Quantum computing for simulating high energy collisions

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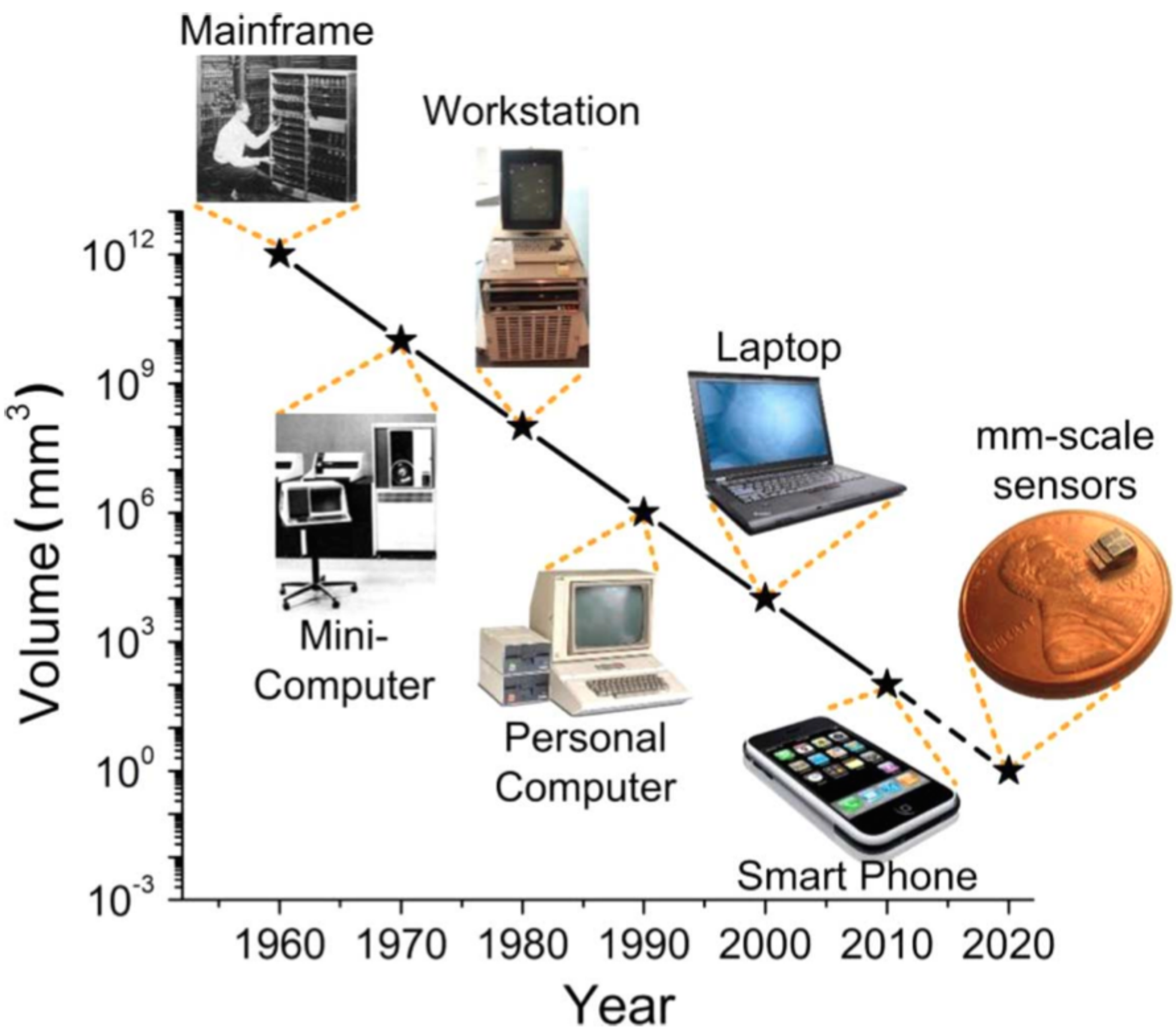
Outline

- Intro to quantum computers
- Review of quantum computers in HEP
- Quantum algorithm for helicity amplitudes
- Quantum algorithm for parton showers
- Future outlook for quantum computers

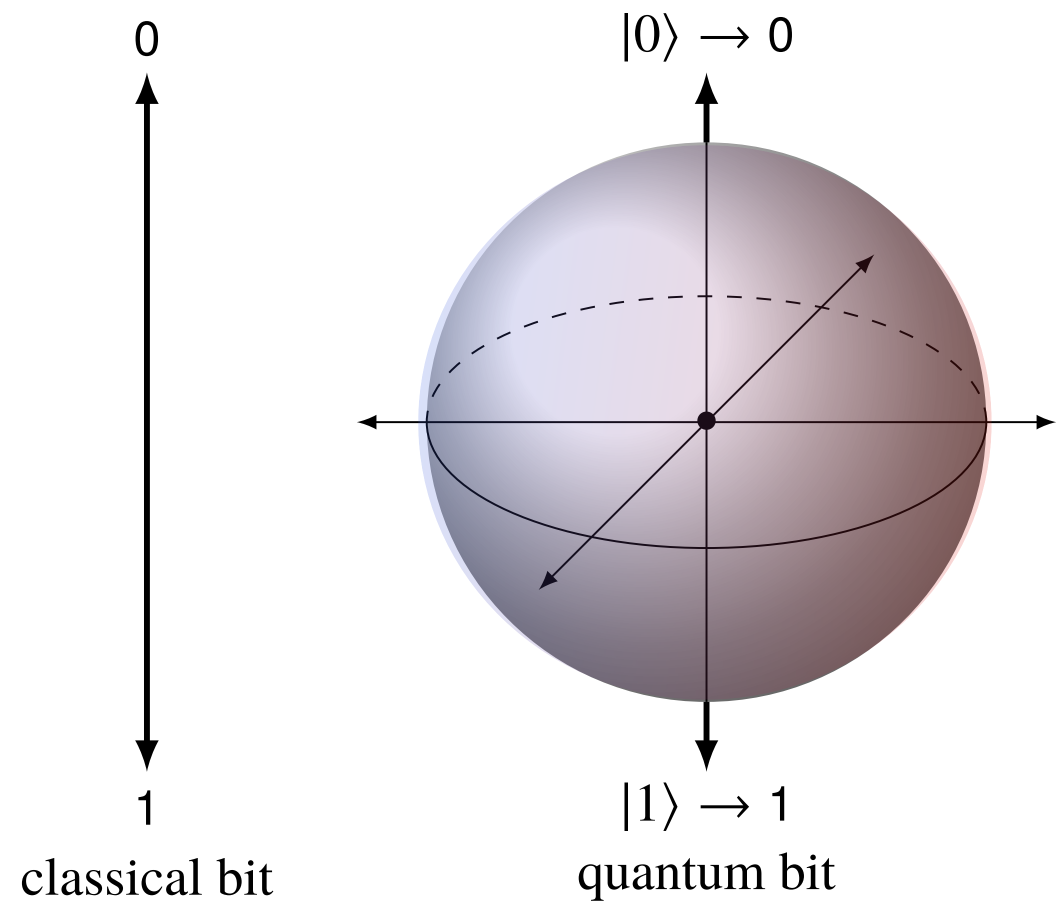
Evolution of classical computer



Classical computers have come a long way since 1950s - size of machines (current size of transistor O(nm)) and complexity of computers

Quantum computing at a similar stage of development as classical computers in 1950s

Bit vs qubit



2-qubit system → 4 basis states $|00\rangle\,|01\rangle\,|10\rangle\,|11\rangle$

N qubits → 2^N dimensional Hilbert space

Power of quantum computing: this exponential increase in size of Hilbert space

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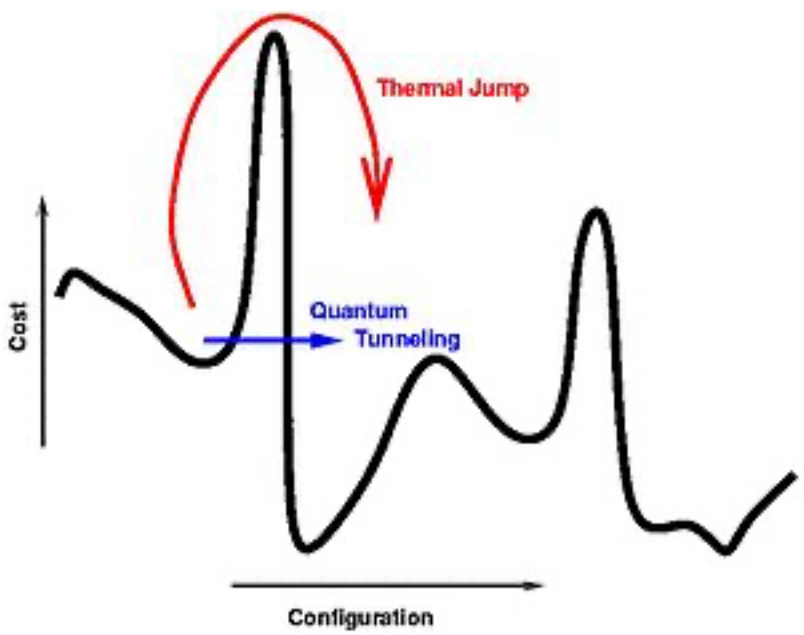
Quantum computing: Two classes/paradigms

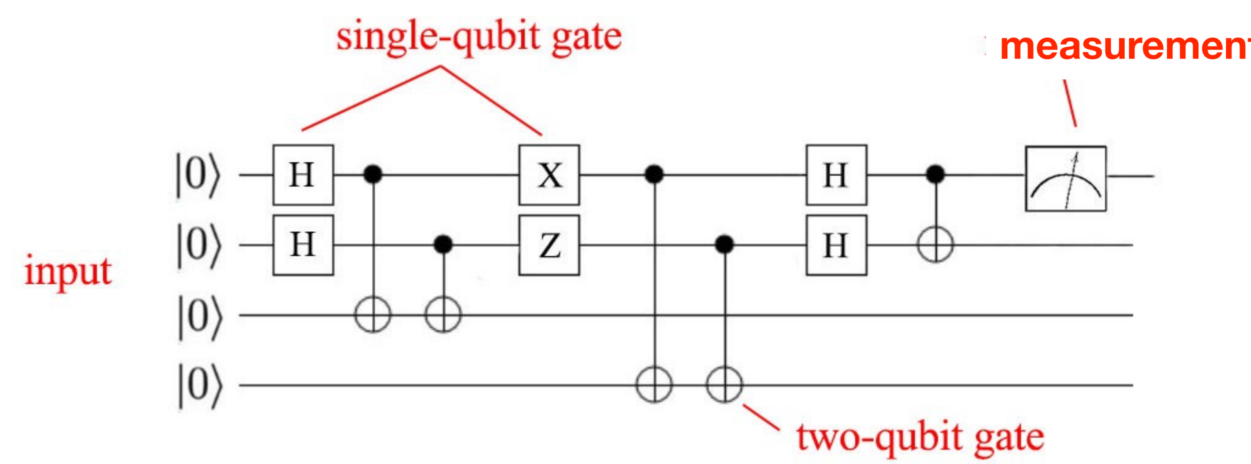
Quantum Annealing



Find ground state of Hamiltonian through continuous-time adiabatic process

- Large number of 'noisy' qubits
- Good for solving specific problems; for instance optimisation, machine learning.
- D-Wave specialises in quantum annealers

Quantum Gate Circuit



Apply unitary transformations to qubits through discrete set of gates

- Small number of qubits but universal quantum computer
- Google, IBM, Microsoft, Rigetti focused on gate-based quantum computing

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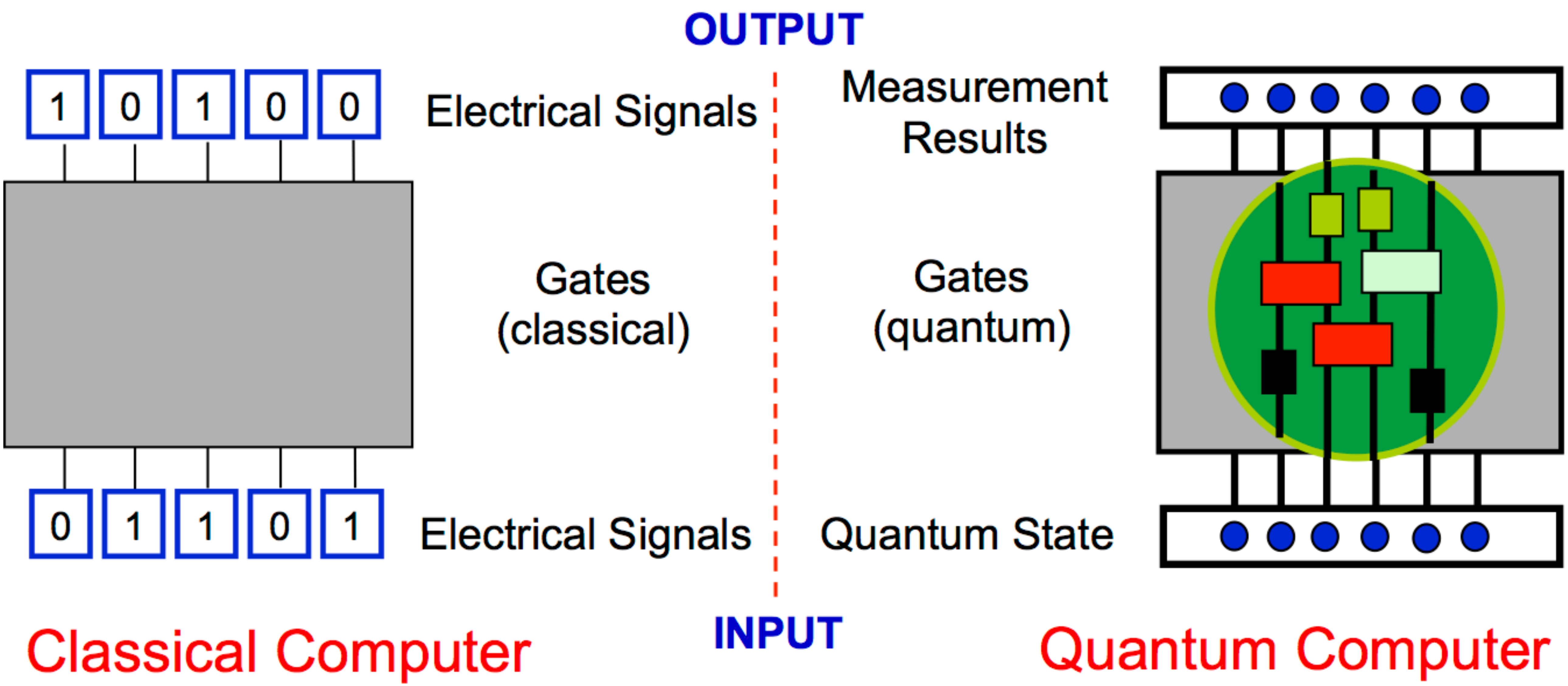
Gate-based quantum computers



