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Intrinsic parton motion soft mechanisms and the longitudinal spin asymmetry A_{LL} in high energy $pp \to \pi X$

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Abstract

The longitudinal double spin asymmetry A_{LL} in the reaction $pp \to \pi X$ has been measured at RHIC with extremely interesting consequences. If the gluon polarization in a proton were as big as needed to resolve the famous 'spin crisis' then A_{LL} would be large and positive. Latest RHIC results indicate that A_{LL} is small and disfavour large positive values of the gluon polarization. We examine whether the soft mechanisms (Collins, Sivers, Boer–Mulders), essential for generating transverse single spin asymmetries, have any significant influence on A_{LL}, and whether they could alter the conclusion that the gluon polarization is necessarily small. It turns out that the contribution from these effects is essentially negligible.

1. Introduction

Large transverse single spin asymmetries (up to 40%) have been observed in a multitude of reactions for over three decades, whereas such asymmetries are tiny ($\lesssim 1\%$) in the standard leading twist QCD parton model. To explain the size of these asymmetries Sivers and Collins [1, 2] introduced new soft mechanisms, utilizing, as an essential ingredient, the intrinsic transverse momentum of partons [3]. Later other similar mechanisms were shown to be possible, for example the Boer–Mulders mechanism [4, 5].

Although these mechanisms were invented in order to produce transverse asymmetries, it turns out that they also contribute to the longitudinal double spin asymmetries and to the total cross-section [6]. For the latter, it has been shown that the effect of the soft functions is negligible. However, it was found that intrinsic transverse momentum *per se* significantly affects the value of the cross-section [7]. That this should be the case at lower energies had already been noted by Field and Feynman [8], and later by Vogelsang and Weber [9] on the grounds that taking into account intrinsic $k_\perp$ is a particular method of including higher twist corrections.

For the longitudinal asymmetries, the question is much more delicate, for the following reason. One of the most important reactions measured at RHIC is the double spin longitudinal asymmetry A_{LL}, which has been found to be very small, and which has been used, based on a leading twist collinear treatment, to confirm the growing belief that the gluon polarization is far too small to explain the 'spin crisis in the parton model' [10].

Given that A_{LL} is so small, and that the implications of this are so important, we felt it necessary to check whether the soft mechanisms can have a significant impact, in particular whether they could influence the above conclusion about the gluon polarization. We have also checked for any significant sensitivity in A_{LL} to intrinsic transverse momentum.

The plan of this paper is the following. In section 2 we briefly recall the formalism used for our calculation, which includes the full non-collinear kinematics of the scattering process. In section 3 we present the 'kernels' for the calculation of each partonic contribution to the polarized cross-sections. In section 4 we show and discuss our phenomenological results for the longitudinal double spin asymmetry in inclusive neutral pion production at RHIC. Finally, in section 5 we draw our conclusions.

2. Formalism

Here we simply sketch the main aspects of the formalism; for details of the approach we refer to [6]. The longitudinal double spin asymmetry A_{LL} for the reaction $pp \to \pi X$ is defined as

$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} = \frac{d\sigma^{++} - d\sigma^{+-}}{2d\sigma^{unp}}, \tag{1}$$

where the labels refer to the helicities of the protons.

The general expression for the differential cross-sections for the polarized hadronic process $(A, S_A) + (B, S_B) \to C + X$ is given by

$$\begin{aligned}\frac{E_C d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} &= \sum_{a,b,c,d,\{\lambda\}} \int \frac{\mathrm{d}x_a\,\mathrm{d}x_b\,\mathrm{d}z}{16\pi^2 x_a x_b z^2 s}\,\mathrm{d}^2\boldsymbol{k}_{\perp a}\,\mathrm{d}^2\boldsymbol{k}_{\perp b}\,\mathrm{d}^3\boldsymbol{k}_{\perp C}\,\delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\\ &\quad\times J(\boldsymbol{k}_{\perp C})\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})\\ &\quad\times \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\delta(\hat{s}+\hat{t}+\hat{u})\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}),\end{aligned} \tag{2}$$

which involves a (factorized) convolution of all possible hard elementary QCD processes, $ab \to cd$, with soft partonic polarized distribution and fragmentation functions. In equation (2) $\hat{s}, \hat{t}$ and $\hat{u}$ are the Mandelstam variables for the partonic reactions. The detailed connection between the hadronic and the partonic kinematical variables is given in full in appendix A of [6]. A discussion of some technical details, like, e.g., the infrared regulators related to small partonic scattering angles, can be found, for example, in [7].

Let us simply recall here, for a better understanding, the physical meaning of the different factors in equation (2):

- $\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}$ is the helicity density matrix of parton a inside the polarized hadron A, with spin state S_A; it describes the parton polarization. $\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a})$ is the number density (or distribution) of unpolarized partons a inside the polarized hadron A, S_A: each parton carries a light-cone momentum fraction x_a and a transverse momentum $\boldsymbol{k}_{\perp a}$. Similarly for parton b inside hadron B with spin S_B.

