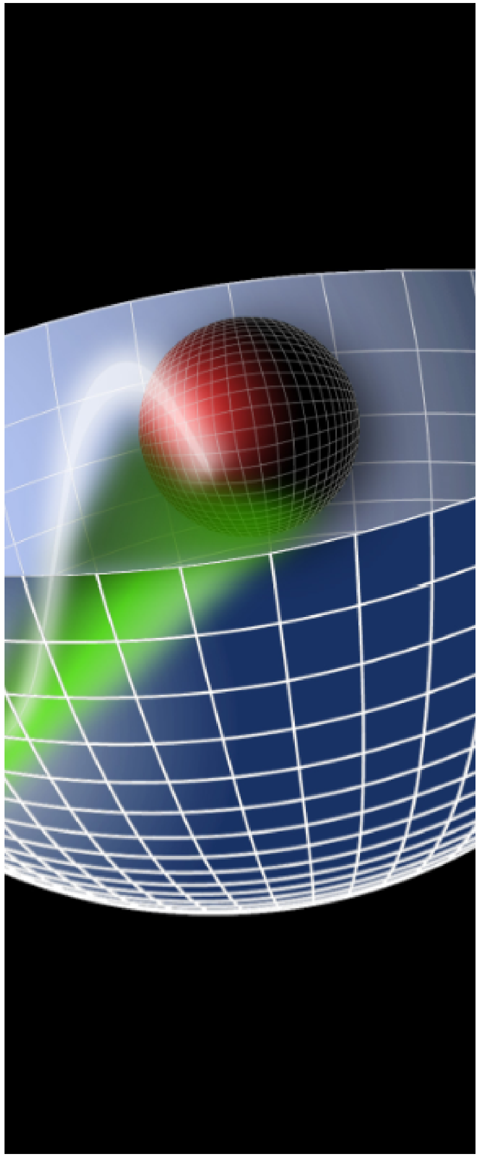
Hadron Spectroscopy and Form Factors in AdS/QCD for Experimentalists



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I. Introduction

- Lattice QCD
- Gravity
- Holographic Correspondence

II. Gauge/Gravity Correspondence and Light-Front QCD

- Higher Spin Modes in AdS Space

III. Light Front Dynamics

- Light-Front Fock Representation
- Semiclassical Approximation to QCD in the Light Front
- Light-Front Holographic Mapping
- Light Meson and Baryon Spectrum

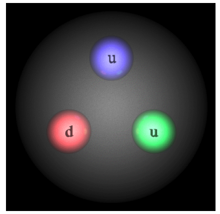
IV. Light-Front Holographic Mapping of Current Matrix Elements

- Electromagnetic Form Factors

V. Higher Fock Components

- Detailed Structure of Space-and Time Like Pion Form Factor

I. Introduction



- QCD fundamental theory of quarks and gluons
- QCD Lagrangian follows from the gauge invariance of the theory

$$\psi(x) \rightarrow e^{i\alpha^a(x)T^a}\psi(x), \qquad \left[T^a, T^b\right] = i f_{abc} T^c$$

- Find QCD Lagrangian

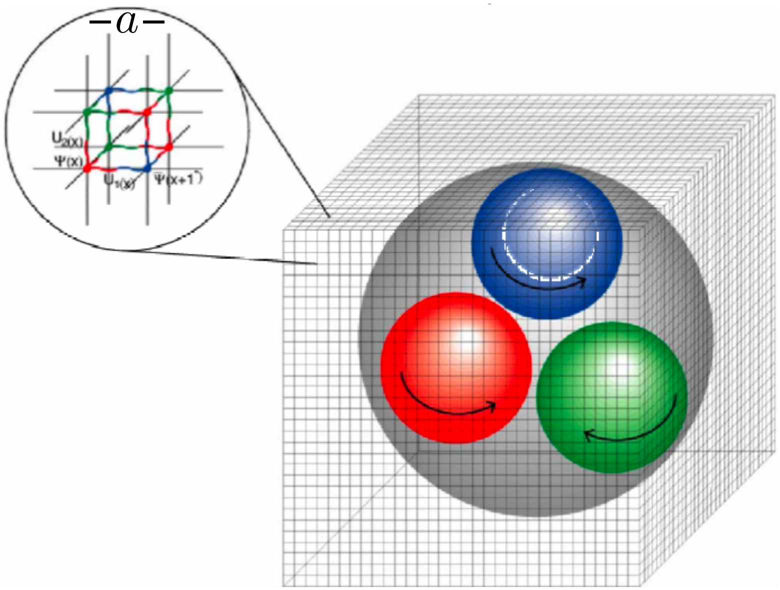
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

where $D_\mu = \partial_\mu - igT^a A^a_\mu, \;\; G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc}A^b_\mu A^c_\nu$

- Quarks and gluons interactions from color charge, but ... gluons also interact with each other: strongly coupled non-abelian gauge theory $\longrightarrow$ color confinement
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom

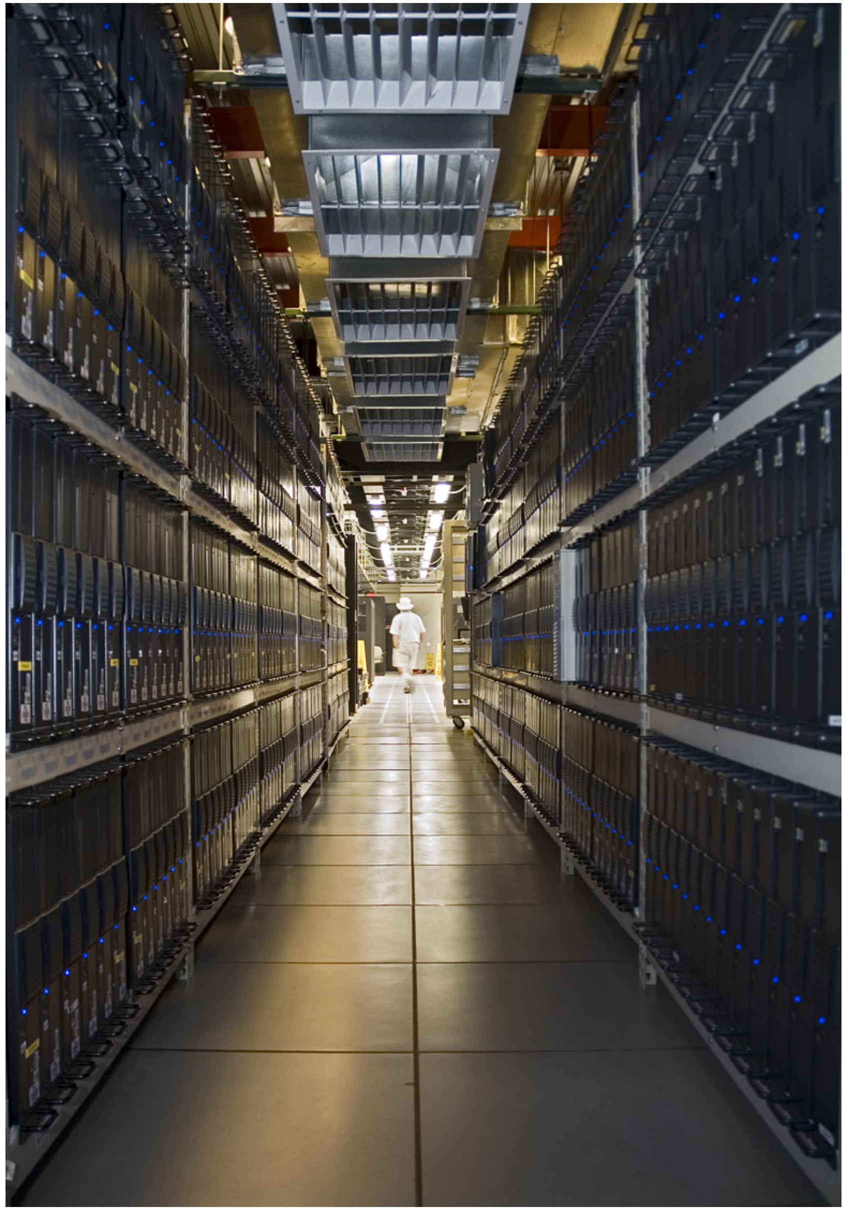
Lattice QCD

- Lattice numerical simulations at the teraflop/sec scale (resolution $\sim L/a$)
- Sums over quark paths with billions of dimensions
- LQCD (2009) $>$ 1 petaflop/sec



$\longleftarrow \quad L \quad \longrightarrow$

- Dynamical properties in Minkowski space-time not amenable to Euclidean lattice computations



Gravity

- Space curvature determined by the mass-energy present following Einstein's equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}}_{\text{geometry}} = \kappa\, \underbrace{T_{\mu\nu}}_{\text{mater}}$$

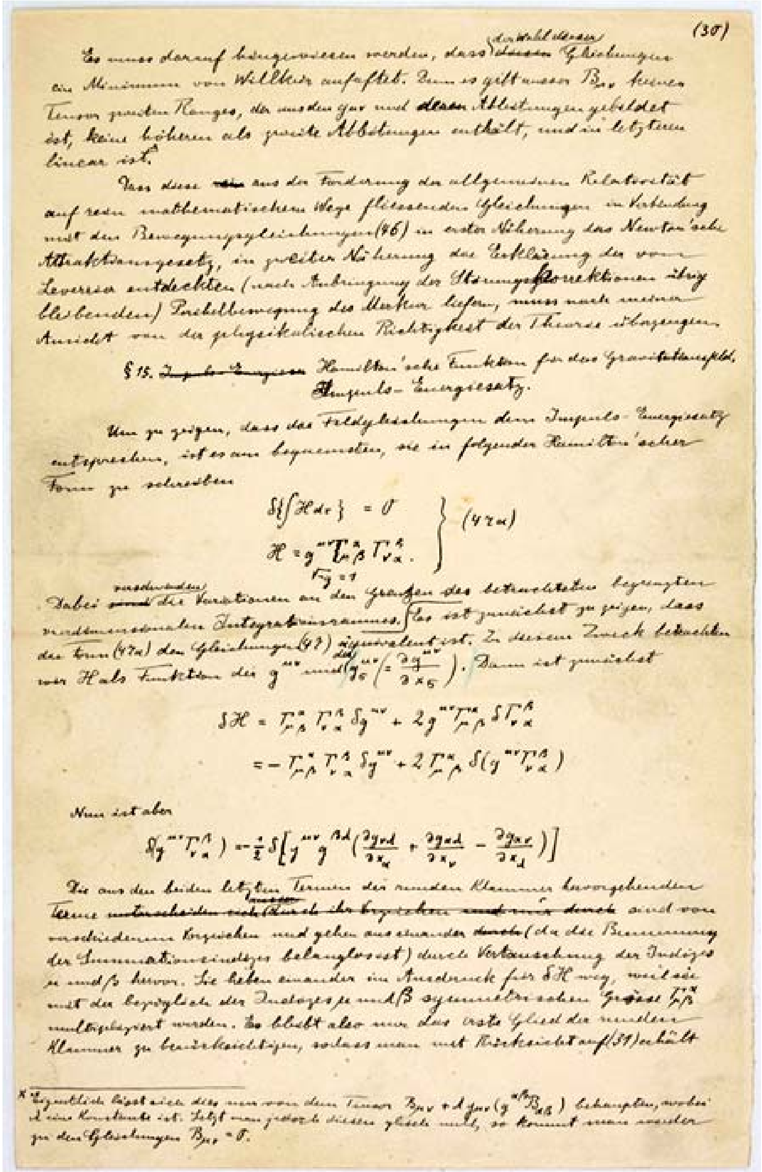
$R_{\mu\nu}$ Ricci tensor , R space curvature