Quantum gates: Hadamard

Hadamard gate

- One of the most frequently used and important gates in quantum computing
- Has no classical equivalent.
- It puts a qubit initialised in the $|0\rangle$ or $|1\rangle$ state into a superposition of states.

$$\mathrm{H}|0\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big), \qquad \mathrm{H}|1\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big).$$

Circuit representation

$$-\!\!\!-\boxed{H}-\!\!\!-$$

Matrix representation

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Quantum gates: CNOT and Toffoli

CNOT

- One of the most important gates in QC
- 2-qubit operation that flips the state of a target qubit based on state of a control qubit.
- This is used to create entangled qubits.

$$\mathrm{CNOT}|00\rangle = |00\rangle, \qquad \mathrm{CNOT}|01\rangle = |01\rangle,$$
$$\mathrm{CNOT}|10\rangle = |11\rangle, \qquad \mathrm{CNOT}|11\rangle = |10\rangle.$$

Circuit representation

Matrix representation

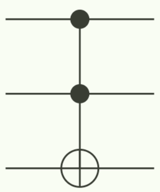
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Toffoli (CCNOT)

- 3-qubit operation, an extension of CNOT gate but on 3 qubits
- Flips the state of a target qubit based on state of the 2 other control qubits

$$\mathrm{CCNOT}|000\rangle = |000\rangle, \qquad \mathrm{CCNOT}|001\rangle = |001\rangle,$$
$$\mathrm{CCNOT}|100\rangle = |100\rangle, \qquad \mathrm{CCNOT}|010\rangle = |010\rangle,$$
$$\mathrm{CCNOT}|110\rangle = |111\rangle, \qquad \mathrm{CCNOT}|111\rangle = |110\rangle.$$

Circuit representation



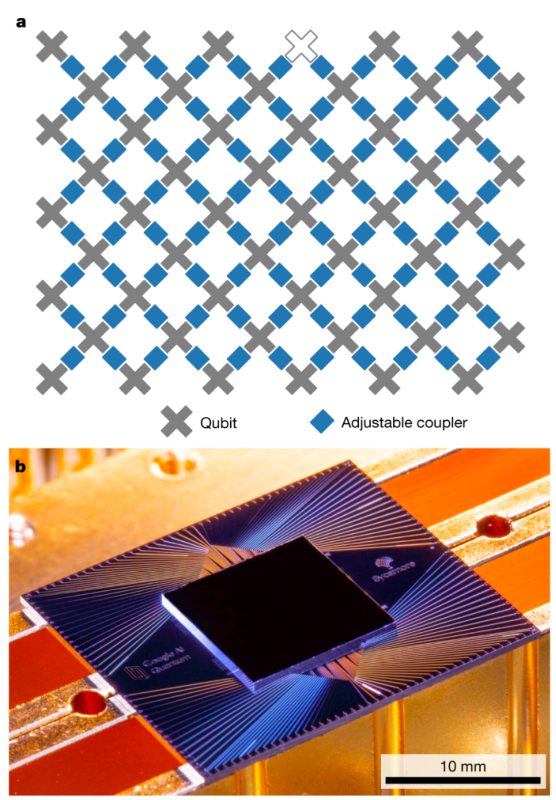
Matrix representation

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Quantum supremacy?

Nature volume 574

- **Google claimed quantum supremacy with 54-qubit quantum computer - performed a random sampling calculation in 3 mins, 20 sec.**
- **They claimed the this would take 10,000 years to do on classical machine.**
- **IBM counterclaim : can be done on classical machine in 2.5 days**



Layout of processor

Sycamore chip

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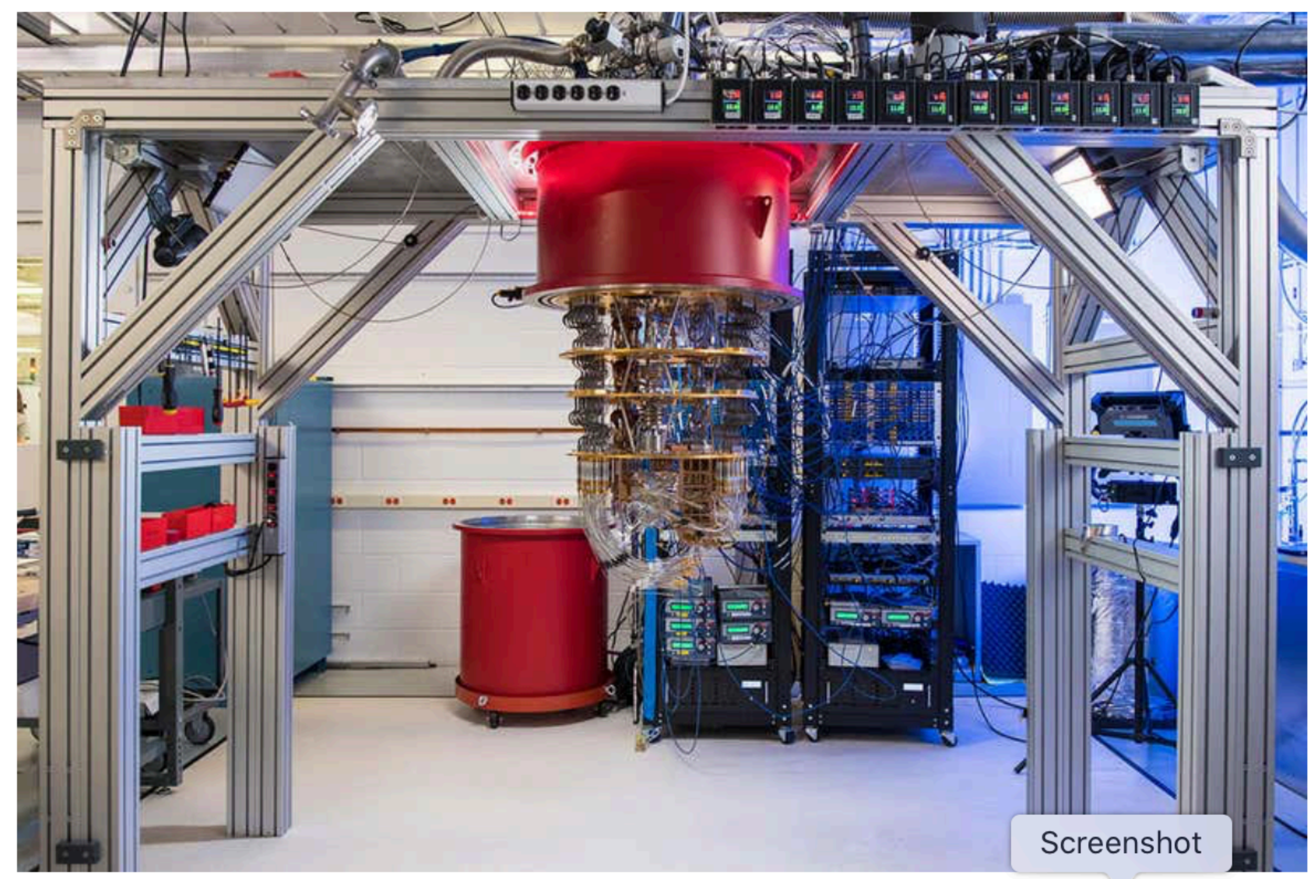
It's official: Google has achieved quantum supremacy

PHYSICS 23 October 2019

By Daniel Cossins



Google's quantum computer is a record-breaker
HANNAH BENET/Google

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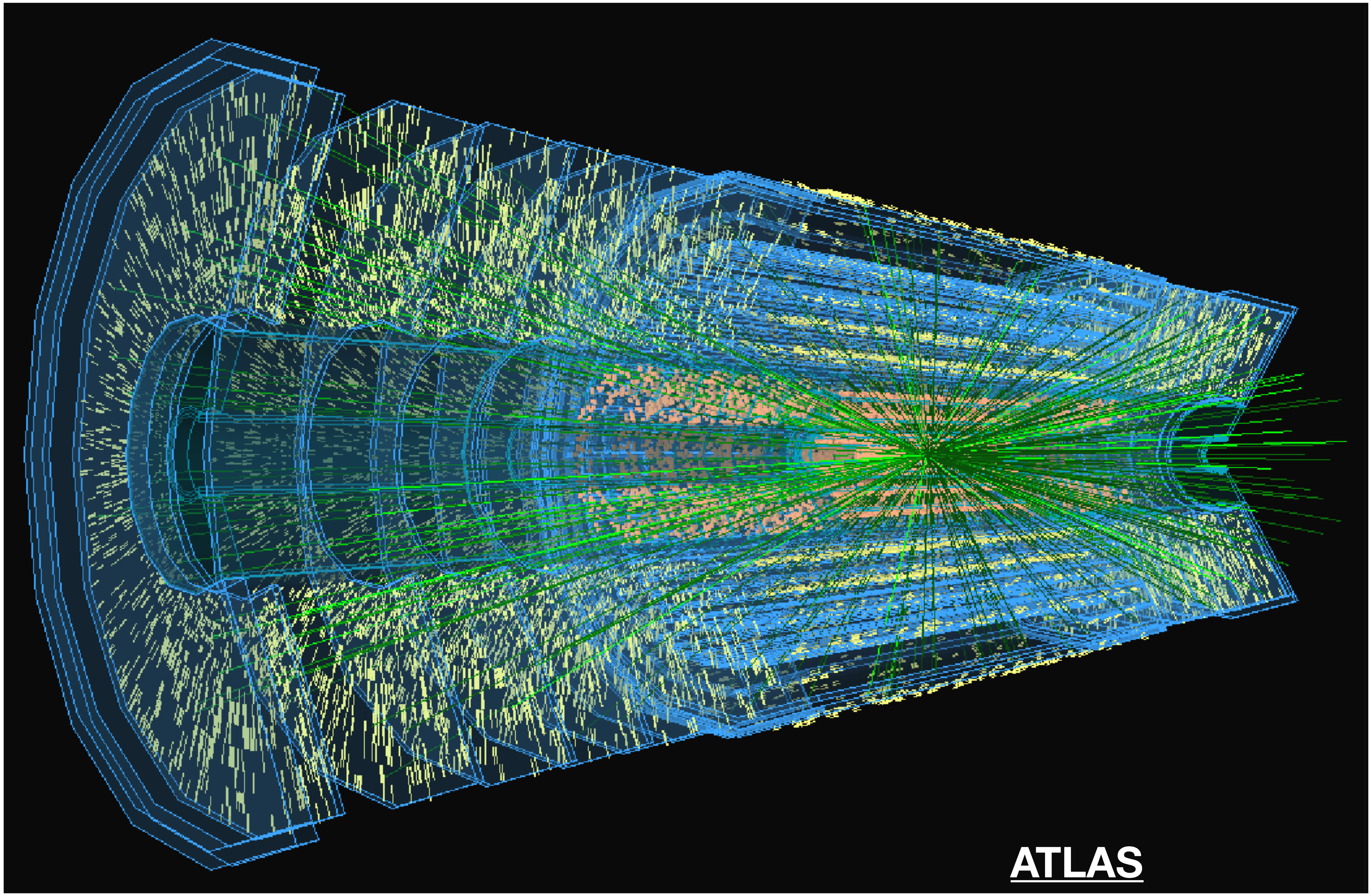
Quantum computing in High Energy Physics

Track reconstruction at HL-LHC

- One of the key challenges at HL-LHC : track reconstruction in a very busy, high pileup environment (140 - 200 overlapping pp collisions)
- Much more CPU and storage needed
- Can quantum computers help?



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Track reconstruction at HL-LHC

arXiv:1902.08324

https://hep-qpr.lbl.gov

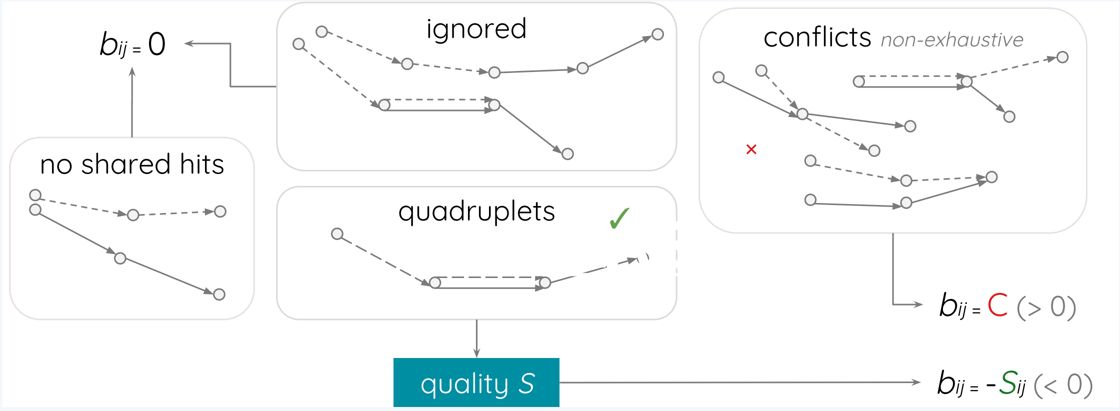
- Express problem of pattern recognition as that of finding the global minimum of an objective function (QUBO)
- Use D-Wave quantum annealer as minimiser (D-Wave 2X (1152 qubits))
- Use triplets (set of 3 hits); which triplets belong to the trajectory of a charged particle.

Minimise function O : equivalent to finding the ground state of the Hamiltonian

$$O(a,b,T) = \sum_{i=1}^{N} a_i T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j \quad T_i, T_j \in \{0,1\}$$

weights quality of individual triplets based on physics properties

encodes relationship between triplets



Minimising O = selecting the best triplets to form track candidates.