- The polarized cross-sections for the elementary partonic process $(a, s_a) + (b, s_b) \to (c, s_c) + d$ are expressed in terms of products of the helicity amplitudes $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$.
- The factor $\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z, \boldsymbol{k}_{\perp C})$ describes, again in the helicity basis, the fragmentation process $c \to C + X$, according to which a polarized parton c fragments into an unpolarized hadron C carrying a light-cone momentum fraction z and a transverse momentum $\boldsymbol{k}_{\perp C}$.
- $J(\boldsymbol{k}_{\perp C})$ is a kinematical factor, numerically very close to 1 for RHIC kinematics. All details can be found in [6]. Throughout this paper, we work in the AB c.m. frame, assuming that hadron A moves along the positive Z_{cm}-axis and hadron C is produced in the $(XZ)_{cm}$ plane, with $(p_C)_{X_{cm}} > 0$.

Equation (2) is written in a factorized form, separating the soft, long distance from the hard, short distance contributions. The hard part is computable in perturbative QCD, while information on the soft one has to be extracted from other experiments or modelled. As already mentioned and discussed in [7, 6], such a factorization with non-collinear kinematics has never been formally proven. Indeed, studies of factorization [11–14], comparing semi-inclusive deep inelastic scattering (SIDIS) and Drell–Yan reactions have indicated unexpected modifications of simple factorization, and the situation for single inclusive particle production in hadron–hadron collisions is not yet resolved. Thus, our approach can only be considered as the natural extension of the collinear case and a reasonable phenomenological model. Of course, the perturbative calculation of the hard part is only reliable if the hard scale—in this case the square of the transverse momentum of the final hadron, p_T^2—is large enough. It turns out that the data on unpolarized cross-sections in hadronic collisions at low-intermediate energy scales suggest [7] an average value of $k_\perp^2 \equiv |\boldsymbol{k}_\perp|^2 \simeq 0.64\ (\mathrm{GeV}/c)^2$ for the intrinsic transverse momentum of the parton distributions. On the other hand, both unpolarized hadronic cross-sections at RHIC energies [15] and the Cahn effect in SIDIS [16] are rather well reproduced by using $\langle k_\perp^2\rangle \simeq 0.25\ (\mathrm{GeV}/c)^2$. We shall therefore study how the contributions to A_{LL} depend on the value of $\langle k_\perp^2\rangle$.

3. Kernels

As we can see from equation (2), the computation of the cross-section corresponding to any polarized hadronic process $(A, S_A)+(B, S_B) \to C+X$ requires the evaluation and integration, for each elementary process $a + b \to c + d$, of the general kernel

$$\Sigma(S_A, S_B)^{ab\to cd} = \sum_{\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) \rho^{b/B,S_B}_{\lambda_b,\lambda'_b} \hat{f}_{b/B,S_B}(x_b, \boldsymbol{k}_{\perp b}) \times \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z, \boldsymbol{k}_{\perp C}). \tag{3}$$

While the hadronic process $(A, S_A) + (B, S_B) \to C + X$ takes place, according to our choice, in the $(XZ)_{cm}$ plane, all the elementary processes involved, $A(B) \to a(b) + X$, $ab \to cd$ and $c \to C + X$ do not, since all parton and hadron momenta, $\boldsymbol{p}_a, \boldsymbol{p}_b, \boldsymbol{p}_C$ have transverse components $\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C}$. This 'out of the $(XZ)_{cm}$ plane' geometry induces phases in the fragmentation process, in the distribution functions and in the elementary interactions, which have to be taken into account. Thus, the independent helicity amplitudes for the elementary pQCD processes $ab \to cd$, with massless partons, can be written as [6]

$$\hat{M}_{+,+;+,+} \equiv \hat{M}^0_1\,\mathrm{e}^{\mathrm{i}\varphi_1} \qquad \hat{M}_{-,+;-,+} \equiv \hat{M}^0_2\,\mathrm{e}^{\mathrm{i}\varphi_2} \qquad \hat{M}_{-,+;+,-} \equiv \hat{M}^0_3\,\mathrm{e}^{\mathrm{i}\varphi_3}, \tag{4}$$

where the amplitudes $\hat{M}^0_{1,2,3}$ are the real planar amplitudes defined in the partonic $ab \to cd$ c.m. frame,

$$\hat{M}_1^0 \equiv \hat{M}^0_{+,+;+,+} = \hat{M}^0_{-,-;-,-} \qquad \hat{M}_2^0 \equiv \hat{M}^0_{-,+;-,+} = \hat{M}^0_{+,-;+,-}$$
$$\hat{M}_3^0 \equiv \hat{M}^0_{-,+;+,-} = \hat{M}^0_{+,-;-,+}, \tag{5}$$

as required by parity invariance. The phases $\varphi_{1,2,3}$ are complicated functions of the polar and azimuthal angles of the transverse momenta, $\boldsymbol{k}_{\perp a}$, $\boldsymbol{k}_{\perp b}$ and $\boldsymbol{k}_{\perp C}$, and their explicit expressions can be found in [6]. The relations

$$\hat{M}_{-,-;-,-} = \hat{M}^*_{+,+;+,+} \qquad \hat{M}_{+,-;+,-} = \hat{M}^*_{-,+;-,+} \qquad \hat{M}_{+,-;-,+} = \hat{M}^*_{-,+;+,-}, \tag{6}$$

follow from equations (4), (5) and from the fact that the phases φ_i change sign by helicity inversion [6]. Note that the + and − subscripts refer to (+1/2) and (−1/2) helicities for quarks, and to (+1) and (−1) helicities for gluons. There are eight elementary contributions $ab \to cd$ which we have to consider separately

$$\begin{aligned}
&q_a q_b \to q_c q_d, \qquad && g_a g_b \to g_c g_d,\\
&qg \to qg, && gq \to gq,\\
&qg \to gq, && gq \to qg,\\
&g_a g_b \to q\bar{q}, && q\bar{q} \to g_c g_d,
\end{aligned} \tag{7}$$

where q can in general be either a quark or an antiquark. The subscripts a, b, c, d for quarks, when necessary, identify the flavour (only in processes where different flavours can be present); for gluons, these labels identify the corresponding hadron ($a \to A, b \to B, c \to C$). By performing the explicit sums in equation (3), we obtain the kernels for each of the elementary processes. Note that the new aspect of our calculation is the appearance of the phases which is a reflection of the non-collinear kinematics.