$g_{\mu\nu}$ metric tensor ($ds^2 = g_{\mu\nu} dx^\mu dx^\nu$)

$T_{\mu\nu}$ energy-momentum tensor

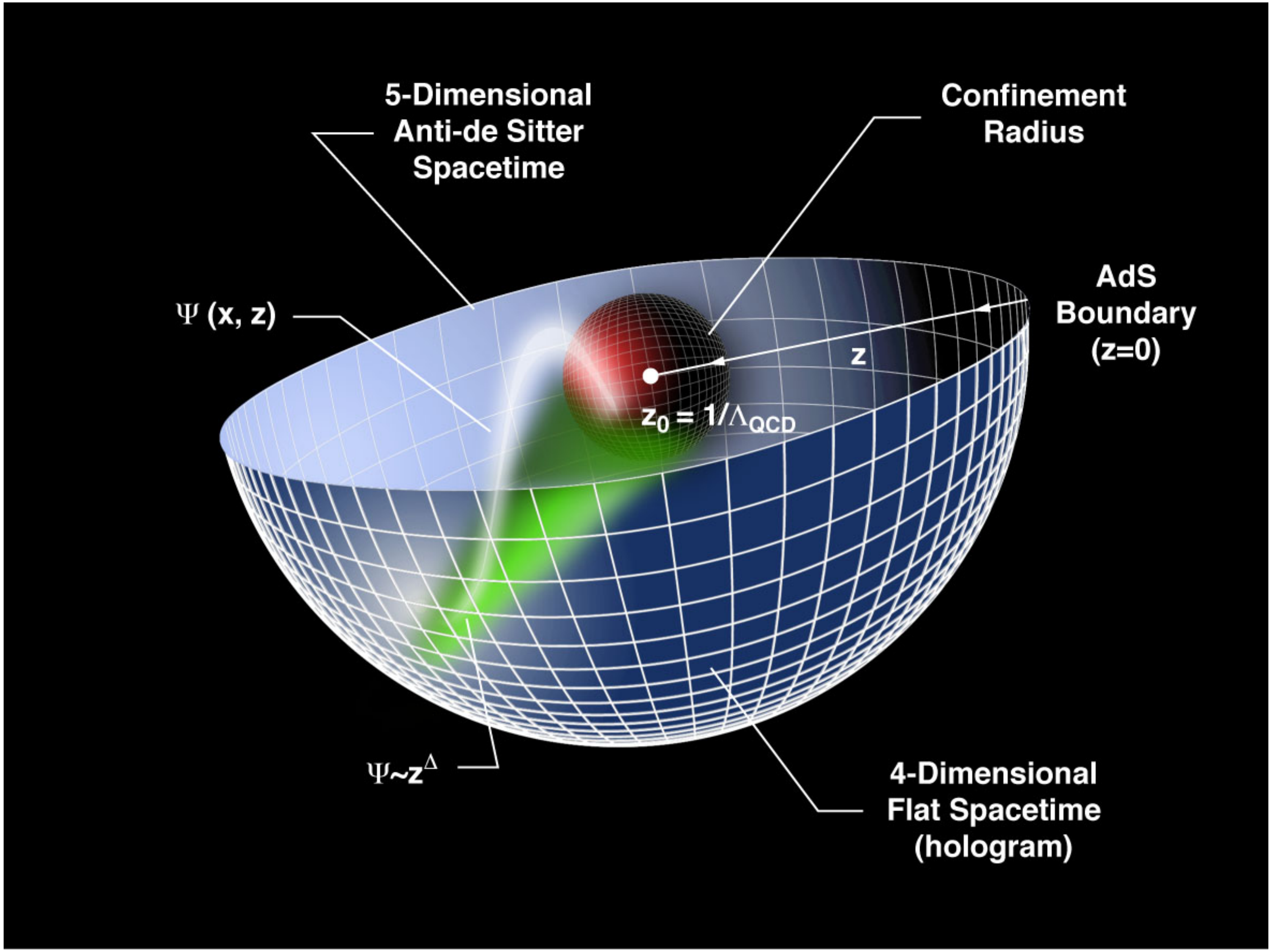
$\kappa = 8\pi G/c^4$,

- Matter curves space and space determines how matter moves !