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Track reconstruction at HL-LHC

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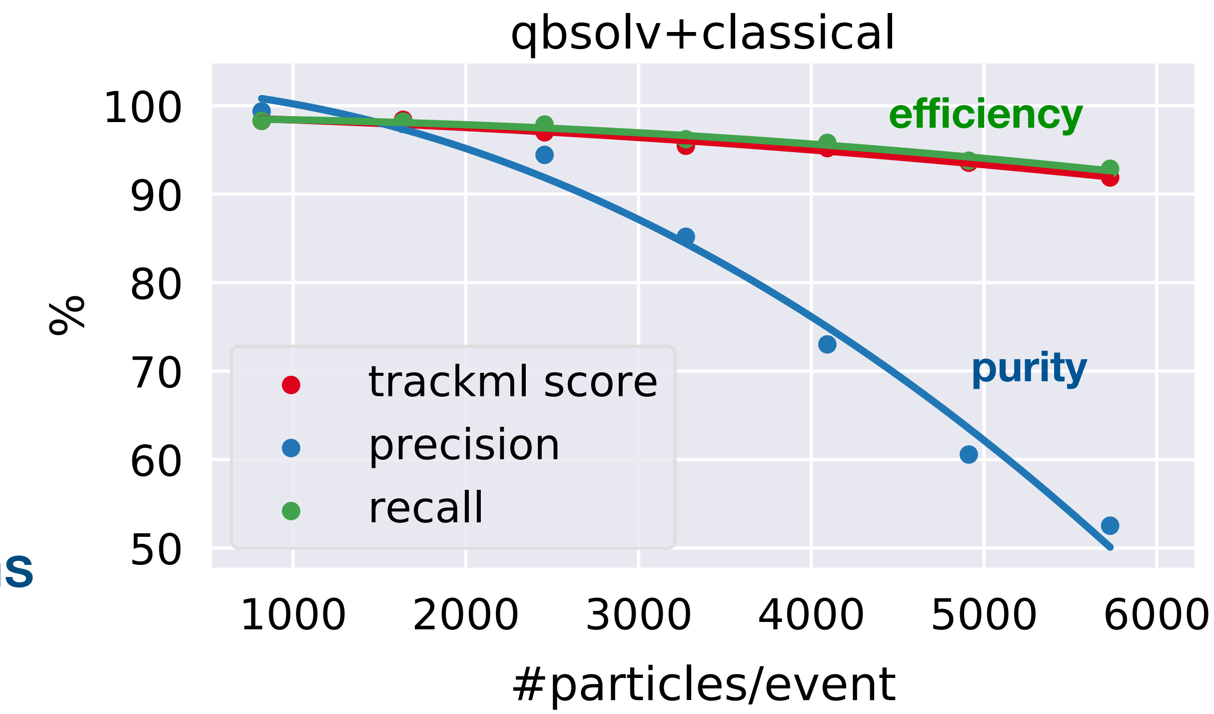
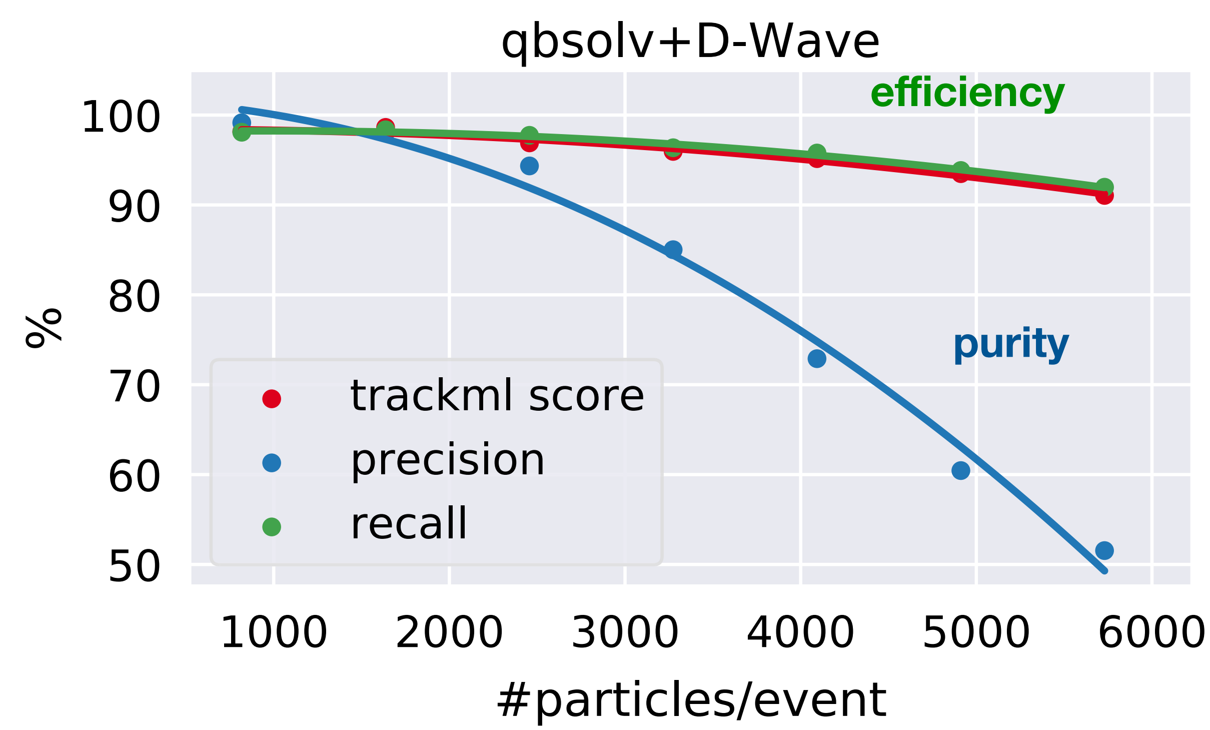
- Use dataset representative of HL-LHC
- Study performance of algorithm as a function of particle multiplicity
- Similar purity and efficiency as current algorithms
- Execution time of algorithm not expected to scale with track multiplicity



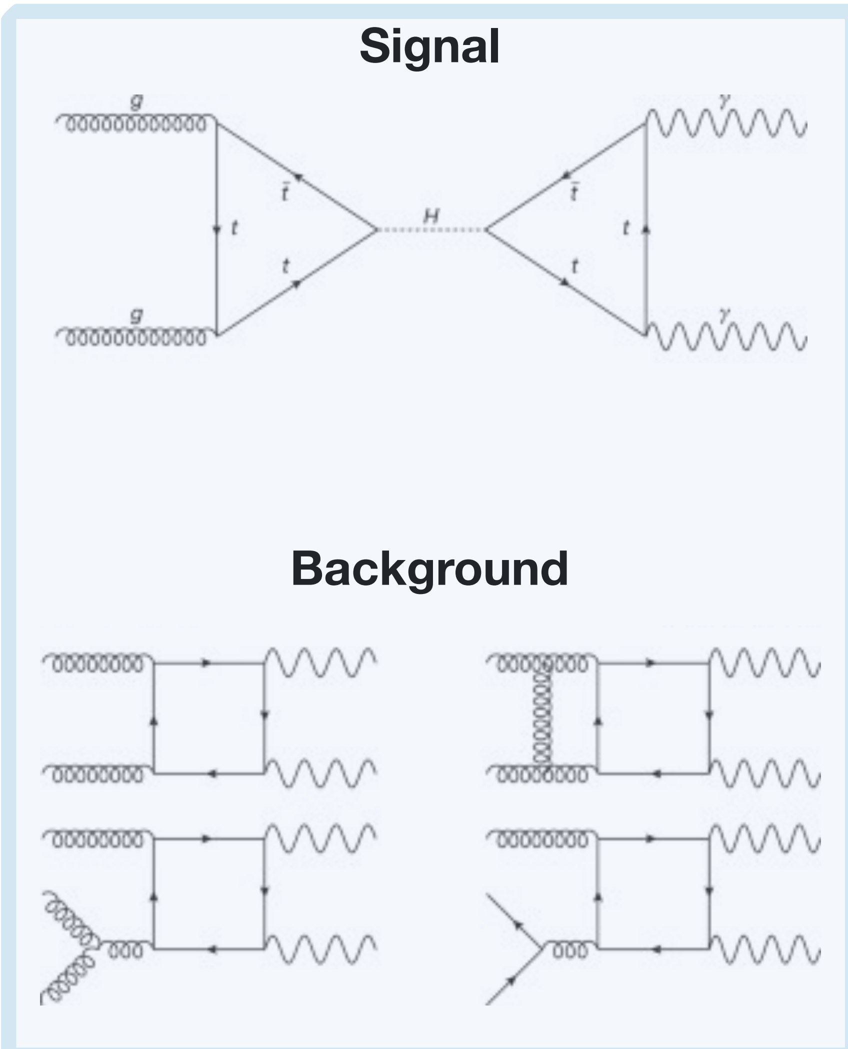


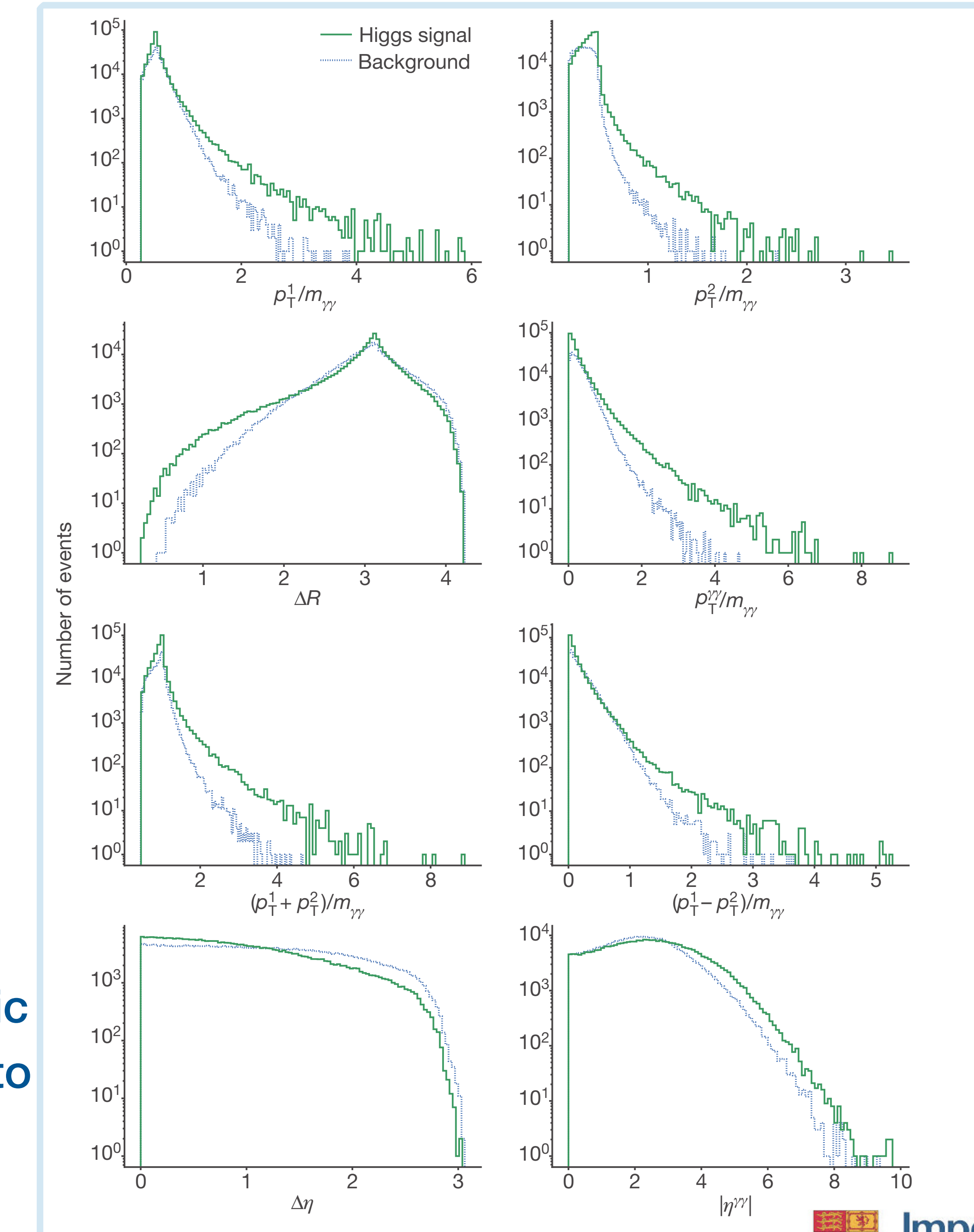
Overall timing still needs to be measured and studied, but physics performance of tracking algorithm similar to classical

Higgs optimisation using D-Wave

Nature volume 550: 375–379(2017)

- Precise measurement of Higgs boson properties requires selecting large and high purity sample of signal events over a large background
- Use quantum and classical annealing to solve a Higgs signal over background machine learning optimisation problem
- Map the optimization problem to that of finding the ground state of a corresponding Ising spin model.



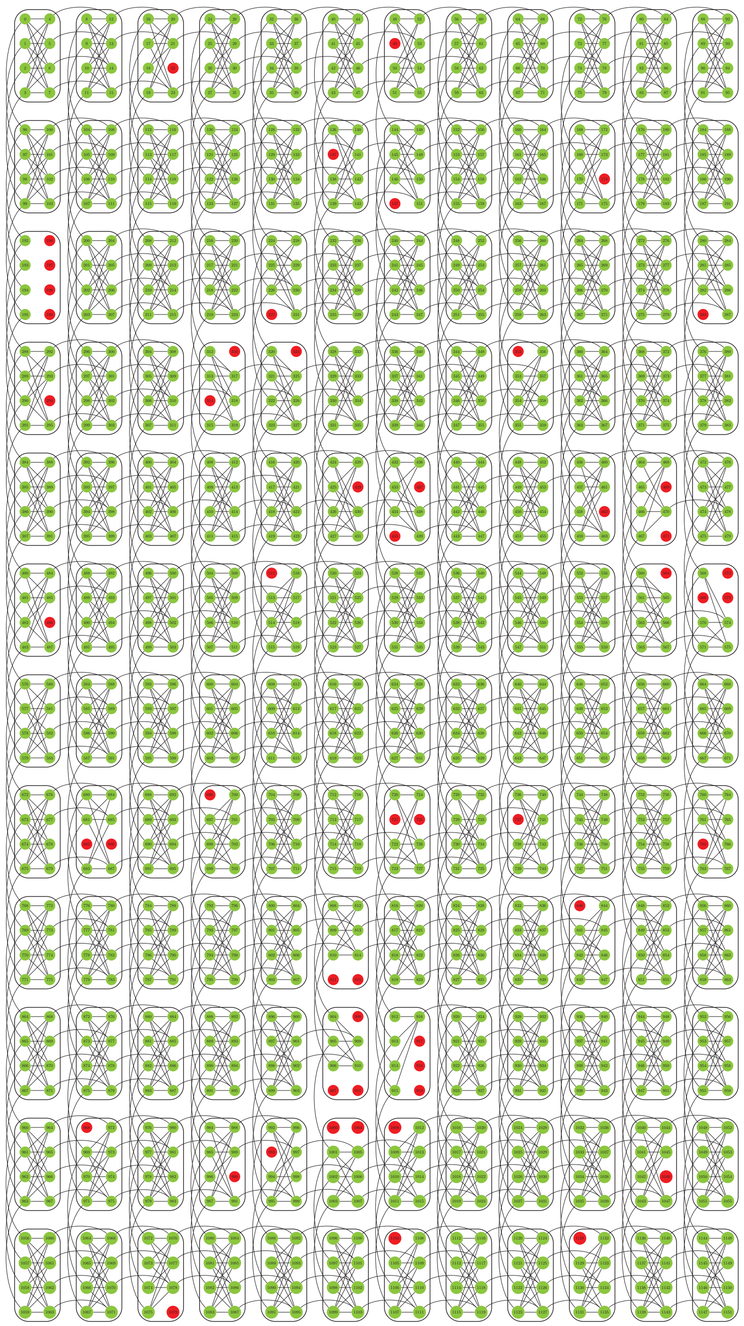


Build a set of weak classifiers from kinematic observables of a $H \to \gamma\gamma$ decay, use these to construct a strong classifier