The computation of the denominator/numerator of A_{LL} in equation (1) requires the evaluation of the kernels $[\Sigma(+,+) \pm \Sigma(+,-)]$, respectively. The expressions for the sums of kernels, which are relevant for the unpolarized cross-section, are given in [6]. Here we give in detail the expressions for the differences. They are calculated from the general kernel given in equation (3). In the following certain terms are underlined: these are terms which vanish after integration over the angles of the momenta $\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C}$ in equation (2); we shall further comment on that at the end of this section. ϕ_C^H is the azimuthal angle of the hadron C in the parton c helicity frame and its expression in terms of the angles of $\boldsymbol{k}_{\perp C}$ is given in appendix A of [6]. Note that all angular dependences of the kernels are explicitly extracted and the parton distribution (PDF) and fragmentation (FF) functions only depend on the magnitudes of the transverse momentum vectors.

- $q_a q_b \to q_c q_d$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{q_a q_b \to q_c q_d}\\
&\quad = \Delta\hat{f}^a_{s_z/+}(x_a, k_{\perp a})\Delta\hat{f}^b_{s_z/+}(x_b, k_{\perp b})\big[\big|\hat{M}_1^0\big|^2 - \big|\hat{M}_2^0\big|^2 - \big|\hat{M}_3^0\big|^2\big]\hat{D}_{C/c}(z, k_{\perp C})\\
&\qquad + \Big[\Delta\hat{f}^a_{s_x/+}(x_a, k_{\perp a})\Delta\hat{f}^b_{s_x/+}(x_b, k_{\perp b})\cos(\varphi_3 - \varphi_2)\\
&\qquad + \underline{\Delta\hat{f}^a_{s_y/A}(x_a, k_{\perp a})\Delta\hat{f}^b_{s_x/+}(x_b, k_{\perp b})\sin(\varphi_3 - \varphi_2)}\Big]\big(2\hat{M}_2^0\hat{M}_3^0\big)\hat{D}_{C/c}(z, k_{\perp C})\\
&\qquad - \underline{\hat{f}_{a/A}(x_a, k_{\perp a})\Delta\hat{f}^b_{s_x/+}(x_b, k_{\perp b})\hat{M}_1^0\hat{M}_3^0\sin\big(\varphi_1 - \varphi_3 + \phi_C^H\big)\Delta^N\hat{D}_{C/c^\uparrow}(z, k_{\perp C})}.
\end{aligned} \tag{8}$$

Note that we have used the relations $\Delta\hat{f}^a_{s_y/+}(x_a, k_{\perp a}) = \Delta\hat{f}^a_{s_y/A}(x_a, k_{\perp a})$ and $\hat{f}_{a/+}(x_a, k_{\perp a}) = \hat{f}_{a/A}(x_a, k_{\perp a})$, see appendix B of [6]. The channels $q\bar{q} \to q\bar{q}$ etc. are formally identical to $qq \to qq$ with amplitudes defined properly in [6].

- $g_a g_b \to g_c g_d$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{g_a g_b \to g_c g_d} \\
&\quad = \Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_1^0\right|^2 - \left|\hat{M}_2^0\right|^2 - \left|\hat{M}_3^0\right|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \\
&\qquad + \Bigg[\Delta \hat{f}^a_{\mathcal{T}_2/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \cos(\varphi_3 - \varphi_2) \\
&\qquad + \underline{\Delta \hat{f}^a_{\mathcal{T}_1/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \sin(\varphi_3 - \varphi_2)} \Bigg] \left(2 \hat{M}_2^0 \hat{M}_3^0\right) \hat{D}_{C/g}(z, k_{\perp C}) \\
&\qquad + \underline{\hat{f}_{a/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \hat{M}_1^0 \hat{M}_3^0 \sin\left(\varphi_1 - \varphi_3 + 2\phi_C^H\right) \Delta^N \hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})}
\end{aligned} \tag{9}$$

- $q\bar{q} \to gg$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{q\bar{q} \to gg} \\
&\quad = -\Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_2^0\right|^2 + \left|\hat{M}_3^0\right|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \\
&\qquad + \Bigg[\Delta \hat{f}^a_{s_x/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_x/+}(x_b, k_{\perp b}) \cos(\varphi_3 - \varphi_2) \\
&\qquad + \underline{\Delta \hat{f}^a_{s_y/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_x/+}(x_b, k_{\perp b}) \sin(\varphi_3 - \varphi_2)} \Bigg] \\
&\qquad \times \left(2 \hat{M}_2^0 \hat{M}_3^0\right) \hat{D}_{C/g}(z, k_{\perp C})
\end{aligned} \tag{10}$$

