The Power of Spin: A scalpel-like probe of theoretical ideas

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Abstract. We survey briefly the important role that spin-dependent measurements have played, time and time again, in helping to shape our current understanding of particle physics.

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INTRODUCTION

My aim is to remind you, or if there are any doubting Thomases, to convince them, that spin dependent measurements have a scalpel-like ability to probe a theory, which, for example, may have been able to fudge the results of ordinary cross-section measurements. Thus the path of spin is strewn with the wreckage of discarded theories. The positive aspect is that better (hopefully) theories arise from the ashes.

THE ANCIENT WORLD (PRE 1988)

I will remind you of two cases where spin played a crucial role in altering our viewpoint.

- Electroweak theory: When I was a student weak interactions were supposed to involve S-T coupling: 1 and $\frac{i}{2}[\gamma_\mu, \gamma_\nu]$. From the role of spin in comparing rates for $\pi \to e\bar{\nu}$ and $\pi \to \mu\bar{\nu}$ and from measurement of the helicity of the neutrino, we eventually learned that the coupling was V-A: $\gamma_\mu(1-\gamma_5)$. The unification of Weak and Electromagnetic interactions is one of the greatest achievements in Particle Physics. Without the discovery of the V-A structure this would have been impossible! The power of spin is vividly illustrated by experiments to determine the structure of the weak Hamiltonian. The most general form, allowing for parity violation, involves TEN coupling constants:

$$H = \sum_{i=1...5} \left[C_i(\bar{\psi}_e \Gamma_i \psi_\mu)(\bar{\psi}_{\nu_\mu} \Gamma_i \psi_{\nu_e}) + C_i'(\bar{\psi}_e \Gamma_i \psi_\mu)(\bar{\psi}_{\nu_\mu} \Gamma_i \gamma_5 \psi_{\nu_e}) \right] \tag{1}$$

  where the Γ_i are the usual five types of Dirac gamma matrix structures. The results of an experiment at SIN to test the V-A structure of the weak Hamiltonian [1] are shown in Fig. 1. With V-A all couplings should be zero except for the one labeled *LL* in the bottom line, which should have the value 1. The agreement is dramatic.
- Regge poles: In the 1960s-70s there was the dramatic discovery that diffraction peaks in $\frac{d\sigma}{dt}$ for elastic cross-sections

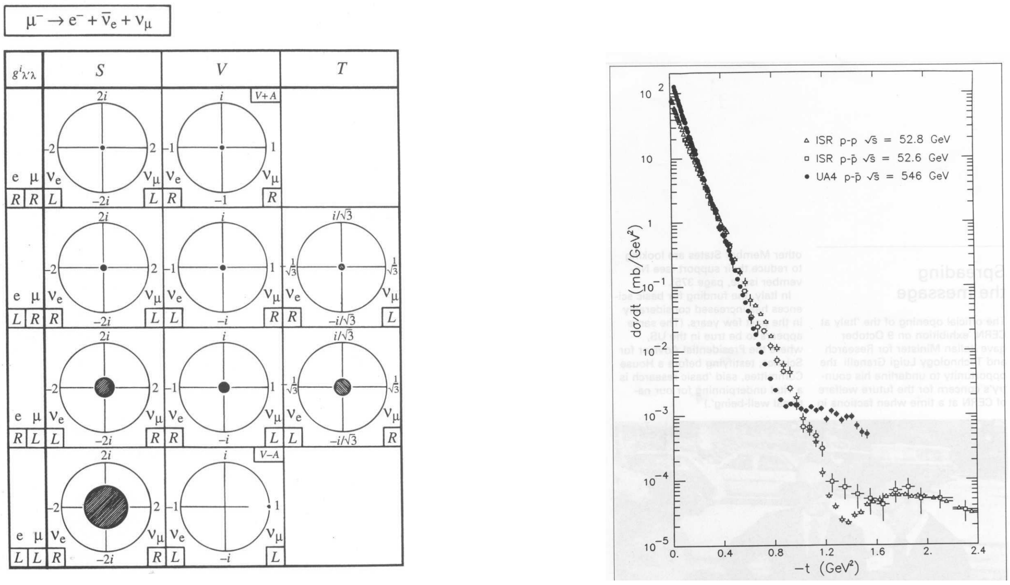

FIGURE 1. Left: V-A structure of the weak Hamiltonian. Right: Shrinking diffraction peaks