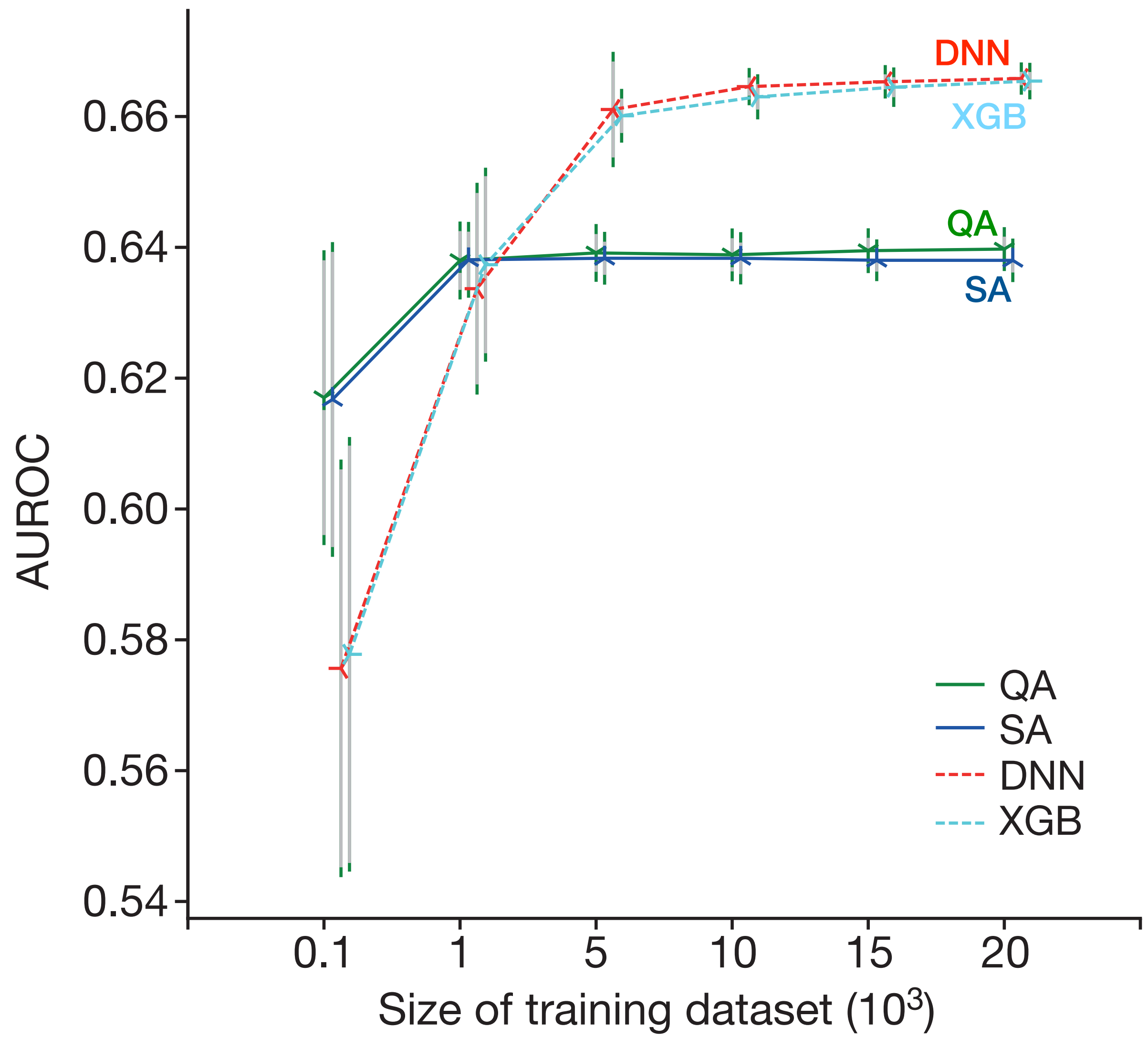
Higgs optimisation using D-Wave

[Nature](#) volume 550: 375–379(2017)

Map a signal vs background optimization problem to that of finding the ground state of a corresponding Ising spin model.



1098 active qubits



Comparable performance to current state of the art machine learning methods, with some advantage for small training datasets

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First application of D-Wave quantum annealing to a scenario in HEP

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arXiv:2010.00046

Quantum algorithm for helicity amplitudes and parton showers

in collaboration with Simon Williams[1],
Khadeeja Bepari[2] and Michael Spannowsky[2]

[1]Imperial College London
[2]IPPP

Collision event at LHC

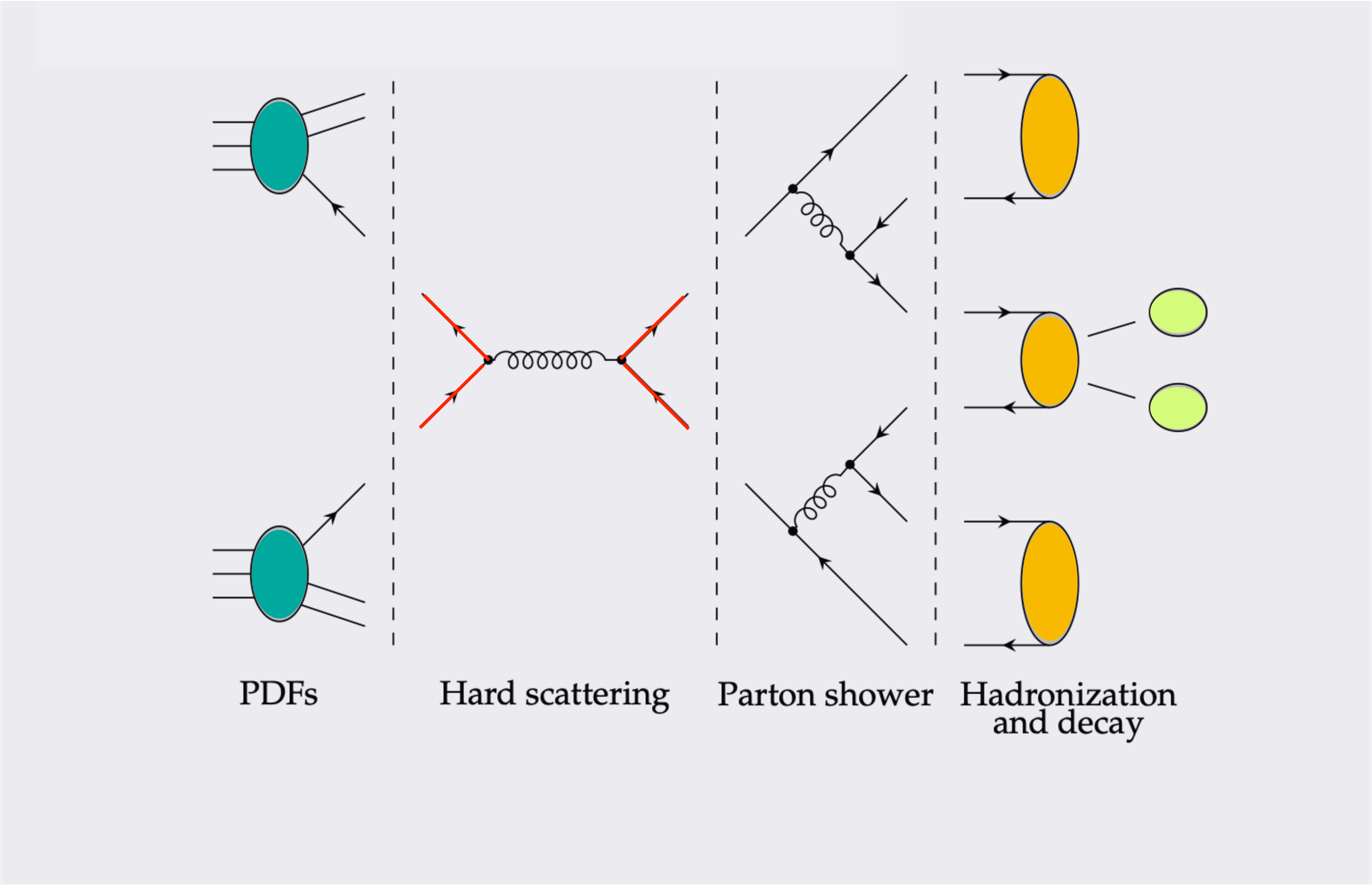
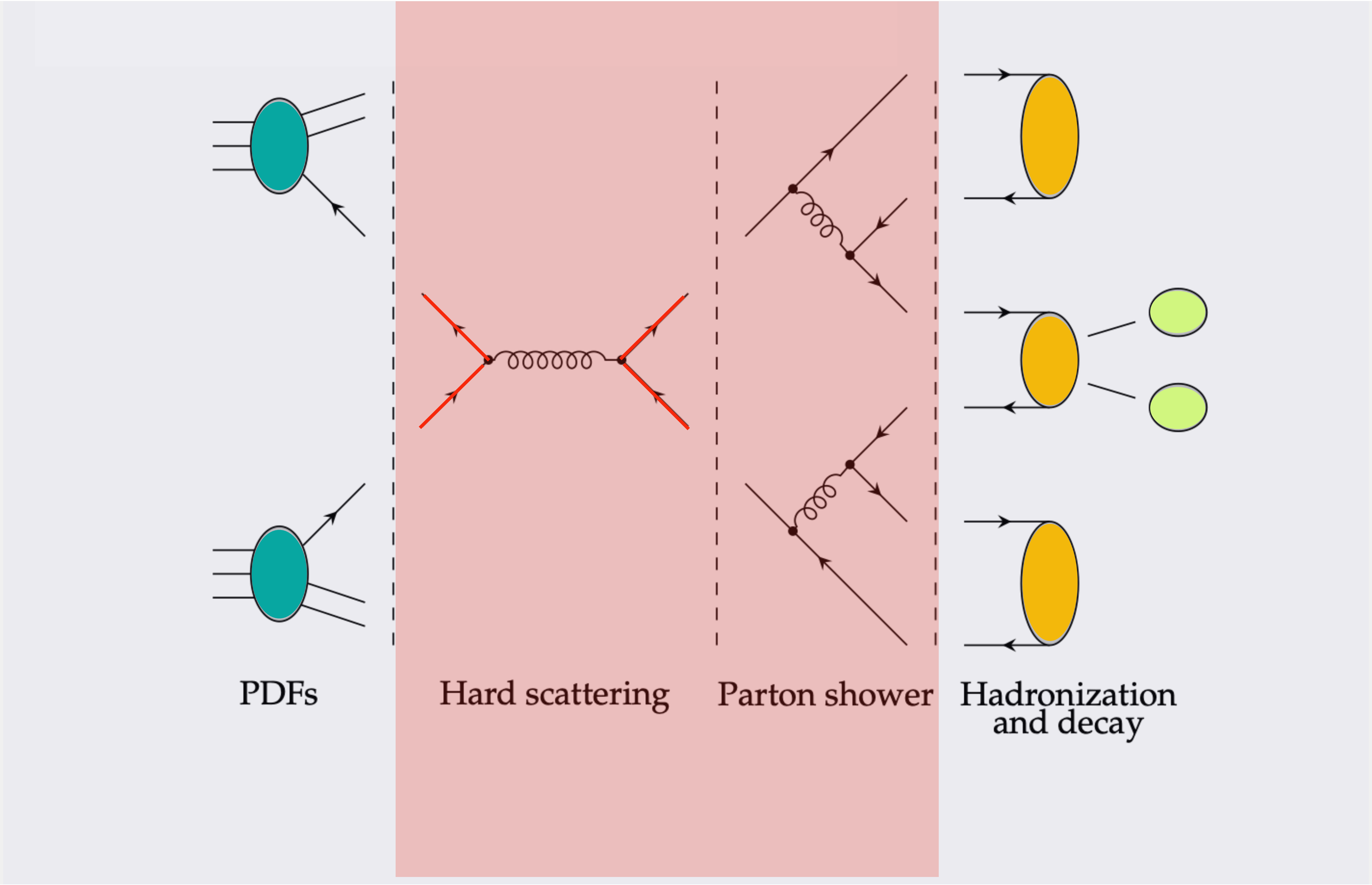


*Diagram taken from Pierpaolo Mastrolia lecture

Collision event at LHC



- Hard interaction + parton shower : can be described perturbatively + independent of non-perturbative processes.
- Most time consuming stages of event simulation

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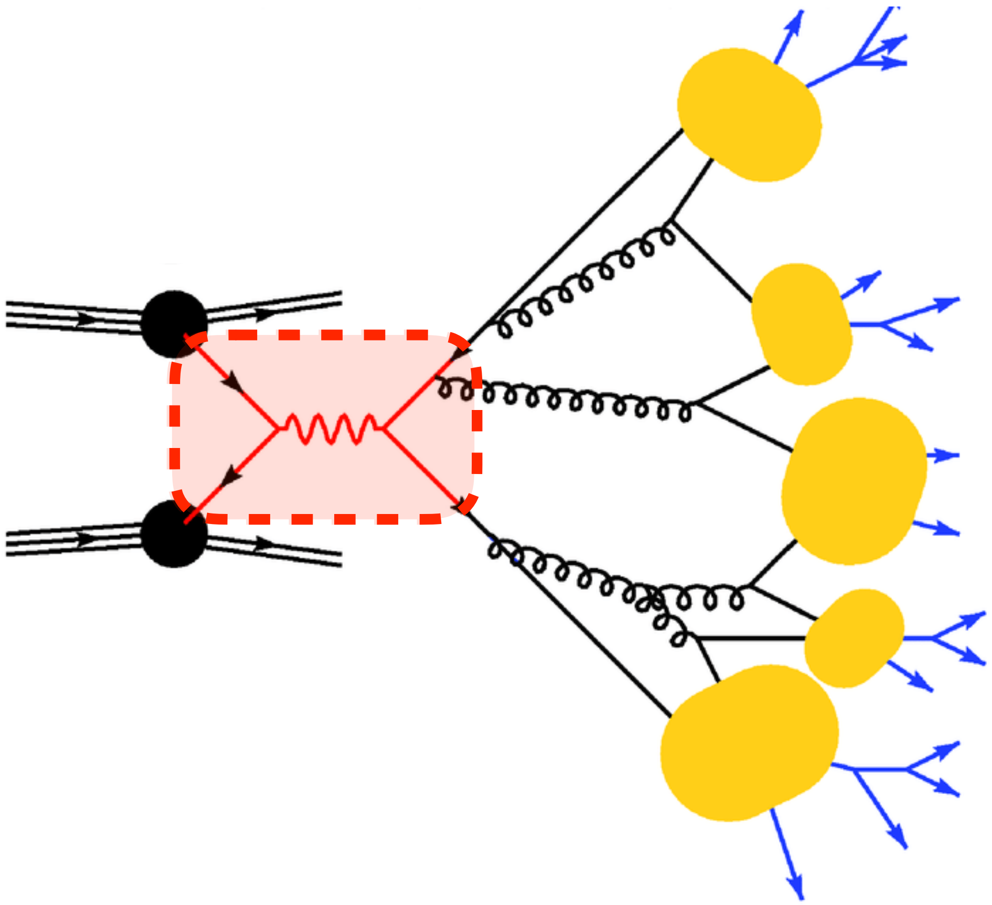
Scattering amplitudes

- Scattering amplitudes - essential for calculating predictions for collider experiments.
- At LHC, collisions dominated by QCD processes, which carry large theoretical uncertainty due to limited knowledge of higher order terms in perturbative QCD
- Improving accuracy of theoretical predictions of cross-sections means computing loop amplitudes and tree level amplitudes of higher multiplicities.