- $g_a g_b \to q\bar{q}/\bar{q}q$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{g_a g_b \to q\bar{q}/\bar{q}q} \\
&\quad = -\Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_2^0\right|^2 + \left|\hat{M}_3^0\right|^2 \right] \hat{D}_{C/c}(z, k_{\perp C}) \\
&\qquad + \Bigg[\Delta \hat{f}^a_{\mathcal{T}_2/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \cos(\varphi_3 - \varphi_2) \\
&\qquad + \underline{\Delta \hat{f}^a_{\mathcal{T}_1/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \sin(\varphi_3 - \varphi_2)} \Bigg] \\
&\qquad \times \left(2 \hat{M}_2^0 \hat{M}_3^0\right) \hat{D}_{C/c}(z, k_{\perp C})
\end{aligned} \tag{11}$$

- $qg \to qg$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{qg \to qg} \\
&\quad = \Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_1^0\right|^2 - \left|\hat{M}_2^0\right|^2 \right] \hat{D}_{C/c}(z, k_{\perp C})
\end{aligned} \tag{12}$$

- $gq \to qg$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{gq \to qg} \\
&\quad = \Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_1^0\right|^2 - \left|\hat{M}_3^0\right|^2 \right] \hat{D}_{C/c}(z, k_{\perp C}) \\
&\qquad - \underline{\hat{f}_{a/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_x/+}(x_b, k_{\perp b}) \hat{M}_1^0 \hat{M}_3^0 \sin\left(\varphi_1 - \varphi_3 + \phi_C^H\right) \Delta^N \hat{D}_{C/c^\uparrow}(z, k_{\perp C})}
\end{aligned} \tag{13}$$

- $qg \to gq$ contribution

$$\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{qg \to gq} \\
&\quad = \Delta \hat{f}^a_{s_z/+}(x_a, k_{\perp a}) \Delta \hat{f}^b_{s_z/+}(x_b, k_{\perp b}) \left[\left|\hat{M}_1^0\right|^2 - \left|\hat{M}_3^0\right|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \\
&\qquad + \underline{\hat{f}_{a/A}(x_a, k_{\perp a}) \Delta \hat{f}^b_{\mathcal{T}_2/+}(x_b, k_{\perp b}) \hat{M}_1^0 \hat{M}_3^0 \sin\left(\varphi_1 - \varphi_3 + 2\phi_C^H\right) \Delta^N \hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})}
\end{aligned} \tag{14}$$

- $gq \to gq$ contribution

$$[\Sigma(+,+) - \Sigma(+,-)]^{gq\to gq} \\ = \Delta\hat{f}^{a}_{s_z/+}(x_a, k_{\perp a})\Delta\hat{f}^{b}_{s_z/+}(x_b, k_{\perp b})\big[\big|\hat{M}^0_1\big|^2 - \big|\hat{M}^0_2\big|^2\big]\hat{D}_{C/g}(z, k_{\perp C}). \tag{15}$$

The physical content of the above expressions is interesting. First note the complete formal symmetry between the $qq \to qq$ kernel in equation (8) and the $gg \to gg$ kernel in equation (9). These kernels contain the largest variety of contributions, and the kernels for all the other partonic processes can be formally read off from these by the suppression of certain terms.

In the second line of both expressions, equations (8) and (9), we recognize the product of the $k_\perp$-dependent helicity distributions, $\Delta\hat{f}^{q}_{s_z/+}(x_q, k_{\perp q}) \equiv \Delta q(x_q, k_{\perp q})$ and $\Delta\hat{f}^{g}_{s_z/+}(x_g, k_{\perp g}) \equiv \Delta g(x_g, k_{\perp g})$ for quarks and gluons respectively, and the unpolarized fragmentation function $\hat{D}_{C/c}(z, k_{\perp C})$, with no azimuthal phases. In the third line of equation (8) we have two parton distribution functions, $\Delta\hat{f}^{q}_{s_x/+}(x, k_\perp)$, referring to quarks transversely polarized, along the x-axis, inside longitudinally polarized nucleons, coupled to the unpolarized fragmentation function. Analogously, in the third line of equation (9) we have two parton distribution functions, $\Delta\hat{f}^{g}_{\mathcal{T}_2/+}(x, k_\perp)$, which are related to the linear polarization of a gluon inside a longitudinally polarized nucleon. Correspondingly the fourth line of equation (8) refers to one quark transversely polarized along the x-axis inside a longitudinally polarized nucleon and the other, $\Delta\hat{f}^{q}_{s_y/A}(x, k_\perp)$, transversely polarized along the y-axis inside an unpolarized nucleon—the latter is the Boer–Mulders function—coupled to the unpolarized fragmentation function. Analogously, in the fourth line of equation (9) we have $\Delta\hat{f}^{g}_{\mathcal{T}_2/+}(x, k_\perp)$ and the 'Boer–Mulders-like' gluon function, $\Delta\hat{f}^{g}_{\mathcal{T}_1/A}(x, k_\perp)$, referring to a linearly polarized gluon inside an *unpolarized* nucleon. For a more complete explanation of the physical meaning of these functions see appendix B of [6]. Finally, the last line of equation (8) contains the Collins fragmentation function, $\Delta^N\hat{D}_{C/c^\uparrow}(z, k_{\perp C})$, coupled to an unpolarized parton density and a transversely polarized one. In the case of the gluon, in the last line of equation (9), there appears a gluonic analogue of the Collins fragmentation function, $\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})$, describing the fragmentation of a linearly polarized gluon into an unpolarized hadron.