$$A(p_1)+B(p_2)\rightarrow A(p_3)+B(p_4) \qquad t=(p_1-p_3)^2 \tag{2}$$

shrink as the energy increases ... a most non intuitive behavior. Fig. 1 shows CERN ISR and SPS UA(4) data. The theory of Complex Angular Momentum (in its simplest form, Regge poles), provided a beautiful explanation! There were many successes in relating different cross-sections, but problems arose with polarizations. Many questions remained unanswered, which could have been resolved by spin-dependent measurements. Our "Critical Tests for Regge Pole Theory" [2] , were, alas, never carried out. The deep level inclusion of spin in Regge theory provided a fantastically rich panoply of possibilities: conspiracies, evasions, daughters... most of which were never tested.

THE RENAISSANCE: THE EUROPEAN MUON COLLABORATION EXPERIMENT

A quick reminder about deep inelastic scattering in the parton model

The polarized cross-section is expressed in terms of two spin-dependent *structure functions*, $g_{1,2}$ (for a recent review and references see [3]). Our interest is in g_1:

$$g_1(x,Q^2)=\frac{1}{2}\sum_{flav} e_j^2[\Delta q_j(x,Q^2)+\Delta\bar{q}_j(x,Q^2)] \tag{3}$$

The key ingredients here are the polarized quark densities. To appreciate the EMC experiment define

$$\Delta q = \int_0^1 dx \Delta q(x) \tag{4}$$

and the important flavor combinations :

$$a_3 = \Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d} = 1.267 \pm 0.0035 \tag{5}$$

$$a_8 = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}) = 0.585 \pm 0.025 \tag{6}$$

$$\Delta\Sigma = \sum_f (\Delta q_f + \Delta \bar{q}_f) \tag{7}$$

where the values of $a_{3,8}$ are known from neutron and hyperon β-decay respectively. Note that

$$\Delta\Sigma = a_8 + 3(\Delta s + \Delta \bar{s}) \tag{8}$$

Ellis and Jaffe [4] had suggested it was safe to ignore $\Delta s + \Delta \bar{s}$ implying

$$\Delta\Sigma \simeq a_8 \simeq 0.59 \tag{9}$$

Now the EMC measurement [5] of

$$\Gamma_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{12}\left[a_3 + \frac{1}{3}(a_8 + 4a_0)\right] \tag{10}$$

implied

$$a_0^{EMC} \simeq 0$$

But in the naive parton model $a_0 = \Delta\Sigma$, which is thus in gross contradiction with the Ellis-Jaffe result Eq. 9.

Moreover, since

$$\Delta\Sigma = 2\langle S_z^{quarks}\rangle \tag{11}$$

this seems to imply $\langle S_z^{quarks}\rangle \simeq 0$ and there appears to be “A crisis in the parton model: where, oh where, is the proton’s spin?" [6].

Resolution (??) of the crisis; the anomalous gluon contribution

The Operator Product Expansion has no gluon operator contributing to the first moment of g_1, but a Feynman diagram approach yields the result:

$$a_0 = \Delta\Sigma - \frac{3\alpha_s(Q^2)}{2\pi}\Delta G(Q^2) \tag{12}$$

It was thus hoped that one could have a reasonable $\Delta\Sigma \simeq 0.6$ and still obtain a very small a_0. But even with present day estimates $a_0 \approx 0.2$ this requires

$$\Delta G \simeq 1.7 \text{ at } Q^2 = 1 GeV^2$$

Is this acceptable? What do we know about ΔG?