- Conventional method of computing an unpolarised cross section involves squaring the amplitude at the beginning and then summing analytically over all possible helicity states using trace techniques
- For complex processes, this approach is not very feasible. For N feynman diagrams for an amplitude, there are N^2 terms in the square of the amplitude

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Spinor helicity formalism

- Tool for calculating scattering amplitudes much more efficiently than conventional approach. Greatly simplifies the calculation of scattering amplitudes for complex processes.

Compute amplitudes of fixed helicity setup which has the advantage:

  - For massless particles, chirality and helicity coincide. Chirality is preserved by gauge interactions, hence helicity is also conserved. Helicity basis an optimal one for massless fermions.

  - Different helicity configurations do not interfere. Full amplitude obtained by summing the squares of all possible helicity amplitudes. $\sum_{\text{helicity}} |M_n|^2,$

- Using recursion relations such as BCFW, it is possible to calculate multi-gluon scattering amplitudes which would be prohibitive using traditional methods

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Equivalence between spinors and qubits

Helicity amplitude calculations based on manipulation of helicity **spinors**

Helicity spinors for massless states can be expressed as :

$$|p\rangle^{\dot{a}} = \sqrt{2E}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}$$

Qubits can be represented on a Bloch sphere as a linear superposition of orthonormal basis states $|0\rangle$ and $|1\rangle$ as:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}$$

Equivalence between spinors and qubits

$$|p\rangle^{\dot{a}} = \sqrt{2E}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix} \longrightarrow |\psi\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}$$

Spinors naturally live in the same representation space as qubits, thus helicity spinors can be represented as qubits

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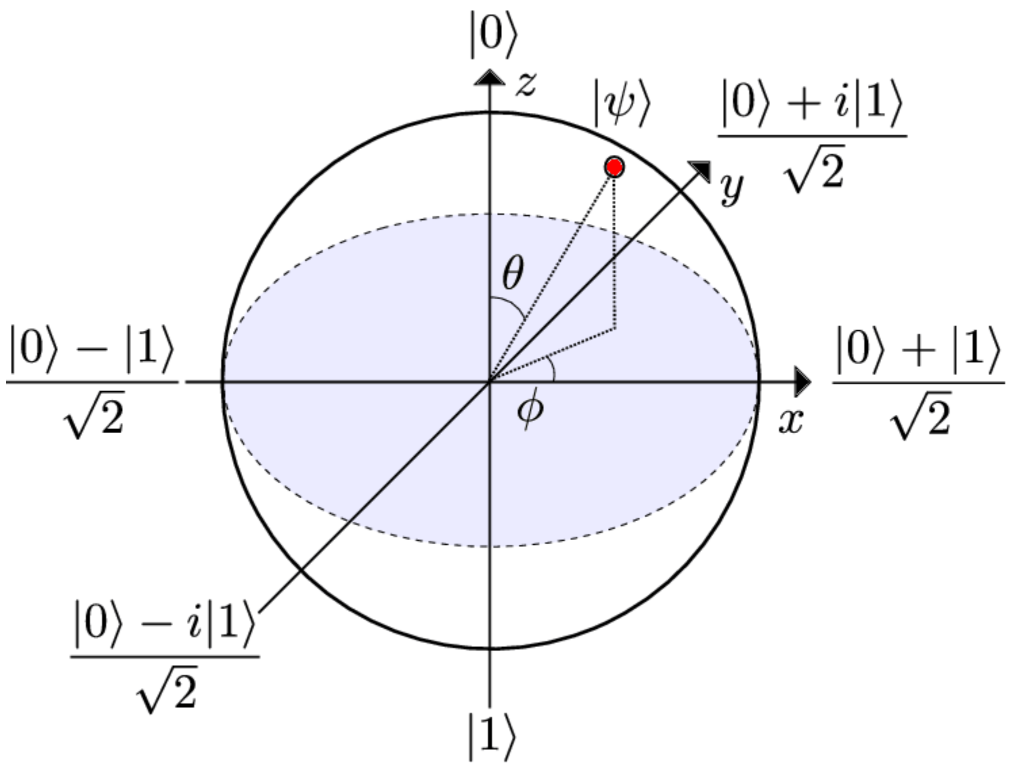
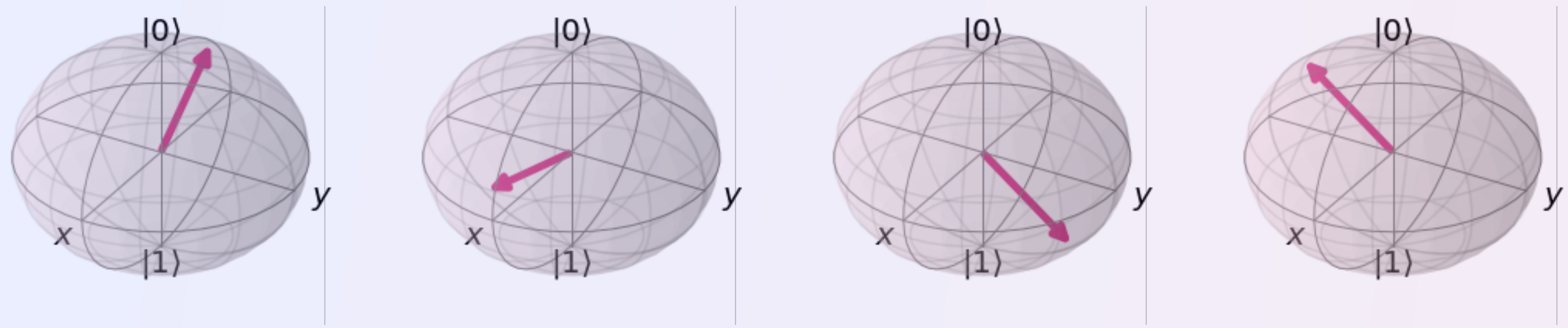
Equivalence between spinors and qubits

Calculation of helicity amplitudes follows same structure as a quantum computing algorithm; quantum operators act on an initial state to transform it into a state that can be measured

- Encode operators acting on spinors as a series of unitary transformations in the quantum circuit
- These unitary operations are applied to qubits to calculate helicity amplitude





(a) $|p\rangle^{\dot{a}}$ (b) $|p]_a$ (c) $(\langle p|_{\dot{a}})^{\mathrm{T}}$ (d) $([p|^{a})^{\mathrm{T}}$

Visualisation of helicity spinors

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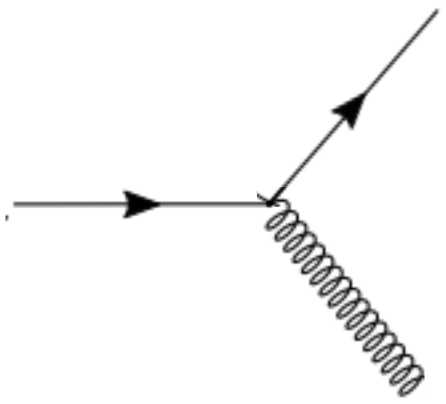
1→2 helicity amplitude calculation

A simple application of the helicity amplitude approach is the calculation of a 1→2 process

$$\mathcal{M}_{gq\bar{q}} = \langle p_f|\bar{\sigma}_\mu|p_{\overline{f}}]\epsilon_\pm^\mu,$$



- Gluon polarisation vectors given by :

$$\epsilon_+^\mu = -\frac{\langle q|\bar{\sigma}^\mu|p]}{\sqrt{2}\langle qp\rangle}, \qquad \epsilon_-^\mu = -\frac{\langle p|\bar{\sigma}^\mu|q]}{\sqrt{2}[qp]}.$$

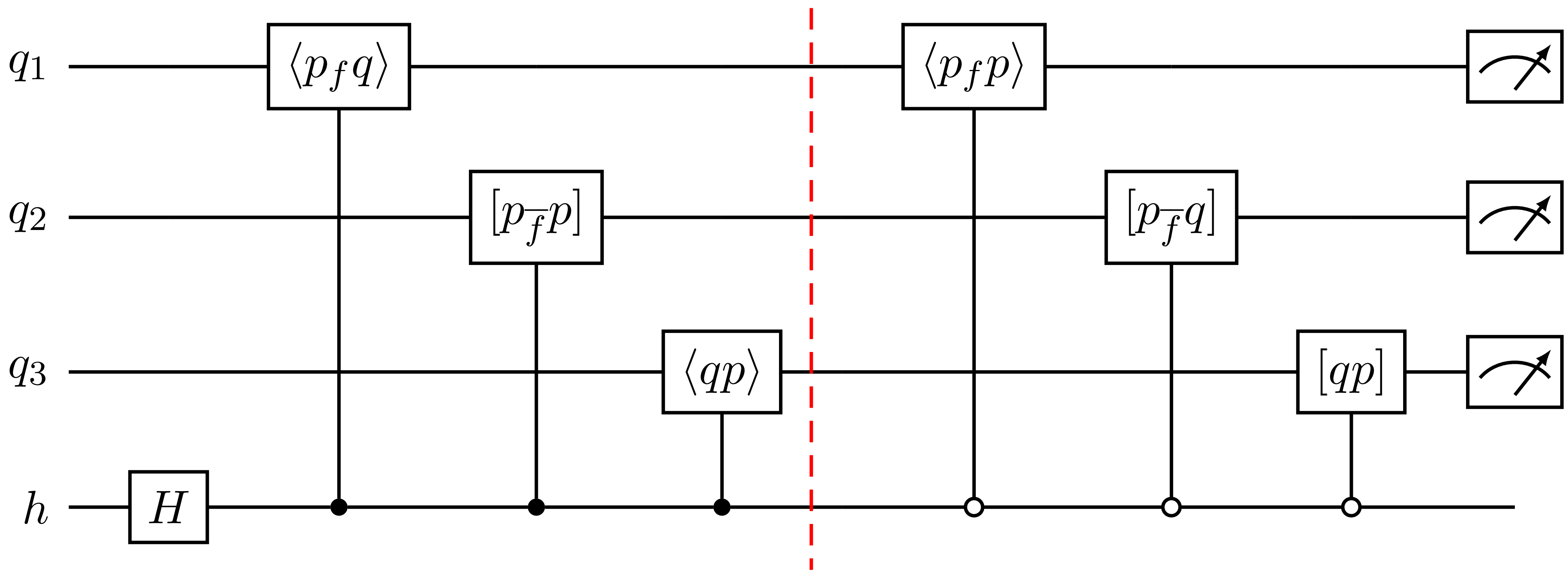
- Can create circuit where each 4-vector calculated individually on 4 qubits - but this will require many qubits and large circuit depth.

- Instead, simplify amplitude using Fierz identity (hence reduce qubits from 10 → 4)

$$\boxed{\mathcal{M}_+ = -\sqrt{2}\frac{\langle p_f q\rangle[p_{\overline{f}}p]}{\langle qp\rangle}, \qquad \mathcal{M}_- = -\sqrt{2}\frac{\langle p_f p\rangle[p_{\overline{f}}q]}{[qp]}.}$$

1→2 helicity amplitude circuit

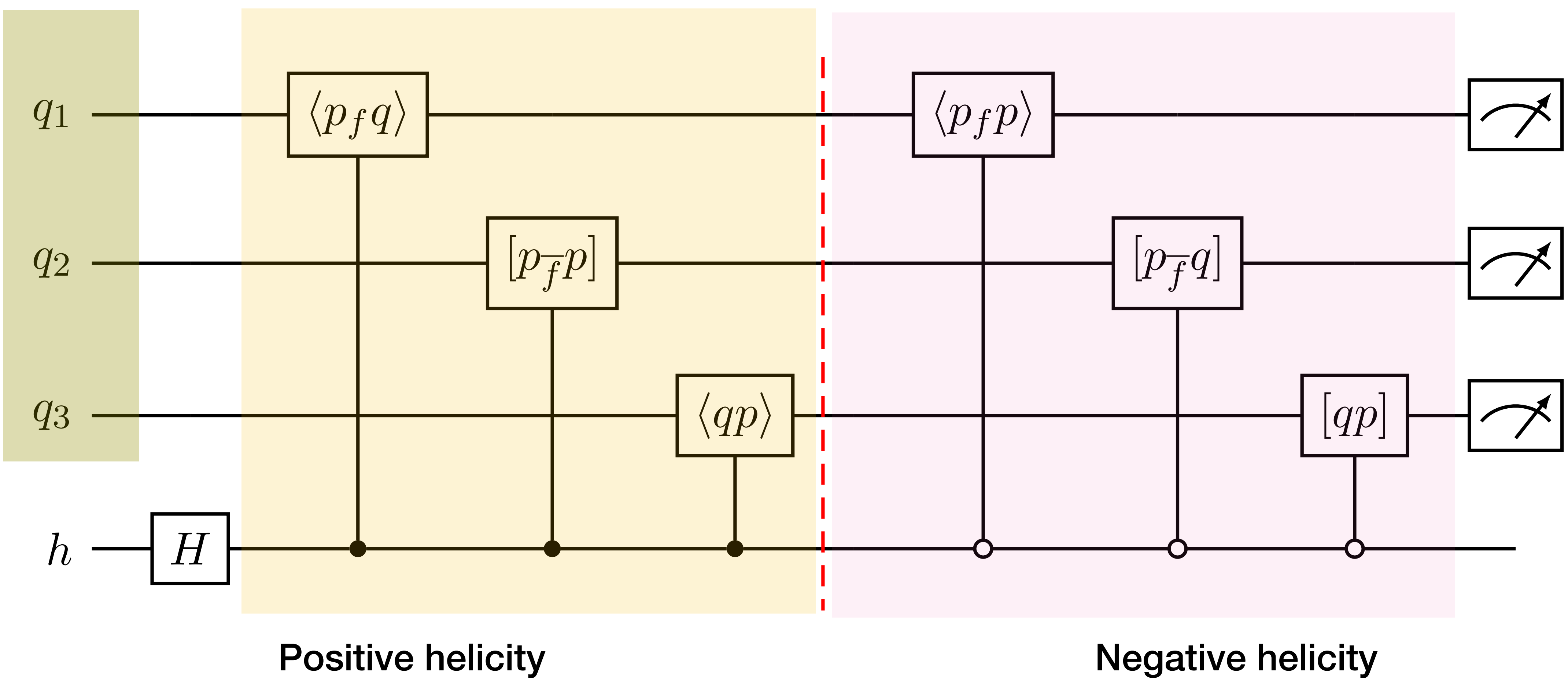
$$\mathcal{M}_+ = -\sqrt{2}\frac{\langle p_f q\rangle [p_{\bar{f}} p]}{\langle qp\rangle}, \qquad \mathcal{M}_- = -\sqrt{2}\frac{\langle p_f p\rangle [p_{\bar{f}} q]}{[qp]}.$$