Ignoring the underlined terms which vanish upon integration, we see that compared to the standard collinear approach, we have extra contributions involving quarks polarized transversely along their x-axis in a longitudinally polarized nucleon, appearing in equations (8), (10) and contributions involving linearly polarized gluons inside a longitudinally polarized nucleon, appearing in equations (9), (11). Note that the processes in equations (12)–(15), initiated by quark–gluon elementary scattering, get contributions only from the usual terms, which survive in the collinear case.

The demonstration of the vanishing upon angular integration of the underlined terms in equations (8)–(15) requires a detailed study of the kinematics and of the relationships between the angular integration variables appearing in $\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C}$ in equation (2) and the phase variables $\varphi_{1,2,3}$ and ϕ_C^H [6]. We have also numerically checked that this is indeed the case.

Note that a parity transformation implies $\varphi_i \to -\varphi_i$ ($i = 1, 2, 3$) and $\phi_C^H \to -\phi_C^H$. Thus the odd sin terms in equations (8)–(15) must vanish if parity is conserved.

Another simple, but interesting example of such a vanishing can be obtained by considering, within the same formalism, the expression of the kernels for the longitudinal *single spin* asymmetry A_L, which we know *must* vanish in a parity conserving theory. The kernels themselves are *not* zero, but under integration do vanish. This is another very stringent test of the correctness of our formalism. For A_L, for the partonic channel $q_aq_b \to q_cq_d$, we have for the numerator of the longitudinal single spin asymmetry the following expression:

$$[\Sigma(+,0)-\Sigma(-,0)]^{q_aq_b\to q_cq_d}$$

$$= \underline{\Delta\hat{f}^{a}_{s_x/+}(x_a,k_{\perp a})\Delta\hat{f}^{b}_{s_y/B}(x_b,k_{\perp b})\big(2\hat{M}_2^0\hat{M}_3^0\big)\sin(\varphi_3-\varphi_2)\hat{D}_{C/c}(z,k_{\perp C})}$$

$$\underline{-\Delta\hat{f}^{a}_{s_x/+}(x_a,k_{\perp a})\hat{f}_{b/B}(x_b,k_{\perp b})\sin\big(\varphi_1-\varphi_2+\phi_C^H\big)\hat{M}_1^0\hat{M}_2^0\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})}, \quad (16)$$

and, again, all terms—being odd functions of φ_i and ϕ_C^H—vanish, as they should, upon angular integration.

4. Phenomenology: A_{LL} at RHIC

The longitudinal double spin asymmetry A_{LL} for inclusive neutral pion and jet production in proton–proton scattering at $\sqrt{s}=200$ GeV has been measured at RHIC in various runs, respectively by the PHENIX [17–19] and STAR [20, 21] Collaborations. The first published PHENIX experimental data [17] showed results for A_{LL} at mid rapidity compatible with negative values. This was quite puzzling, since A_{LL} is a positive quantity in the collinear parton model [22], at least at low p_T where it is dominated by $gg\to gg$ elementary scattering processes, see equation (9) in which it can be shown that $|\hat{M}_1^0|^2-|\hat{M}_2^0|^2-|\hat{M}_3^0|^2>0$. More recent and precise data from both collaborations [18, 20] exclude the possibility of a large and negative A_{LL}: in two subsequent RHIC runs, Run 5 [21, 19] and Run 6 (results from Run 6 have only been presented as 'preliminary' [23, 24]), they confirm and reinforce the statement that A_{LL} is very small and compatible with zero over the whole p_T range covered.

An earlier comparison of present RHIC data with collinear next-to-leading order (NLO) QCD calculations of A_{LL} [25] disfavoured large positive values for Δg, definitely excluding scenarios where Δg is as large as the unpolarized gluon distribution function, g, at low scale. Instead the data were in better agreement with the predictions obtained by assuming $\Delta g=0$ or even $\Delta g=-g$ at the initial scale [26]. A recent statistical analysis shows that the PHENIX Run 5 data are compatible with both $\Delta g=0$ and the 'standard' GRSV parametrization [27], while it rules out the $\Delta g=-g$ (at the initial scale) hypothesis [19]. A newest update of this analysis, which includes the preliminary data from PHENIX Run 6, favours the $\Delta g=0$ scenario over the standard GRSV [24]. (Note that in this section we have adopted the common, short-hand notations $\Delta f^{q}_{s_z/+}\equiv\Delta q$ and $\Delta f^{g}_{s_z/+}\equiv\Delta g$ for the helicity distribution functions, while $f_{q/p}\equiv q$ and $f_{g/p}\equiv g$ for the unpolarized distribution functions, for quarks and gluons respectively.)

Our goal is to explore whether the new mechanisms permitted by the presence of partonic intrinsic transverse momenta, obtained in a general and fully non-collinear kinematics, could affect the above conclusions, which are based on the analysis of A_{LL} in the collinear configuration, i.e. taking into account only the terms proportional to $\Delta q(x)$ and $\Delta g(x)$. Could the 'new' contributions shown in equations (8)–(11) turn the longitudinal double spin asymmetry A_{LL} into a very small (or even slightly negative) quantity without the need to assume Δg to be zero or negative?