Annalen der Physik **49** (1916) p. 30

Holographic Correspondence



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II. Gauge Gravity Correspondence and Light-Front QCD

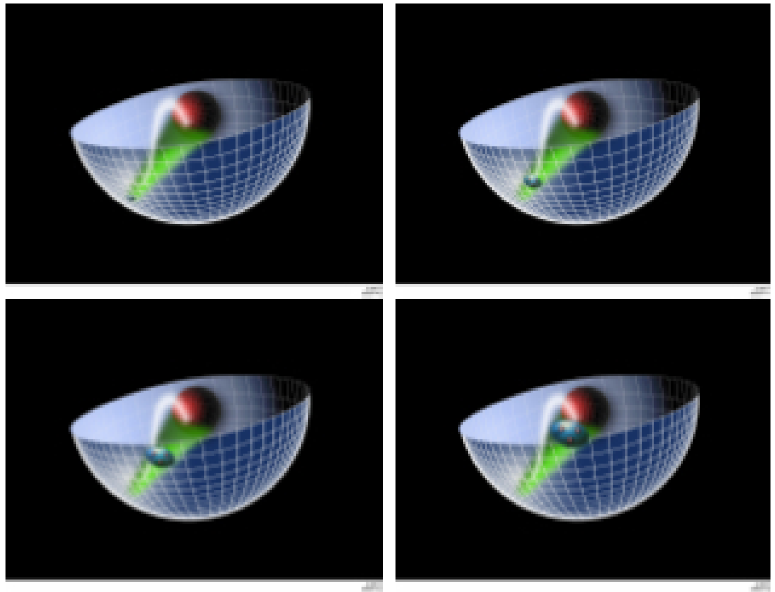
- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in physical spacetime has led to a semiclassical approximation for strongly-coupled QCD, which provides analytical insights into the confining dynamics of QCD
- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...
- Light-front holography provides a remarkable connection between the equations of motion in AdS and the bound-state LF Hamiltonian equation in QCD [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

Isometry group: most general group of transformations which leave invariant the distance between two points

Dim isometry group of AdS$_{d+1}$ is $\frac{(d+1)(d+2)}{2}$

- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\Big(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\Big)$$



- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} \, L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space

- Short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS$_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z

- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

- Nonconformal metric dual to a confining gauge theory

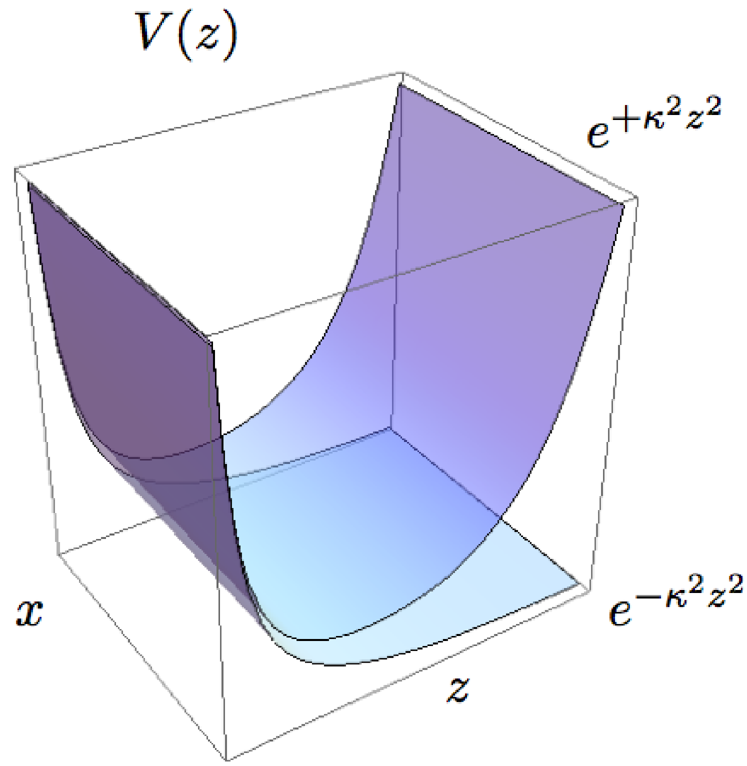
$$ds^2 = \frac{R^2}{z^2}\, e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R\, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Higher Spin Modes in AdS Space

(Frondsal, Fradkin and Vasiliev)

- Lagrangian for scalar field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\quad \left(x^M = (x^\mu, z) \right)$

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left(g^{MN} \partial_M \Phi^* \partial_N \Phi - \mu^2 \Phi^* \Phi \right)$$

- Factor out plane waves along 3+1: $\quad \Phi_P(x^\mu, z) = e^{-iP \cdot x} \Phi(z)$

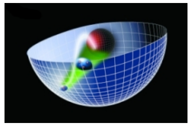
$$\left[-\frac{z^{d-1}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

where $P_\mu P^\mu = \mathcal{M}^2$ invariant mass of physical hadron with four-momentum P_μ

- Define spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 and shifted dimensions $\Phi_J(z) \sim z^{-J} \Phi(z)$

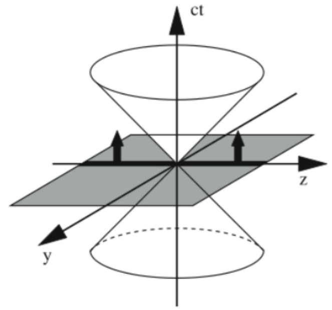
- Find AdS wave equation

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



III. Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



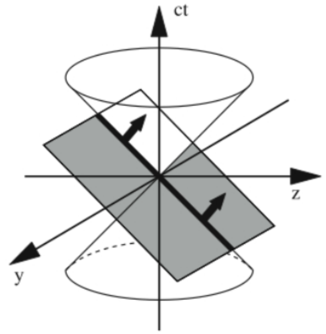
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