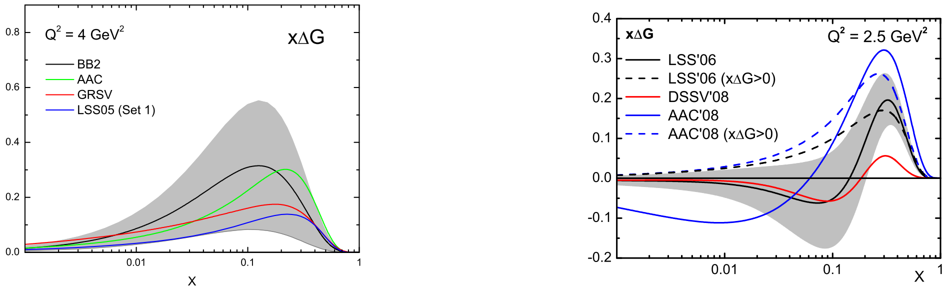

FIGURE 2. Polarized gluon density as in 2006 (left) and 2008 (right)

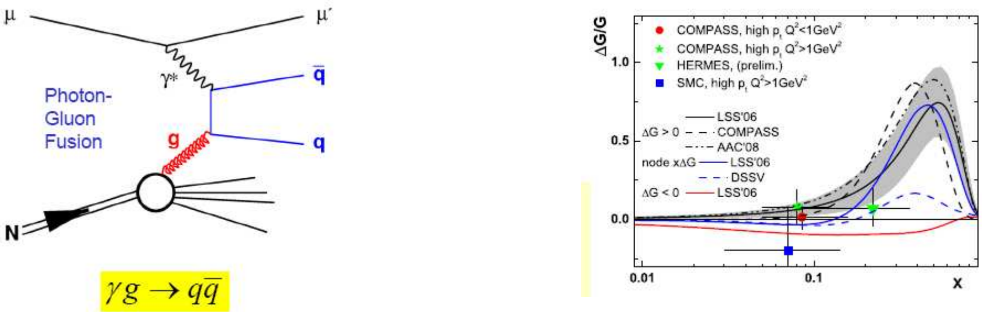

FIGURE 3. Left: Feynman diagram for $c\bar{c}$ production. Right: Direct measurements of ΔG compared with DIS results

THE PRESENT (NEW CHALLENGES)

Attempts to measure ΔG

There are three ways to access $\Delta G(x)$:

(i) In polarized DIS parametrize the polarized quark and gluon densities and fit data on $g_1(x,Q^2)$.The main role of the gluon is in the *evolution* with Q^2, but the range of Q^2 is very limited so the determination of $\Delta G(x)$ is imprecise. Fig. 2 shows the world results on $\Delta G(x)$ in 2006 and how this had changed by 2008.

Typically one has $\Delta G \approx 0.29 \pm 0.32$ ... much smaller than the desired 1.7 !

(ii) $c\bar{c}$ production in DIS. This requires a high energy lepton beam, ideal for COMPASS at CERN. Given that the nucleon has no *intrinsic* charm, the $c\bar{c}$ are produced via “gluon-photon fusion" (see Fig. 3).

Detecting *both* charmed particles would be an absolutely clean signal for the mechanism! But the intensity is too low—-a factor of 30 in rate is lost in detecting the second charmed meson—- so one relies on single charm production and also on back-to-back jets. Fig. 3 shows results from COMPASS, HERMES and SMC, suggesting a very small ΔG compatible with the DIS result quoted above.

(iii) A_{LL} with polarized protons: uniquely at RHIC.
There are several reactions e.g.

$$\vec{p}+\vec{p} \longrightarrow \pi^0 + X \text{ which needs Fragmentation Functions}$$

and

$$\vec{p}+\vec{p} \longrightarrow Jet + X$$

The dominant partonic reactions are

$$\vec{g}+\vec{g} \longrightarrow g+g \text{ : dominates at smaller } p_T^2$$

$$\vec{g}+\vec{q} \longrightarrow g+q\text{: dominates at larger } p_T^2$$

As a test, it turns out that PQCD describes the cross-sections quite well.
The results are (see Fig. 4) that A_{LL} is small, consistent with zero gluon polarization! The spin crisis is still with us.