We have studied A_{LL} at RHIC, for the PHENIX kinematics, $\sqrt{s}=200$ GeV and $|\eta|<0.35$ (numerical calculations are performed at $\eta=0$) and evaluated each separate contribution to A_{LL}, according to equations (8)–(15). Since we have no knowledge of the parton densities $\Delta\hat{f}^{q}_{s_x/+}$ and $\Delta\hat{f}^{g}_{\mathcal{T}_2/+}$ we maximized them in order to see whether, in principle, they can have a significant effect on A_{LL}. We thus used for them the corresponding unpolarized parton densities and adjusted the signs so that all contributions add up coherently.

For the helicity distributions we have used the sets GRSV2000 [27] and LSS05 [28]. The unpolarized cross-section and the maximized contributions to the numerator of A_{LL} have

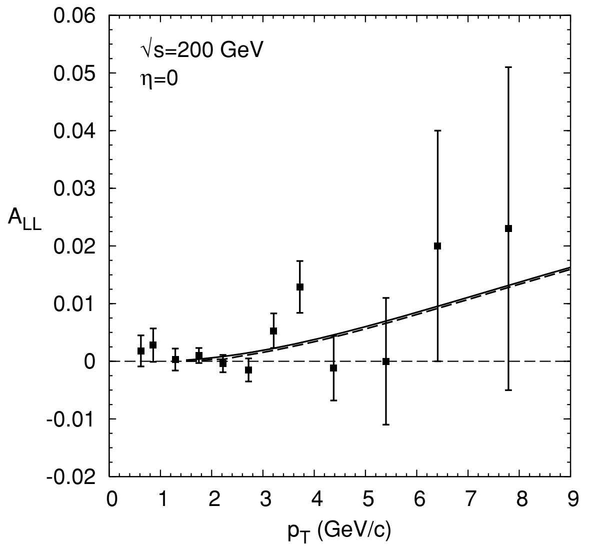

Figure 1. A_{LL} for the process $pp \to \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as a function of p_T, calculated with different choices of $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0$ in the PDF/FFs, compared to PHENIX data, Run 5 [19]. The solid line corresponds to the choice $k_0 = 0.01$ GeV/c in both PDFs and FFs. The dashed line corresponds to $k_0 = 0.8$ GeV/c in PDF/FFs. The PDF sets are LSS05 [28] and MRST01 [30], the FF set is KKP [31] and the factorization scale is $Q = p_T$. Note that the changes in A_{LL} induced by varying the value of $\langle k_\perp^2 \rangle$ are much smaller than those obtained by choosing different sets of distribution functions and/or factorization scales, see figure 3.

been calculated using the GRV98 set [29] and the MRST01 set [30] respectively. For the fragmentation functions we have used the KKP set [31] and, for comparison, the Kretzer set [32]. The transverse momentum dependence has been included by means of a factorized Gaussian smearing, for all the parton distribution and fragmentation functions

$$\hat{f}(x, k_\perp) = f(x) \frac{\mathrm{e}^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}, \tag{17}$$

$$\hat{D}(z, k_{\perp C}) = D(z) \frac{\mathrm{e}^{-k_{\perp C}^2/\langle k_{\perp C}^2 \rangle}}{\pi \langle k_{\perp C}^2 \rangle}, \tag{18}$$

with a constant and flavour independent parameter $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0$, assumed to be the same for all quark flavours and for gluons; we shall study the effect of changes in the value of k_0. Guided by our previous work, we compared the results obtained using three different values for k_0: $k_0 = 0.8$ GeV/c from studies on the unpolarized pp scattering cross-sections and single spin asymmetries [7], $k_0 = 0.5$ GeV/c from fitting the Cahn effect in SIDIS [16], and $k_0 = 0.01$ GeV/c to recover the collinear configuration. For the fragmentation functions, we take $\langle k_{\perp C}^2 \rangle = \langle k_\perp^2 \rangle$ everywhere. We have checked that variations in $\langle k_{\perp C}^2 \rangle$ induce negligible changes in A_{LL}.

It turns out that the new non-collinear soft contributions containing the PDFs $\Delta \hat{f}^q_{s_x/+}(x, k_\perp)$ and $\Delta \hat{f}^g_{\mathcal{T}_2/+}(x, k_\perp)$, even if maximized, are totally negligible. In fact, in the RHIC kinematical regime considered their maximized contribution does not exceed, in

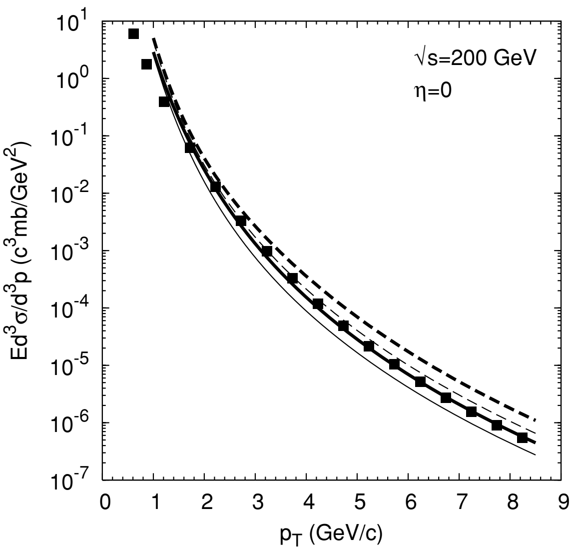

Figure 2. The invariant unpolarized cross-section for the process $pp \to \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as a function of p_T, calculated with different FF sets and factorization scales. The thick, solid and dashed lines correspond to the choice of the KKP FF set [31], at the factorization scale $Q = p_T$ and $Q = p_T/2$ respectively. The thin, solid and dashed lines correspond to the choice of the Kretzer FF set [32], at the factorization scale $Q = p_T$ and $Q = p_T/2$ respectively. The PDF set is GRV98 [29]. $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0 = 0.5$ GeV/c for both PDFs and FFs. The experimental data are from the PHENIX collaboration at RHIC, Run 5 [19].

