.

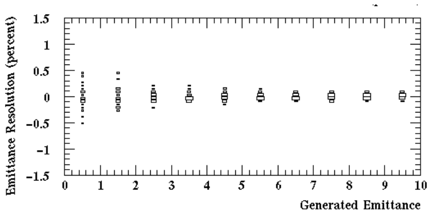

Figure 4: Emittance measurement with the offset removed.

in Figures 1 and 2, for a muon beam at the lowest emittance that is expected in MICE, $\epsilon_{rms} = 2.5\pi$ mm rad. Note that the spatial distribution is non-gaussian due to the sevenfold ganging of the fibres. In Table 1 the resolution of the

Coordinate	Resolution
x	2.58 %
y	2.10 %
P_x	11.5 %
P_y	8.52 %
E	13.8 %

Table 1: Spectrometer resolution for various phase space variables.

upstream and downstream spectrometers in the transverse phase space variables and energy, E, is shown. Note that all five phase space variables have a resolution below the required resolution of 14%. In MICE, timing information will be provided by Time Of Flight counters, the resolution of which is not presented here.

In Figure 3 the offset transverse emittance is shown, calculated from the covariance matrices $\mathbf{V}^{\mathbf{meas}}$ for beams with various transverse emittances, in the upstream tracker.

In Figure 4 the true emittance calculated using the covariance matrix $\mathbf{V}^{\mathbf{true}}$ is shown. Note that in this case the spread in the emittance measurement is less than the required resolution on emittance of 10^{-3}, although a good knowledge of the probability distribution of the errors on the phase space variables has been assumed.

CONCLUSIONS

The determination of the MICE experimental resolution has been explained. It has been shown that a finite resolution on the MICE spectrometer generates an offset in the emittance measurement. This has been used to define a resolution requirement for the MICE spectrometer. A method to remove such an offset has been derived. Finally, a Monte Carlo study has been used to show that the emittance measurement with an accuracy of 10^{-3} should be achievable using the MICE scintillating fibre spectrometers.

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