1→2 helicity amplitude circuit

$$\boxed{\mathcal{M}_+ = -\sqrt{2}\frac{\langle p_f q\rangle [p_{\bar{f}} p]}{\langle qp\rangle}, \qquad \mathcal{M}_- = -\sqrt{2}\frac{\langle p_f p\rangle [p_{\bar{f}} q]}{[qp]}.}$$

qubits calculate the 3 scalar products for each helicity amplitude



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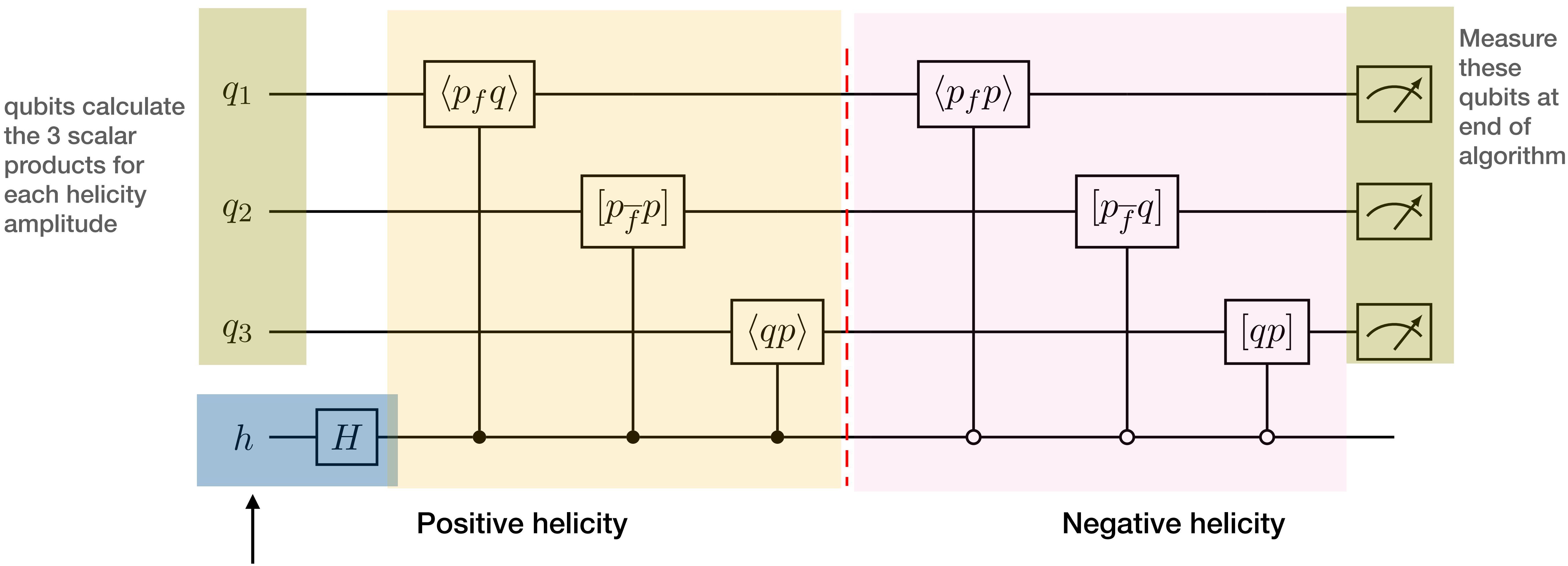
1→2 helicity amplitude circuit

$$\mathcal{M}_+ = -\sqrt{2}\frac{\langle p_f q\rangle [p_{\bar{f}} p]}{\langle qp\rangle}, \qquad \mathcal{M}_- = -\sqrt{2}\frac{\langle p_f p\rangle [p_{\bar{f}} q]}{[qp]}.$$



- Helicity register controls the helicity of each particle. Using a Hadamard gate, we introduce a superposition between the helicity states $|+\rangle = |1\rangle$ and $|-\rangle = |0\rangle$
- Hence, calculate the helicity of each particle involved simultaneously!

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1→2 helicity amplitude calculation

Run algorithm on:

- IBM Q 32-qubit simulator (10,000 shots) without noise profile
- IBM Q 5-qubit Santiago quantum computer (819,200 shots)

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1→2 helicity amplitude calculation

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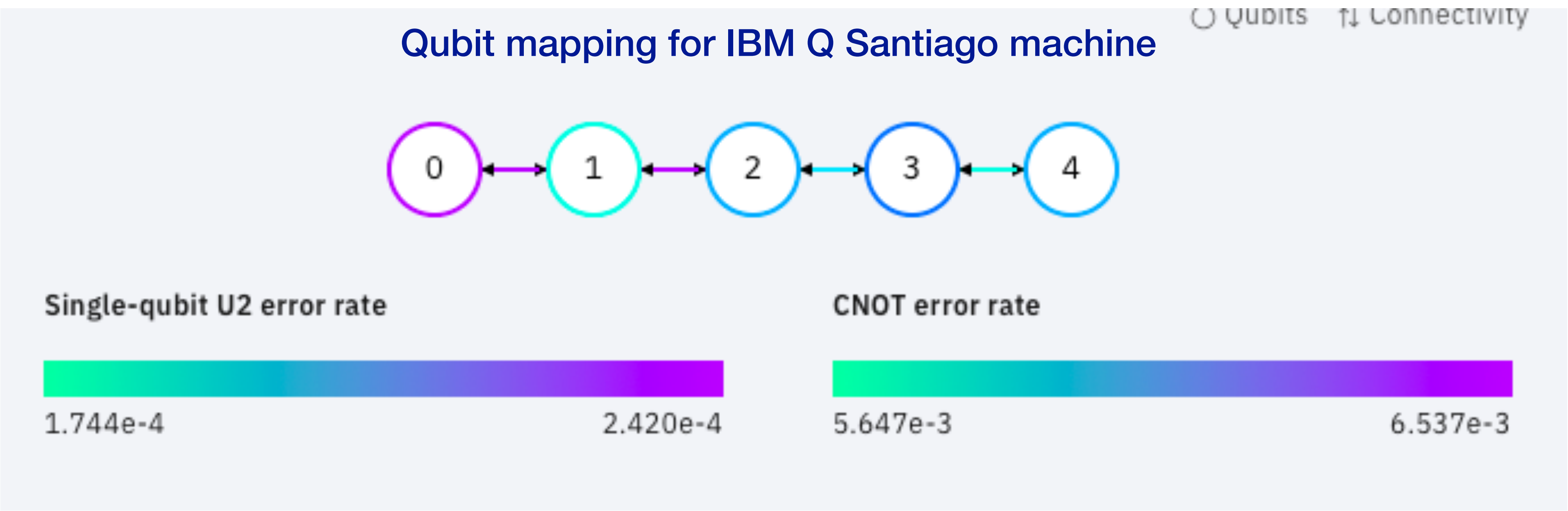
Qubit mapping for IBM Q Santiago machine



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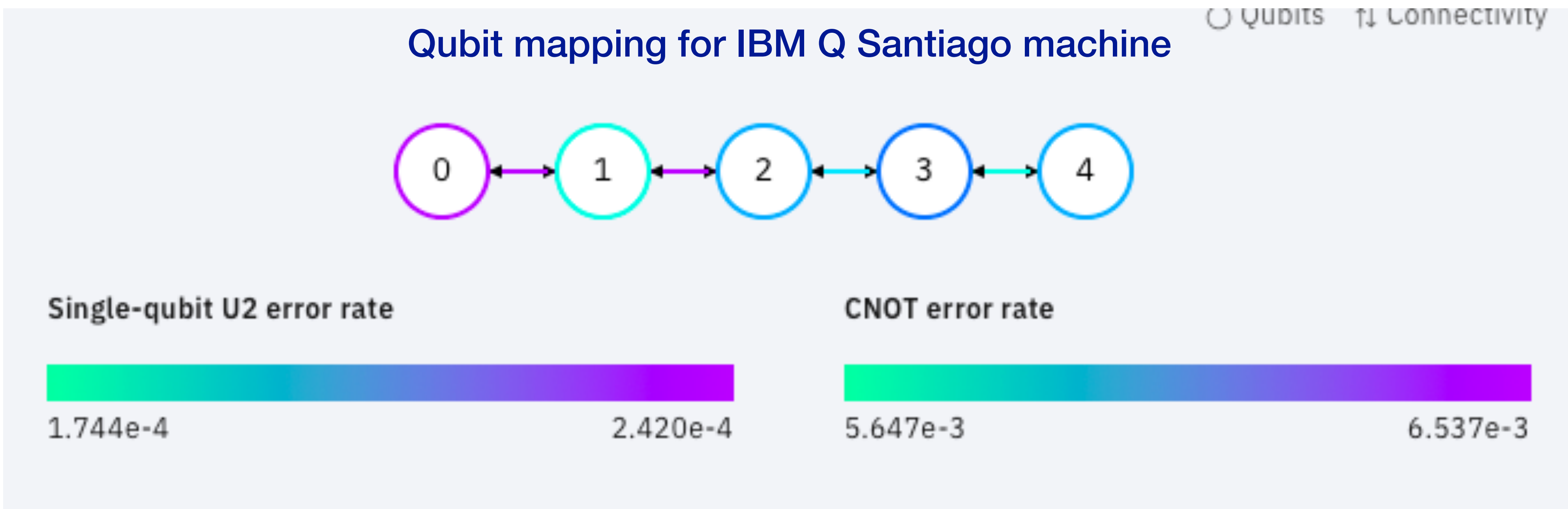
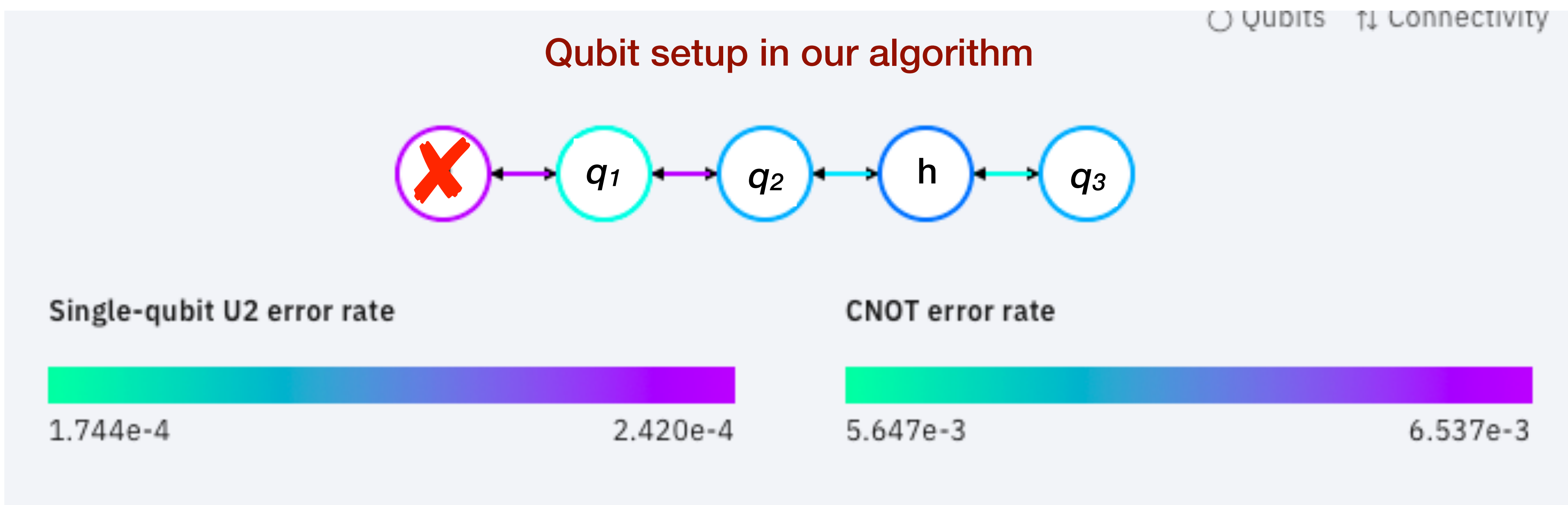


Qubit mapping for IBM Q Santiago machine



Qubit setup in our algorithm

Optimal qubit setup to reduce CNOT errors and limit the number of SWAP operations