the lowest p_T range, few percent of the usual terms (already present in the collinear case), becoming much smaller at larger p_T. We have checked that this result remains true also at lower energies. Let us remark that a similar situation holds also for the unpolarized cross-section [7]. Although the two additional terms (with respect to that already present in the collinear case) are of course different in this case [6], involving respectively the convolution of two Boer–Mulders functions with an unpolarized fragmentation function and the convolution of a Boer–Mulders function and an unpolarized distribution with the Collins fragmentation function, their total maximized contribution reaches at most 1% of the usual term, being even smaller on the average.

Therefore, we conclude that there is no way for the extra contributions induced by the presence of partonic intrinsic transverse momenta to alter the size of A_{LL}. We have checked that this conclusion is not sensitive to the choice of the mean intrinsic transverse momentum k_0. In fact, figure 1 shows that in general A_{LL} depends very little on the different choices of k_0; in particular, A_{LL} decreases when increasing the width of the gaussian, but compared to data this variation is quite negligible. This result can be understood because the $k_\perp$ dependence is given by the same gaussian for all distribution and fragmentation functions and at mid rapidity the $\hat{M}$ amplitudes depend very mildly on $k_\perp$.

It is interesting to note that the corresponding unpolarized cross-section is also almost independent of the value assigned to the average intrinsic transverse momentum k_0, while it turns out to be more sensitive to the choice of the factorization scale and of the fragmentation function set, as we show in figure 2, consistently with the NLO collinear pQCD calculations. The comparison with PHENIX data [19] is well satisfactory. The solid lines correspond to the

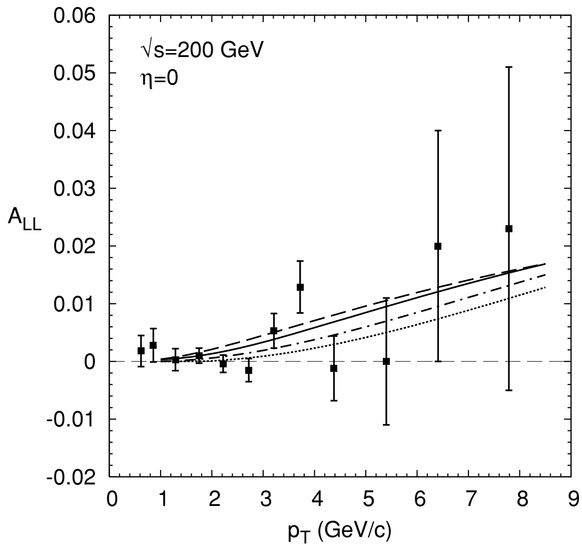

Figure 3. A_{LL} for the process $pp \to \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as a function of p_T, calculated with different PDF sets and factorization scales. The solid and dashed lines correspond to the choice of PDFs GRSV2000/GRV98, at the factorization scale $Q = p_T/2$ and $Q = p_T$ respectively. The dotted and dash-dotted lines correspond the choice of PDFs LSS05/MRST01, at the factorization scale $Q = p_T/2$ and $Q = p_T$ respectively. The FF set is KKP. $\sqrt{\langle k_\perp^2\rangle} = 0.01$ GeV$/c$ for both PDF/FF. The experimental data are from the PHENIX collaboration at RHIC, Run 5 [19].

factorization scale $Q = p_T$, the dashed lines to $Q = p_T/2$, using the GRV98 [29] PDF set and the KKP [31] (thick lines) or the Kretzer [32] (thin lines) FF sets. Results for the STAR and BRAHMS kinematical regimes at $\sqrt{s} = 200$ GeV can be found in [15] and show similar agreement with data when adopting the same average $k_\perp$'s as in [16].

Contrary to what happens for the $k_\perp$ dependence, A_{LL} is sensitive to the choice of the PDF set and of the scale. In figure 3 we show A_{LL} calculated in an almost collinear configuration, $\sqrt{\langle k_\perp^2\rangle} = 0.01$ GeV$/c$, and for two choices of scale, $Q = p_T$ and $Q = p_T/2$. Using the LSS05/MRST01 PDFs, the dotted line corresponds to $Q = p_T/2$ and the dash-dotted line to $Q = p_T$. Using the GRSV2000/GRV98 PDFs, the solid line corresponds to $Q = p_T/2$, and the dashed line to $Q = p_T$. As can be seen, the variations induced by different choices of PDF sets and scale are quite large, larger than those produced by changes in the k_0 value; nevertheless, all these curves are compatible with present experimental data (we have checked that these same conclusions hold also when adopting $Q = 2p_T$). However, very precise data on A_{LL} in the future might be able to distinguish between various sets of PDFs. Data collected at different energies [23] will also be very useful to cover presently unexplored regions of the Bjorken x variable.