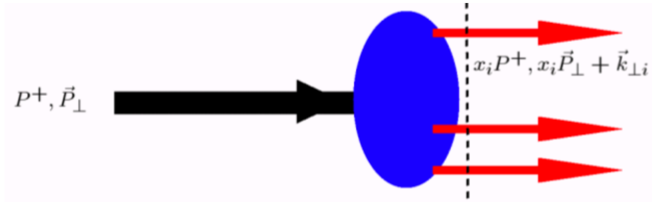
$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Light-Front Fock Representation



- LF Lorentz invariant Hamiltonian equation for the relativistic bound state system

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \qquad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \cdots \}$$

with $k_i^2 = m_i^2, \; k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each constituent i in state n

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+/P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = \mathcal{M}^2\langle\psi(P')|\psi(P)\rangle$$

- Find

$$\mathcal{M}^2 = \sum_n \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right] \sum_\ell \left(\frac{\mathbf{k}_{\perp\ell}^2 + m_\ell^2}{x_q}\right) \left|\psi_n(x_i, \mathbf{k}_{\perp i})\right|^2 + \text{interactions}$$

- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \ldots, k_n) \to \phi_n\Big(\underbrace{(k_1 + k_2 + \cdots + k_n)^2}_{\mathcal{M}_n^2}\Big)$$

with $k_i^2 = m_i^2$ for each constituent

- Functional dependence of Fock state $|n\rangle$ given by invariant mass

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^{n} k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a}$$

Key variable controlling bound state: off-energy shell $\mathcal{E} = \mathcal{M}^2 - \mathcal{M}_n^2$

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

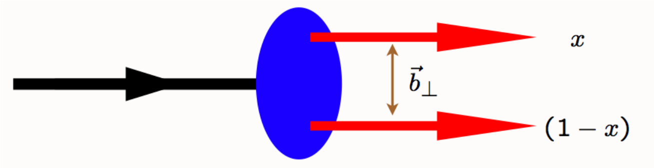
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable conjugate to invariant mass

$$\zeta = \sqrt{\frac{x}{1-x}} \Big| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \Big|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = L^z)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

Light-Front Holographic Mapping $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]
- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=\mathcal{M}^2\Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$

with

$$U(\zeta)=\frac{1}{2}\varphi''(z)+\frac{1}{4}\varphi'(z)^2+\frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

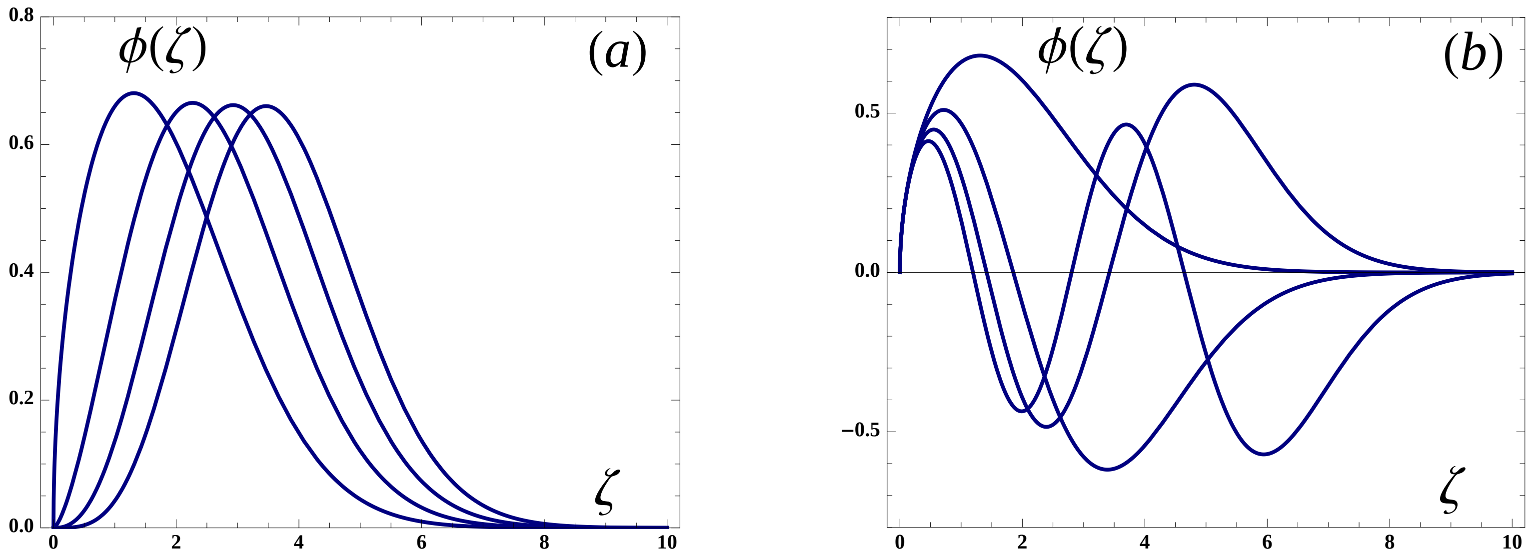
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode Φ_J is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD

- Positive dilaton background $\varphi = \kappa^2 z^2$: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, |\phi(z)^2| = 1$

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

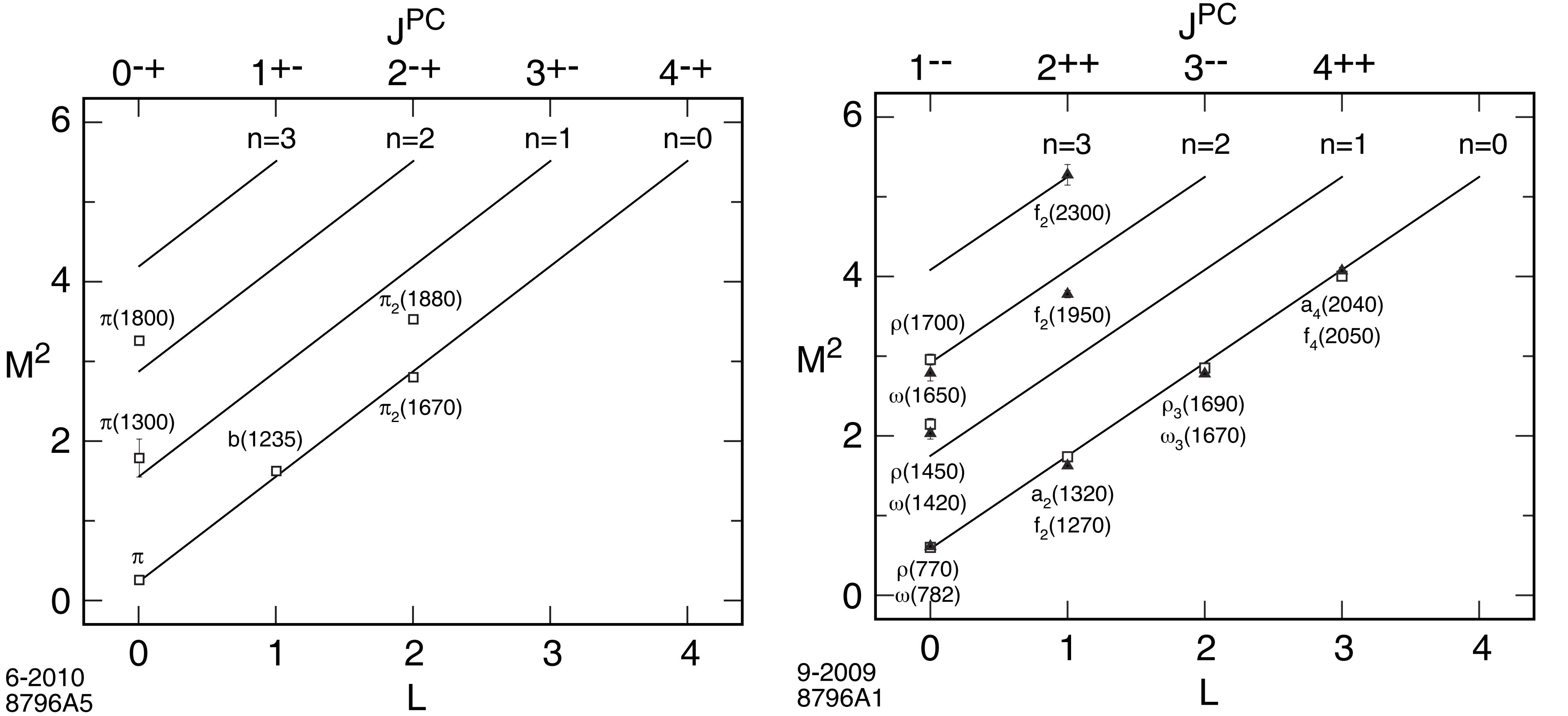
$$\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$$



LFWFs $\phi_{n,L}(\zeta)$ in physical spacetime for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