Transverse single-spin asymmetries

Consider hadronic reactions like

$$p^{\uparrow} + p \rightarrow \pi + X \tag{13}$$

where $p^{\uparrow}$ means a transversely polarized proton. The asymmetry under reversal of the direction of polarization is

$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \tag{14}$$

The partonic mechanism is shown in Fig. 4. In the simple collinear Parton Model $A_N \approx 0$. To get an idea of the size, at parton level

$$\hat{a}_N = \alpha_s \frac{m_q}{\sqrt{s}} f(\theta^*) \tag{15}$$

where $f(\theta^*)$ is of order 1. This gives asymmetries of a fraction of a percent. The data strongly contradict this, as shown in Fig. 5 [7]

This was a profound challenge to the theory. In fact it was a major problem that had been swept under the carpet for decades. The solution is to go beyond the simple parton model, by inventing new soft mechanisms which require the inclusion of the intrinsic transverse momentum $\boldsymbol{k}_T$ of the partons.

(i) In the *Sivers* mechanism the number density of quarks with momentum $x\boldsymbol{P} + \boldsymbol{k}_T$ depends on the polarization $\mathscr{P}$ of the parent hadron:

$$q(x, \boldsymbol{k}_T) = A + B_S\, \mathscr{P}\cdot(x\boldsymbol{P} \times \boldsymbol{k}_T)$$

But one can can show that this violates Parity and Time Reversal invariance *if*

$$hadron \longrightarrow quark + X$$

is treated as an independent reaction——-as it is in the parton model. To avoid this requires *initial* or *final* state interactions, thereby spoiling straightforward *universality* of the parton model.

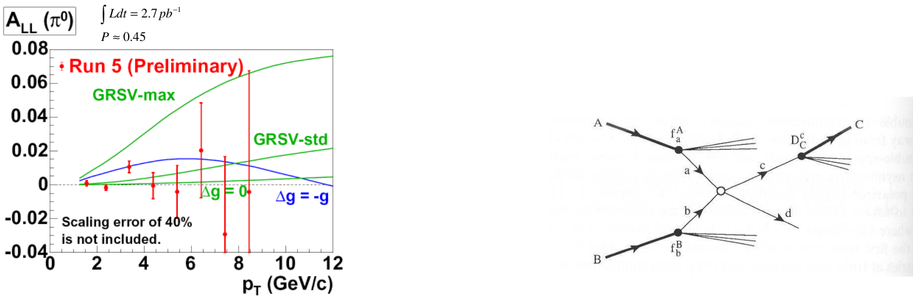

FIGURE 4. Left: PHENIX preliminary results on A_{LL} for π^0 production. Right: Partonic mechanism for hadronic $A + B \longrightarrow C + X$

FIGURE 5. Left: E704 data on the transverse spin asymmetries for pions. Right: One photon diagram for $ep \longrightarrow ep$

(ii) In the *Collins* mechanism, in the fragmentation of a quark of momentum $\boldsymbol{p}$ into a hadron, the hadron has intrinsic transverse momentum $\boldsymbol{k}_T$ relative to the quark. The number density of hadrons with momentum $\boldsymbol{P}_h = \frac{1}{z}\boldsymbol{p} + \boldsymbol{k}_T$ depends on the polarization $\mathscr{P}$ of the fragmenting quark.

$$D(z, \boldsymbol{P}_h) = A + B_C\, \mathscr{P}\cdot(\boldsymbol{p} \times \boldsymbol{P}_h)$$

Again, this vanishes if the fragmentation

$$q \rightarrow hadron + X$$

is treated as an independent reaction, as it is in the parton model. So again one loses straightforward universality. In summary, spin has caused a major rethink of the theory!