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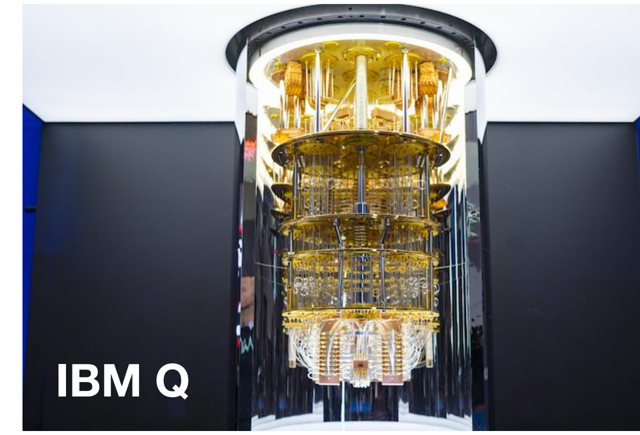
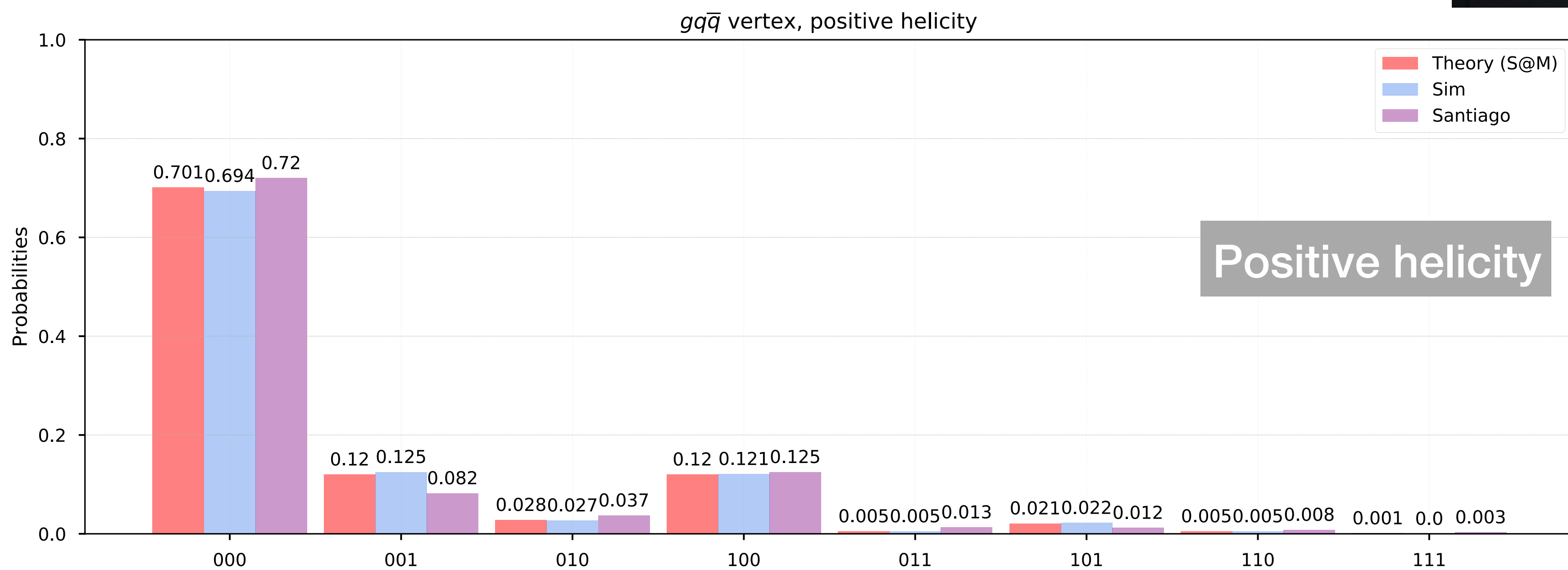
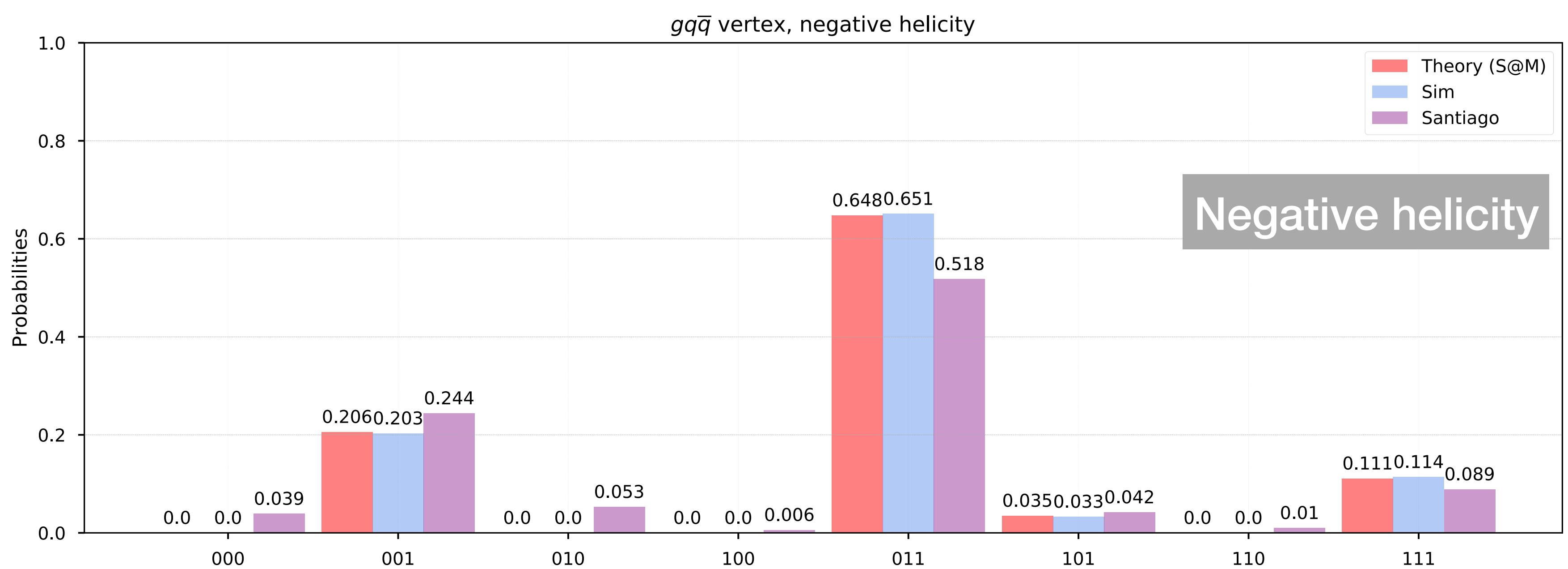
1→2 helicity amplitude calculation

Run algorithm on:

- IBM Q 32-qubit simulator (10,000 shots) without noise profile
- IBM Q 5-qubit Santiago quantum computer (819,200 shots)
- Compare with theoretical calculation







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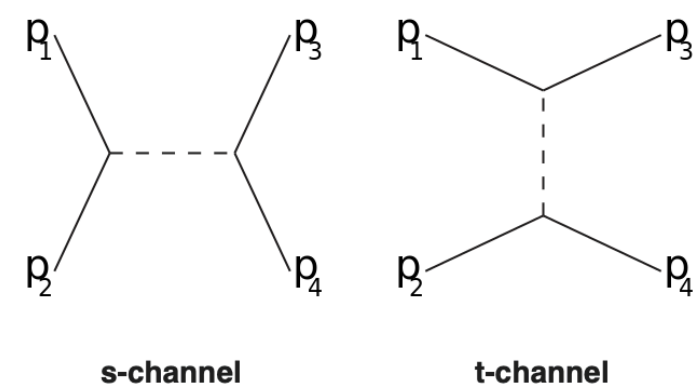
Extending from the 1 → 2 process, we consider the 2 → 2 scattering case of $q\bar{q} \to q\bar{q}$



Amplitudes for the s and t-channel:

$$\mathcal{M}_{s(+-+-)} = -\langle 2|\bar{\sigma}^\mu|1]\frac{1}{s_{12}}[3|\sigma_\mu|4\rangle, \qquad \mathcal{M}_{s(+--+)} = -\langle 2|\bar{\sigma}^\mu|1]\frac{1}{s_{12}}\langle 3|\bar{\sigma}_\mu|4]$$

$$\mathcal{M}_{t(++--)} = -\langle 3|\bar{\sigma}^\mu|1]\frac{1}{s_{13}}[2|\sigma_\mu|4\rangle, \qquad \mathcal{M}_{t(+--+)} = -\langle 3|\bar{\sigma}^\mu|1]\frac{1}{s_{13}}\langle 2|\bar{\sigma}_\mu|4]$$

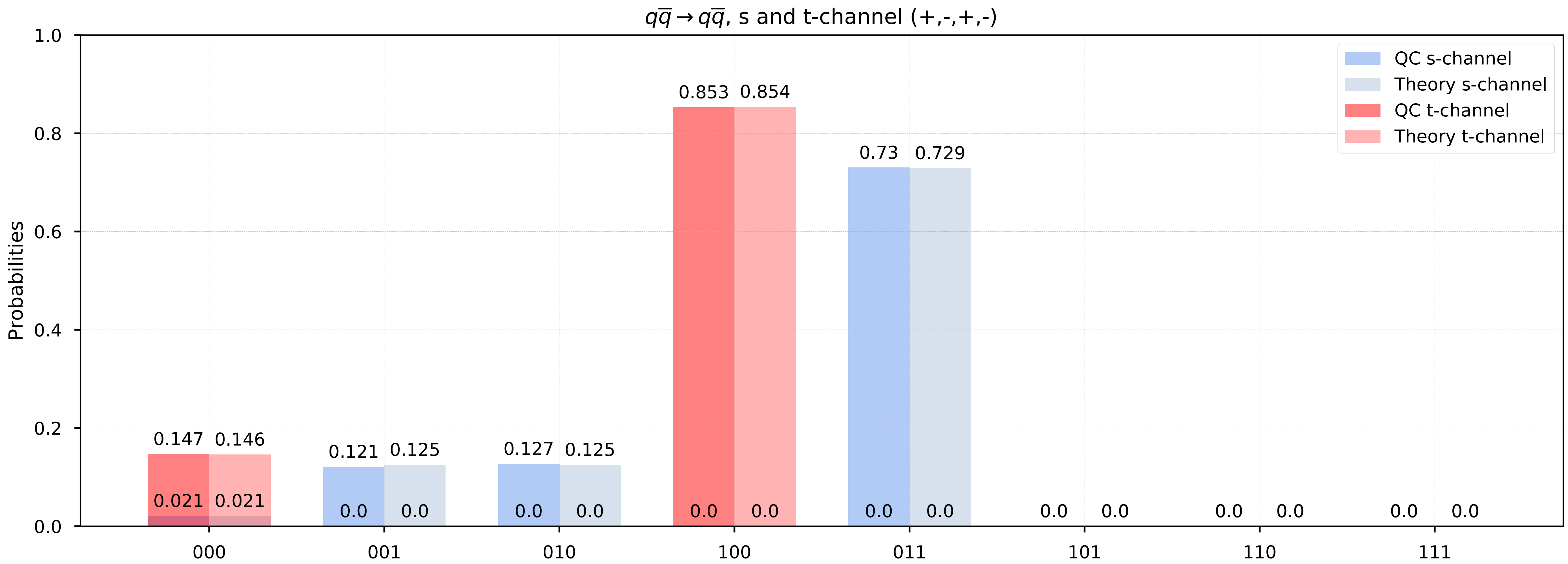
Again using the Fiery identity, can simplify these to (reduce # of qubits needed from 17 to 12) :

$$\mathcal{M}_{t_{(++--)}} = 2\frac{\langle 34\rangle[21]}{\langle 13\rangle[31]}, \qquad \mathcal{M}_{t_{(+--+)}} = 2\frac{\langle 32\rangle[41]}{\langle 13\rangle[31]}.$$

$$\mathcal{M}_{s_{(+-+-)}} = 2\frac{\langle 24\rangle[31]}{\langle 12\rangle[21]}, \qquad \mathcal{M}_{s_{(+--+)}} = 2\frac{\langle 23\rangle[41]}{\langle 12\rangle[21]}$$

2→2 helicity amplitude calculation

Run algorithm on:
- **IBM Q 32-qubit simulator (10,000 shots)**
- **Compare with theoretical calculation**



- Algorithm calculates the positive and negative helicity of each particle involved AND the s and t-channels simultaneously!

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2→n helicity amplitude calculation

- Proposed algorithm can be generalised to calculating helicity amplitudes for multi-particle final states.

- Using BCFW recursion formula, scattering amplitudes for massless partons can be reduced to combination of scalar products between helicity spinors

e.g. Parke-Taylor formula for 2→n gluon scattering process

$$\mathcal{A}_n[1^+ \cdots i^- \cdots j^- \cdots n^+] = (-g_s)^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}.$$

- The number of calculation qubits and helicity qubits needed in the algorithm both scale linearly with the number of final state particles.

- Each scalar product requires two spinor operations, the circuit depth scales linearly with number of scalar products.

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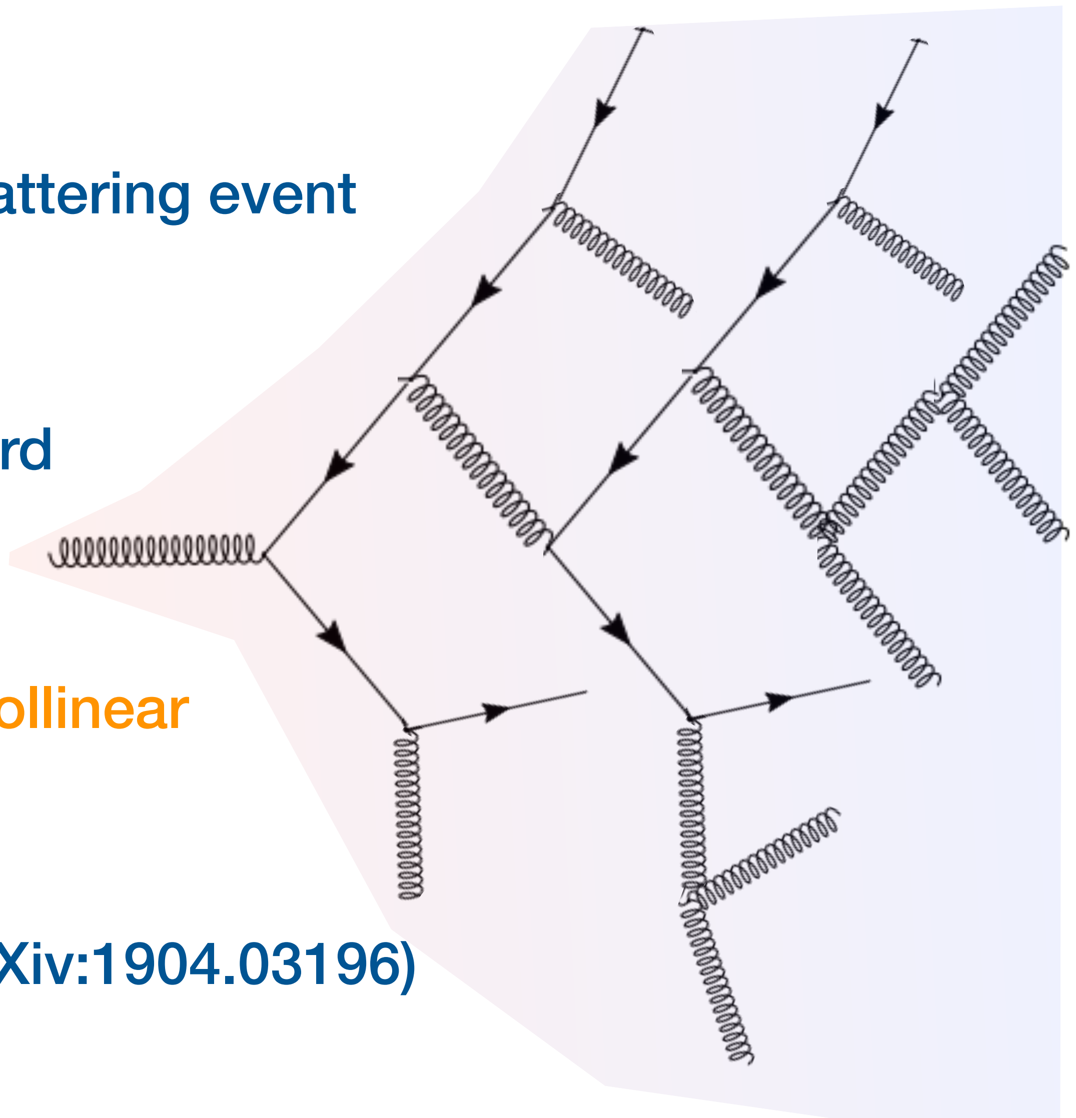
Parton shower