Light Meson and Baryon Spectrum

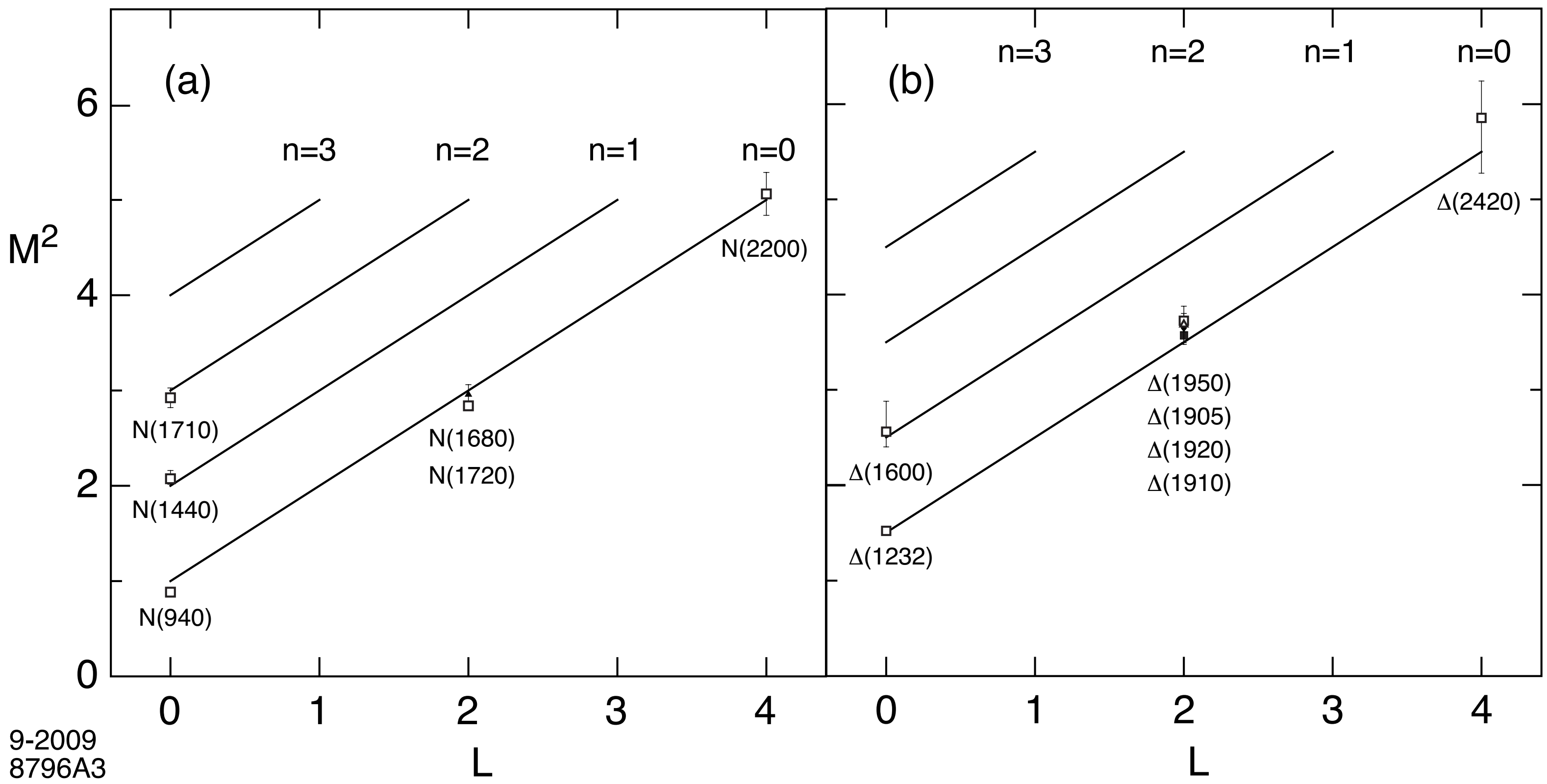
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I=1$ ρ-meson and $I=0$ ω-meson families ($\kappa = 0.54$ GeV)

Same multiplicity of states for mesons and baryons!

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

- Δ spectrum identical to Forkel, Beyer and Frederico and Forkel and Klempt

IV. Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle\psi(P')|J^\mu|\psi(P)\rangle = (P+P')F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q}\gamma^\mu q$

- EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode $\Phi(x,z)$

$$\int d^4x\, dz\, \sqrt{g}\, A^\ell(x,z)\Phi^*_{P'}(x,z)\overleftrightarrow{\partial}_\ell \Phi_P(x,z)$$

- Are the transition amplitudes related ?

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

  [Polchinski and Strassler (2002)]

- Mapping at fixed light-front time: $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} V(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\,\partial_z - z^2 Q^2\right] V(Q,z) = 0$$

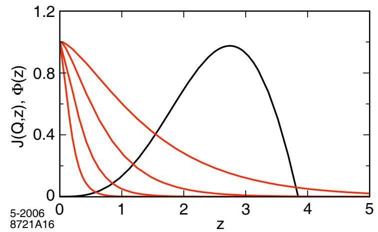
- Solution $\;V(Q,z) = zQK_1(zQ)$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \qquad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q,z)\,\Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

Electromagnetic Form-Factor

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006); PRD **77**, 056007 (2008)]

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\Big) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Consider a two-quark π^+ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q\!=\!0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space

$$F(Q^2) = R^3 \int \frac{dz}{z^3} \, V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} \, J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) \Phi^2_{\pi^+}(z)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} \Phi(\zeta), \qquad \zeta \to z$$

- Same results from mapping of gravitational form factor [S. J. Brodsky and GdT, PRD **78**, 025032 (2008)]

- Expand $X(x)$ in Gegenbauer polynomials (DA evolution equation [Lepage and Brodsky (1980)])

$$X(x) = \sqrt{x(1-x)} = x(1-x) \sum_{n=0}^{\infty} a_n C_n^{3/2}(2x-1)$$

- Normalization

$$\int_0^1 \frac{dx}{x(1-x)} X^2(x) = \sum_n P_n = 1$$

$$\begin{aligned} \langle C_n^\lambda | C_m^\lambda \rangle &= \int_0^1 dx\, x^{\lambda-1/2}(1-x)^{\lambda-1/2} C_n^\lambda(2x-1) C_m^\lambda(2x-1) \\ &= \frac{2^{1-4\lambda}\pi\Gamma(n+2\lambda)}{n!(n+\lambda)\Gamma^2(\lambda)} \end{aligned}$$

- Compute asymptotic probability $P_{n=0}$ $(Q \to \infty)$

$$P_{n=0} = \frac{\pi^2}{32} \simeq 0.3 \qquad \left(a_0 = \frac{3\pi}{4} \right)$$

V. Higher Fock Components

(S. Brodsky and GdT)