The biggest surprise of all: $ep \rightarrow ep$

The reaction is usually considered as a one photon exchange process (see Fig. 5) The photon-proton vertex is given by:

$$\bar{u}(p')[\gamma_\mu F_1^{EM}(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}\kappa F_2^{EM}(Q^2)]u(p) \tag{16}$$

where $q = p' - p$ $\quad \kappa$ = anomalous magnetic moment $\quad Q^2 = -q^2$

$F_{1,2}$ are the Dirac and Pauli EM form factors. The Sachs form factors are more convenient: $G_E = F_1 - \kappa\tau F_2 \quad G_M = F_1 + \kappa F_2$ where $\tau = \frac{Q^2}{4M^2}$. They are normalized to

$$G_E(0) = 1 \qquad G_M(0) = \text{total magnetic moment}(\mu) = 2.79 \tag{17}$$

The differential cross-section in the LAB is given by the *Rosenbluth* formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)'_{Mott}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2(\theta/2)\right] \tag{18}$$

In the textbooks one is told that both G_E and G_M drop with increasing Q^2, and that experimentally

$$G_M(Q^2) \approx \mu G_E(Q^2)$$

as seen in Fig. 6.

However, a totally new kind of measurement, the polarization transfer to the proton from a longitudinally polarized electron colliding with an unpolarized target has produced startling results [8, 9, 10]! The *longitudinal* polarization of the recoil proton, in one-photon exchange, is given by

$$\mathscr{P}_L \propto \left[\frac{E+E'}{M}\right]\sqrt{\tau(1+\tau)}\, G_M^2 \tan^2(\theta/2) \tag{19}$$

The *transverse* (in scattering plane) polarization of the recoil proton is given by

$$\mathscr{P}_T \propto -2\sqrt{\tau(1+\tau)}\, G_E\, G_M \tan(\theta/2) \tag{20}$$

Fig. 6 compares $\frac{\mu G_E}{G_M}$ for the proton extracted from polarization and from Rosenbluth measurements. The disagreement implies that the assumed dynamical mechanism is incorrect.

This is one of the oldest, simplest reactions studied in particle physics. For decades

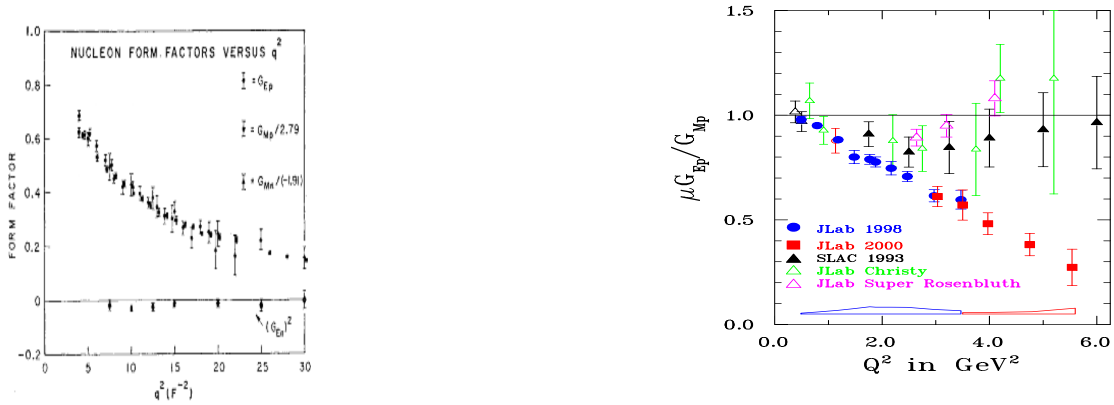

FIGURE 6. Left: Early data on the EM form factors of the nucleon. Right: Ratio of μG_E to G_M for protons, from Rosenbluth and from polarization measurements

it was believed to be totally understood. *Once again a spin-dependent measurement has upset the apple cart!!!*

CONCLUSION: A PUZZLE

Manifestly, spin-dependent measurements have played a crucial role in shaping theories of elementary particles. Why, then, has it *not* attracted a large following? Why is it *not* a subject of major interest to everyone in Particle Physics? There are possibly two reasons:

(i) Practical: Polarization measurements are notoriously difficult. Sources, acceleration, depolarizing resonances etc etc are a headache.

(ii) Pedagogical-psychological: Spin had a difficult birth! There was the conflict between the Stern-Gerlach experiment and the fine structure of hydrogen (the spin-orbit coupling). In most textbooks one finds words like:"Mysterious effects too complicated to explain in an undergraduate text". Here are some examples. The emphases are mine.