Concerning the dependence of A_{LL} on the set of fragmentation functions, we have checked, adopting again the KKP and Kretzer sets, that this is almost negligible over the whole p_T range considered. Only at the largest p_T values, where A_{LL} data show large experimental errors, there is some residual dependence. This result can be understood, since both the numerator and the denominator of A_{LL} contain the unpolarized FF.

5. Conclusions

We have examined, at leading order in perturbative QCD, the effect on the longitudinal double spin asymmetry A_{LL} of allowing the partons to have non-zero intrinsic transverse momentum, and of including in A_{LL} the contributions arising from the new soft functions that play a crucial role in transverse single spin asymmetries. The study was carried out in the hope that such effects might negate the conclusion that the very small measured values of A_{LL} automatically imply that the polarized gluon density is very small. Our analysis indicates that the contribution from these effects is negligible and we are forced, at the present stage, to accept the conclusion that the polarized gluon density is much too small to explain the 'spin crisis in the parton model' [10].

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References

[1] Sivers D 1990 *Phys. Rev.* D **41** 83
Sivers D 1991 *Phys. Rev.* D **43** 261
[2] Collins J C 1993 *Nucl. Phys.* B **396** 161
[3] Anselmino M, Boglione M and Murgia F 1995 *Phys. Lett.* B **362** 164
Anselmino M, Boglione M and Murgia F 1999 *Phys. Rev.* D **60** 054027
[4] Mulders P J and Tangerman R D 1996 *Nucl. Phys.* B **461** 197
Mulders P J and Tangerman R D 1997 *Nucl. Phys.* B **484** 538 (erratum)
Boer D and Mulders P J 1998 *Phys. Rev.* D **57** 5780
[5] Boer D 1999 *Phys. Rev.* D **60** 014012
[6] Anselmino M, Boglione M, D'Alesio U, Leader E, Melis S and Murgia F 2006 *Phys. Rev.* D **73** 014020
[7] D'Alesio U and Murgia F 2004 *Phys. Rev.* D **70** 074009
[8] Field R G and Feynman R P 1977 *Phys. Rev.* D **15** 2590
[9] Vogelsang W and Weber A 1992 *Phys. Rev.* D **45** 4069
[10] Leader E and Anselmino M 1988 *Z. Phys.* C **41** 239
[11] Bacchetta A, Bomhof C J, Mulders P J and Pijlman F 2005 *Phys. Rev.* D **72** 034030
Bomhof C J, Mulders P J and Pijlman F 2006 *Eur. Phys. J.* C **47** 147
[12] Collins J C and Metz A 2004 *Phys. Rev. Lett.* **93** 252001
[13] Ji X, Ma J-P and Yuan F 2005 *Phys. Rev.* D **71** 034005
Ji X, Ma J-P and Yuan F 2004 *Phys. Lett.* B **597** 299
[14] Collins J and Qiu J-W 2007 *Phys. Rev.* D **75** 114014
[15] Boglione M, D'Alesio U and Murgia F 2008 *Phys. Rev.* D **77** 051502
[16] Anselmino M, Boglione M, D'Alesio U, Kotzinian A, Murgia F and Prokudin A 2005 *Phys. Rev.* D **71** 074006
[17] Adler S S *et al* (PHENIX Collaboration) 2004 *Phys. Rev. Lett.* **93** 202002
[18] Adler S S *et al* (PHENIX Collaboration) 2006 *Phys. Rev.* D **73** 091102
[19] Adare A *et al* (PHENIX Collaboration) 2007 *Phys. Rev.* D **76** 051106
[20] Abelev B I *et al* (STAR Collaboration) 2006 *Phys. Rev. Lett.* **97** 252001
[21] Abelev B I *et al* (STAR Collaboration) 2008 *Phys. Rev. Lett.* **100** 232003
[22] Jäger B, Stratmann M, Kretzer S and Vogelsang W 2004 *Phys. Rev. Lett.* **92** 121803
[23] Aoki K (PHENIX Collaboration) 2007 *Talk presented at the 23rd International Nuclear Physics Conference (INPC2007), (Tokyo, Japan, 3-8 June 2007)* (arXiv:0709.0147)
[24] Saito N 2008 *Proc. of the International School of Physics Enrico Fermi on Strangeness and Spin in Fundamental Physics* ed M Anselmino, T Bressani, A Feliciello and Ph G Ratcliffe (Amsterdam: IOS Press) p 379

[25] Jäger B, Schafer A, Stratmann M and Vogelsang W 2003 *Phys. Rev.* D **67** 054005
[26] Stratmann M and Vogelsang W 2007 *J. Phys. Conf. Ser.* **69** 012035
[27] Gluck M, Reya E, Stratmann M and Vogelsang W 2001 *Phys. Rev.* D **63** 094005
[28] Leader E, Sidorov A V and Stamenov D B 2006 *Phys. Rev.* D **73** 034023
[29] Gluck M, Reya E and Vogt A 1998 *Eur. Phys. J.* C **5** 461
[30] Martin A D, Roberts R G, Stirling W J and Thorne R S 2002 *Phys. Lett.* B **531** 216
[31] Kniehl B A, Kramer G and Pötter B 2000 *Nucl. Phys.* B **582** 514
[32] Kretzer S 2000 *Phys. Rev.* D **62** 054001