- LF Lorentz invariant Hamiltonian equation for the relativistic bound state system

$$P_\mu P^\mu |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle,$$

where $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 \equiv H_{LF}$

- H_{LF} sum of kinetic energy of partons H^0_{LF} plus an interaction H_I, $\;H_{LF} = H^0_{LF} + H_I$
- Expand in Fock eigenstates of H^0_{LF}: $\;|\psi\rangle = \sum_n \psi_n |n\rangle$,

$$\left(\mathcal{M}^2 - \sum_{i=1}^{n} \frac{\mathbf{k}^2_{\perp i} + m^2}{x_i}\right) \psi_n = \sum_m \langle n|V|m\rangle \psi_m$$

an infinite number of coupled integral equations

- Only interaction in AdS/QCD is the confinement potential
- In QFT the resulting LF interaction is the 4-point effective interaction $H_I = \overline{\psi}\psi V(\zeta^2)\overline{\psi}\psi$ wich leads to $qq \to qq$, $q\overline{q} \to q\overline{q}$, $q \to qq\overline{q}$ and $\overline{q} \to \overline{q}q\overline{q}$
- Create Fock states with extra quark-antiquark pairs. No mixing with $q\overline{q}g$ Fock states $(g_s \overline{\psi}\gamma \cdot A\psi)$
- Explain the dominance of quark interchange in large angle elastic scattering

Detailed Structure of Space-and Time Like Pion Form Factor

- Holographic variable for n-parton hadronic bound state

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- Form factor in soft-wall model expressed as $N-1$ product of poles along vector radial trajectory

[Brodsky and GdT (2008)] $\left(\mathcal{M}_\rho{}^2 \to 4\kappa^2(n+1/2)\right)$

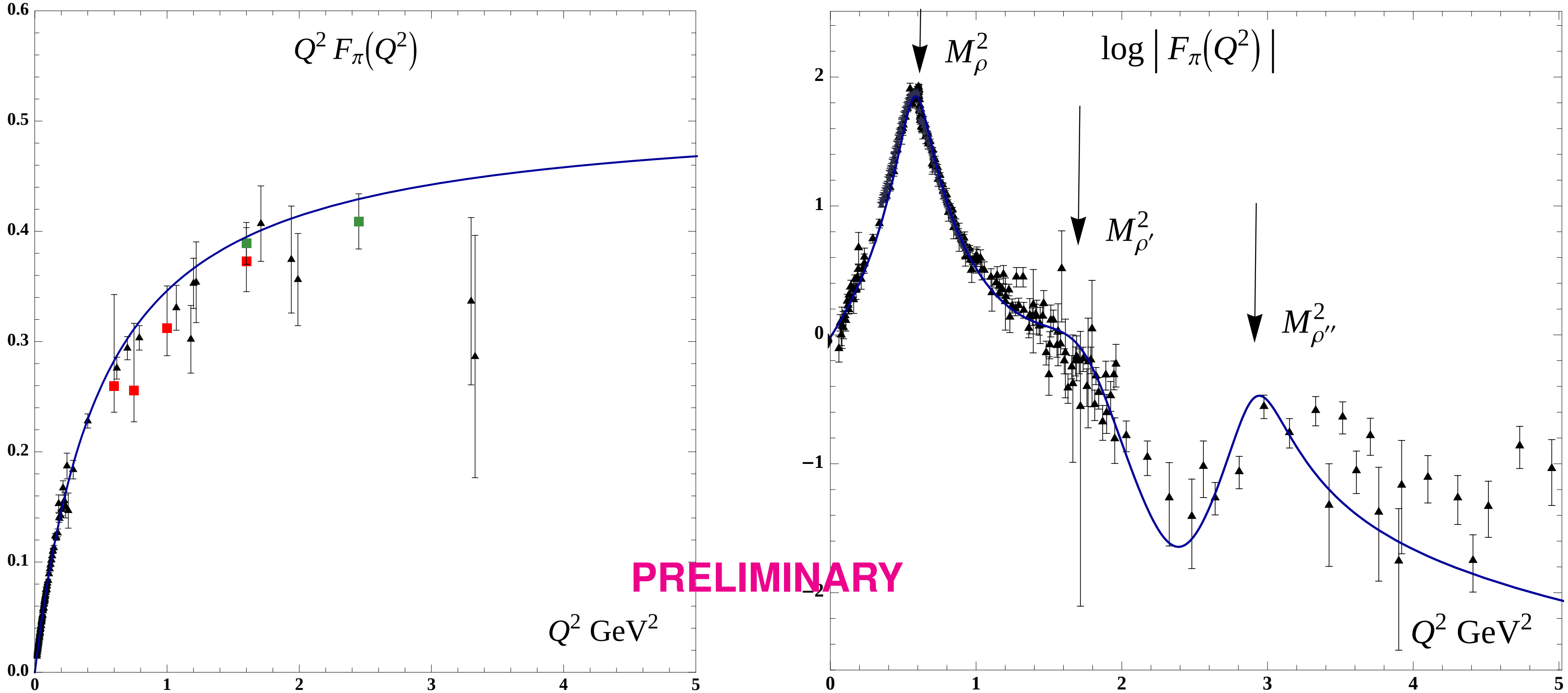
$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho'}}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}$$

- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Expansion of LFWF up to twist 4 (monopole + tripole)

$$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130,\ \Gamma_{\rho'} = 400,\ \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$$



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
M_ρ^2
$\log |F_\pi(Q^2)|$
$M_{\rho'}^2$
$M_{\rho''}^2$
PRELIMINARY
Q^2 GeV^2