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$$H_{magn} = \frac{1}{m^2c^2}\frac{1}{r}\frac{dV}{dr}\boldsymbol{L.S} \qquad (18.100) \qquad (21)$$

When the actual calculation is made with the proper Lorentz transformations for the fields, it is found that owing to purely kinematic effects we must add a term to the energy, which has the same form as (18.100) but a different coefficient. Known as the Thomas term, this contribution to the hamiltonian is

$$H_{Thomas} = -\frac{1}{2}H_{magn} \qquad (18.101) \qquad (22)$$

....

However, the kinematics used above is nonrelativistic. Relativistically, the electron also precesses about the nucleus (this is called the Thomas precession) with a certain frequency. The net upshot of this precession is that the magnetic field " seen" by the electron is only half as large as the one assumed in the derivation of equation (19.2), and therefore................

NB: Relativity produces a factor of 1/2 !!!!!!!!!!!!!

....

Note that a direct calculation of the spin-orbit coupling, with the usual formulations of special relativity , gives a value twice as large as (13.22), and therefore a fine-structure splitting twice too large. This is why Pauli, at the end of 1925, did not believe in the idea of spin, and called it "irrleher" (heresy) in a letter to Niels Bohr. However, in March 1926, L.H.Thomas remarked that the rest frame of of the electron is not an inertial frame, and that a correct calculation introduces a factor of 1/2 in the formula (the Thomas precession). This convinced Pauli of the validity of the spin-1/2 concept.

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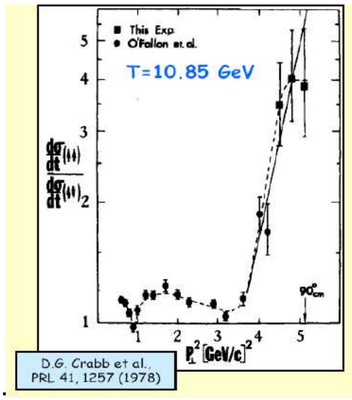
Thus spin was an unwanted child, its adolescence made even more difficult by the following. In the 1960s there was great excitement at the possibility of calculating strong interaction amplitudes reliably for the first time ever using the Mandelstam Representation. It gave rise to a massive effort to study the analytic properties of Feynman diagrams, a horribly complicated task, made much worse by the inclusion of spin. Hence the motto: *Spin is an inessential complication*...a dangerous generalization! Indeed, grossly incorrect as seen in Fig. 7 for the double spin asymmetry in elastic *pp* scattering.

Without doubt, spin-dependent measurements provide a very sharp and subtle probe of dynamical theories. They are scalpel-like in exposing theoretical weaknesses, far more so than are cross-section measurements. We have seen this on many occasions historically and I predict we will see it again often in the future.



The greatest asymmetry in hadron physics ever seen by a human being

(Brodsky)

FIGURE 7. Asymmetries can be enormous!

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REFERENCES

1. W. Fetscher, H. J. Gerber, and K. F. Johnson (1986), in *Heidelberg 1986, Proceedings, Weak and electromagnetic interactions in nuclei* 812-815. (see Conference Index).
2. E. Leader, and R. C. Slansky, *Phys. Rev.* **148**, 1491–1501 (1966).
3. S. E. Kuhn, J. P. Chen, and E. Leader (2008), to appear in Prog. Part. Nucl. Phys., `hep-ph/0812.3535`.
4. J. R. Ellis, and R. L. Jaffe, *Phys. Rev.* **D9**, 1444 (1974).
5. J. Ashman, et al., *Phys. Lett.* **B206**, 364 (1988).
6. E. Leader, and M. Anselmino, *Z. Phys.* **C41**, 239 (1988).
7. D. L. Adams, et al., *Phys. Lett.* **B264**, 462–466 (1991).
8. V. Punjabi, et al., *Phys. Rev.* **C71**, 055202 (2005), `nucl-ex/0501018`.
9. O. Gayou, et al., *Phys. Rev.* **C64**, 038202 (2001).
10. O. Gayou, et al., *Phys. Rev. Lett.* **88**, 092301 (2002), `nucl-ex/0111010`.