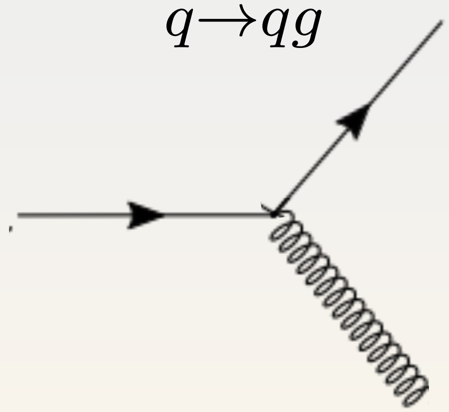
- After the hard interaction, the next step in simulating a scattering event at LHC is the parton shower
- Parton shower evolves the scattering process from the hard interaction scale down to the hadronisation scale
- Propose a quantum computing algorithm that simulates collinear emission in a 2-step parton shower
- This algorithm builds on previous work by Bauer et. al. (arXiv:1904.03196)
- To comply with capability of quantum computers we had access to, consider a simplified model of the parton shower consisting of only one flavour of quark



Parton shower

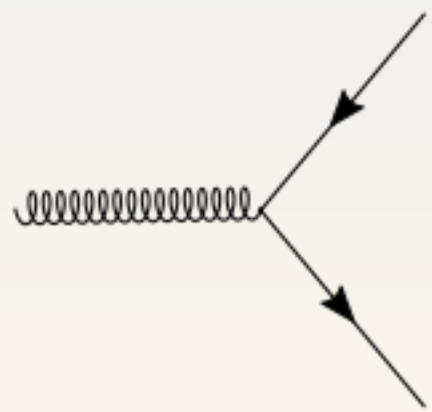
- Collinear emission occurs when a parton splits into two massless particles which have parallel 4-momenta
- The total momentum, P, of the parton is distributed between the particles as: $p_i = zP, \quad p_j = (1-z)P$
- Emission probabilities are calculated using collinear splitting functions, which at LO are given by:

$q \to qg$



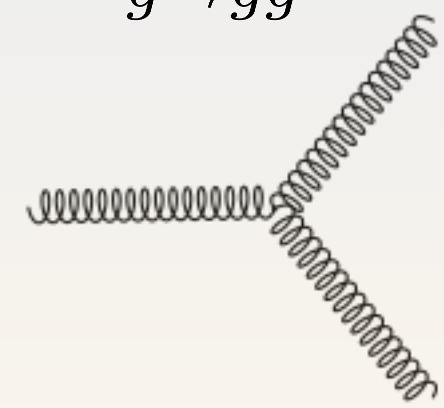
$$P_{q\to qg}(z) = C_F \frac{1+(1-z)^2}{z},$$

$g \to q\bar{q}$



$$P_{g\to q\bar{q}}(z) = n_f T_R (z^2 + (1-z)^2),$$

$g \to gg$



$$P_{g\to gg}(z) = C_A \Big[2\frac{1-z}{z} + z(1-z) \Big]$$

Non-emission probability calculated using Sudakov factors

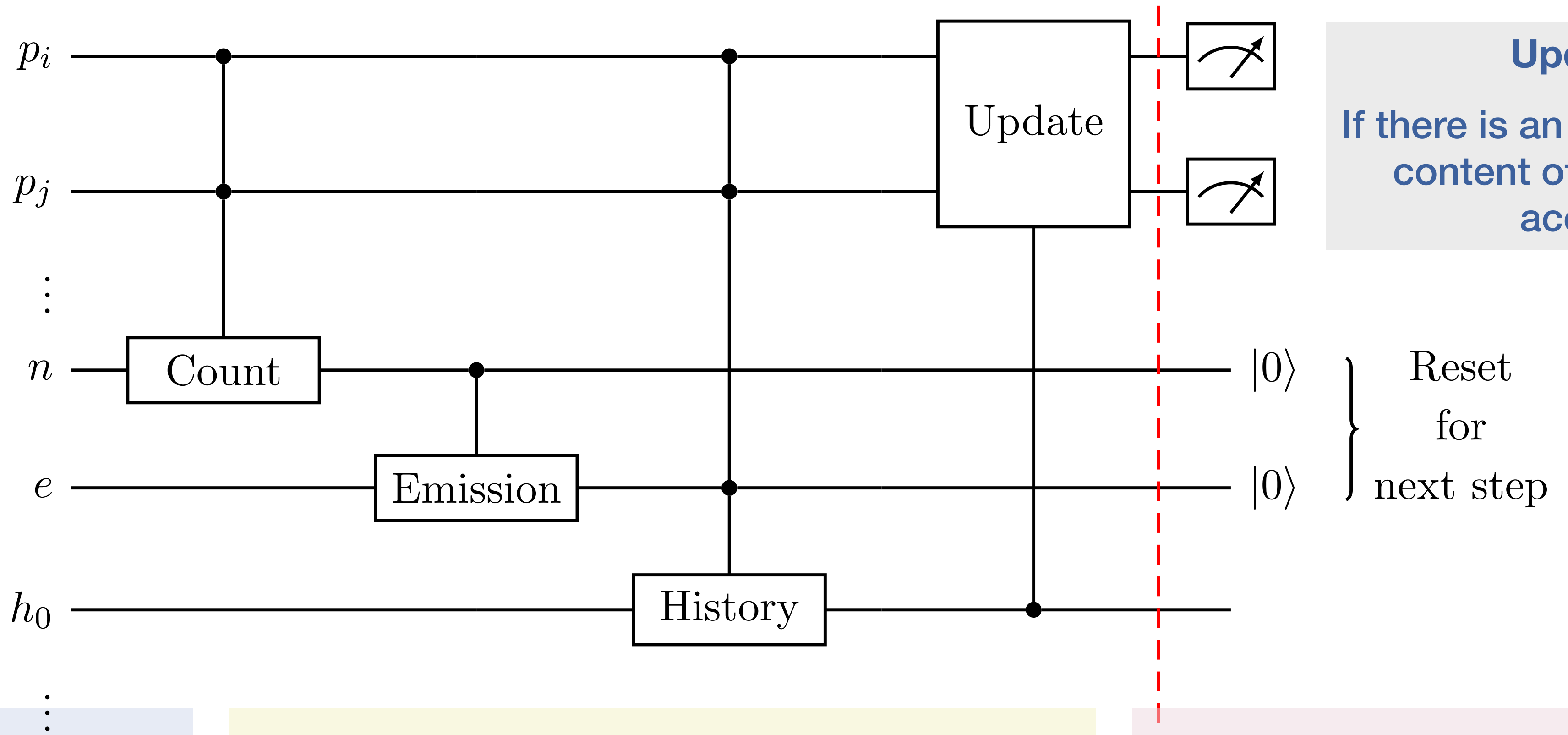
$$\Delta_{i,k}(z_1, z_2) = \exp\left[-\alpha_s^2 \int_{z_1}^{z_2} P_k(z') dz' \right],$$

Probability of a splitting is given by,

$$\mathrm{Prob}_{k\to ij} = \big(1 - \Delta_k\big) \times P_{k\to ij}(z).$$

Circuit for parton shower algorithm

- Circuit comprises of particle registers, emission registers, and history registers and uses a total of 31 qubits

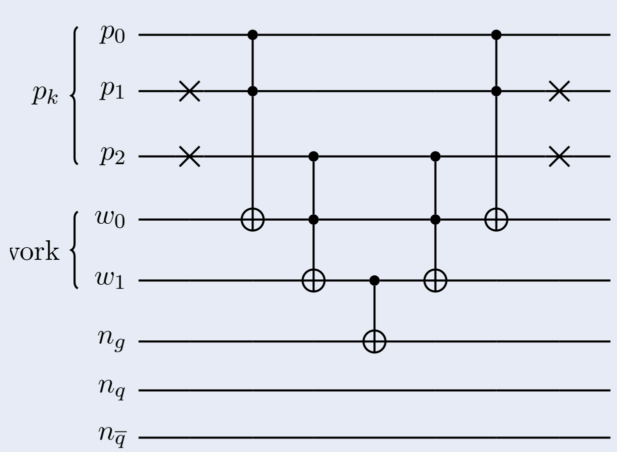


Update gate

If there is an emission, changes content of particle counts accordingly

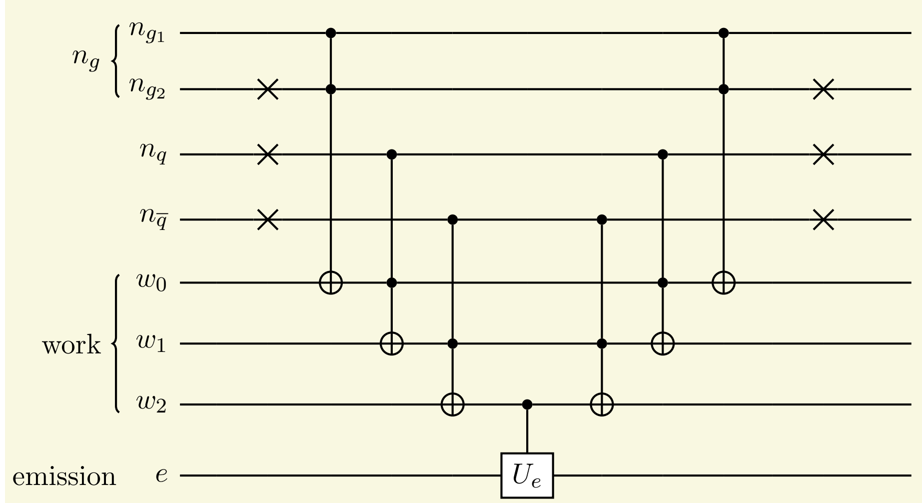
Count gate

Uses series of NOT, CNOT and Toffoli (CCNOT) gates to count number of each type of particle



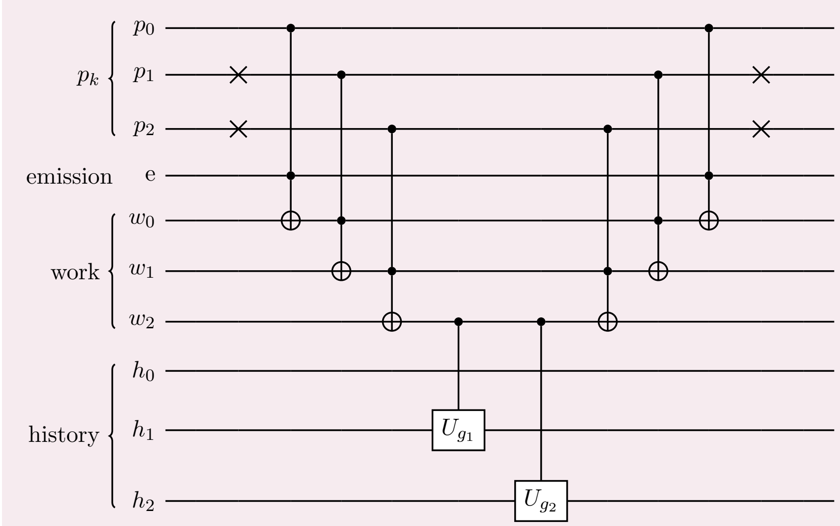
Emission gate

Implements the Sudakov factors using a rotation, which changes the state of the emission gate to $|1\rangle$ if emission, $|0\rangle$ if not.



History gate

Determines which emission has occurred



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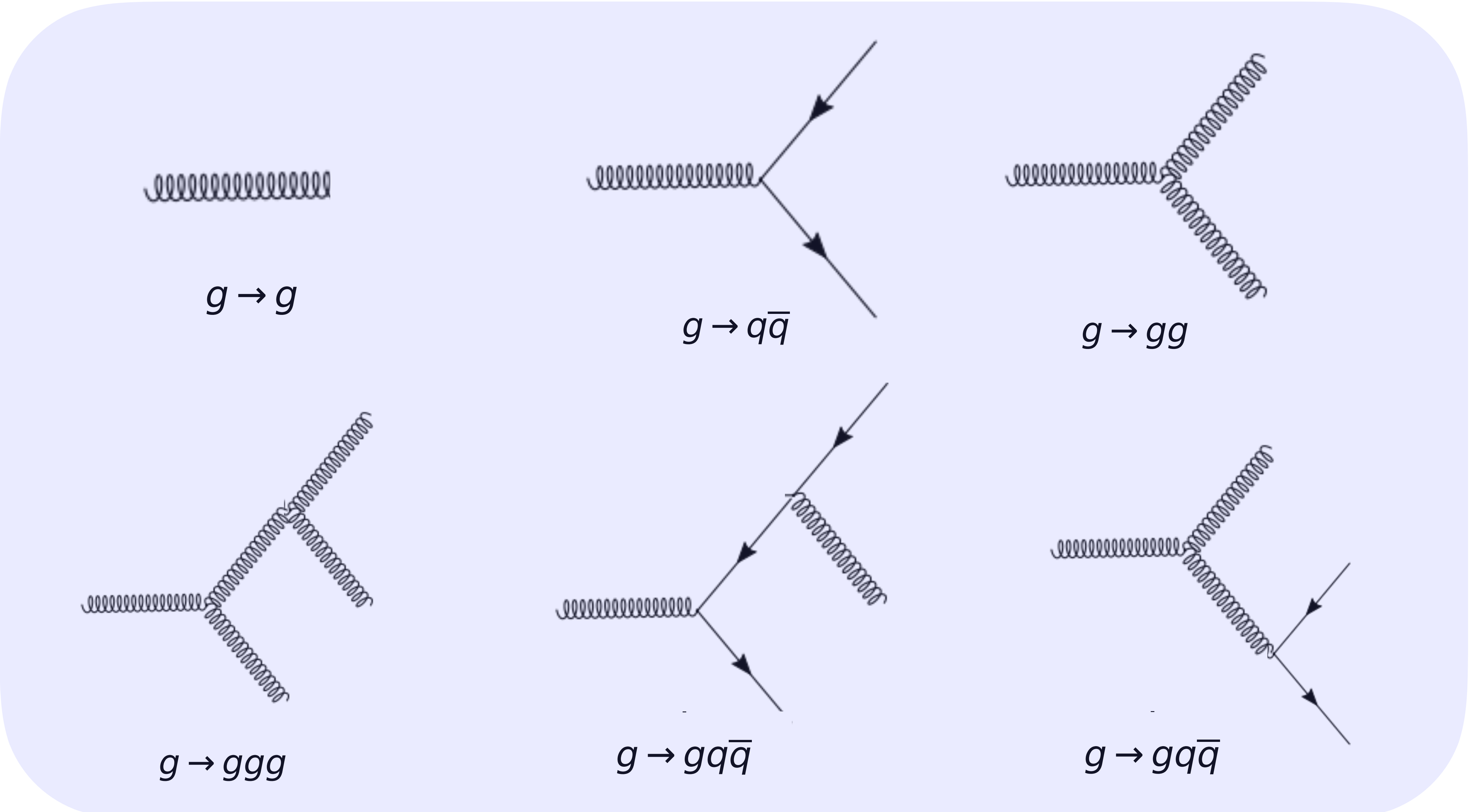
2-step parton shower: initial state a gluon



- Classical Monte Carlo methods need to manually keep track of individual shower histories, which must be stored on a physical memory device.
- Quantum computing algorithm constructs a wavefunction for the whole process and calculates all possible shower histories simultaneously!

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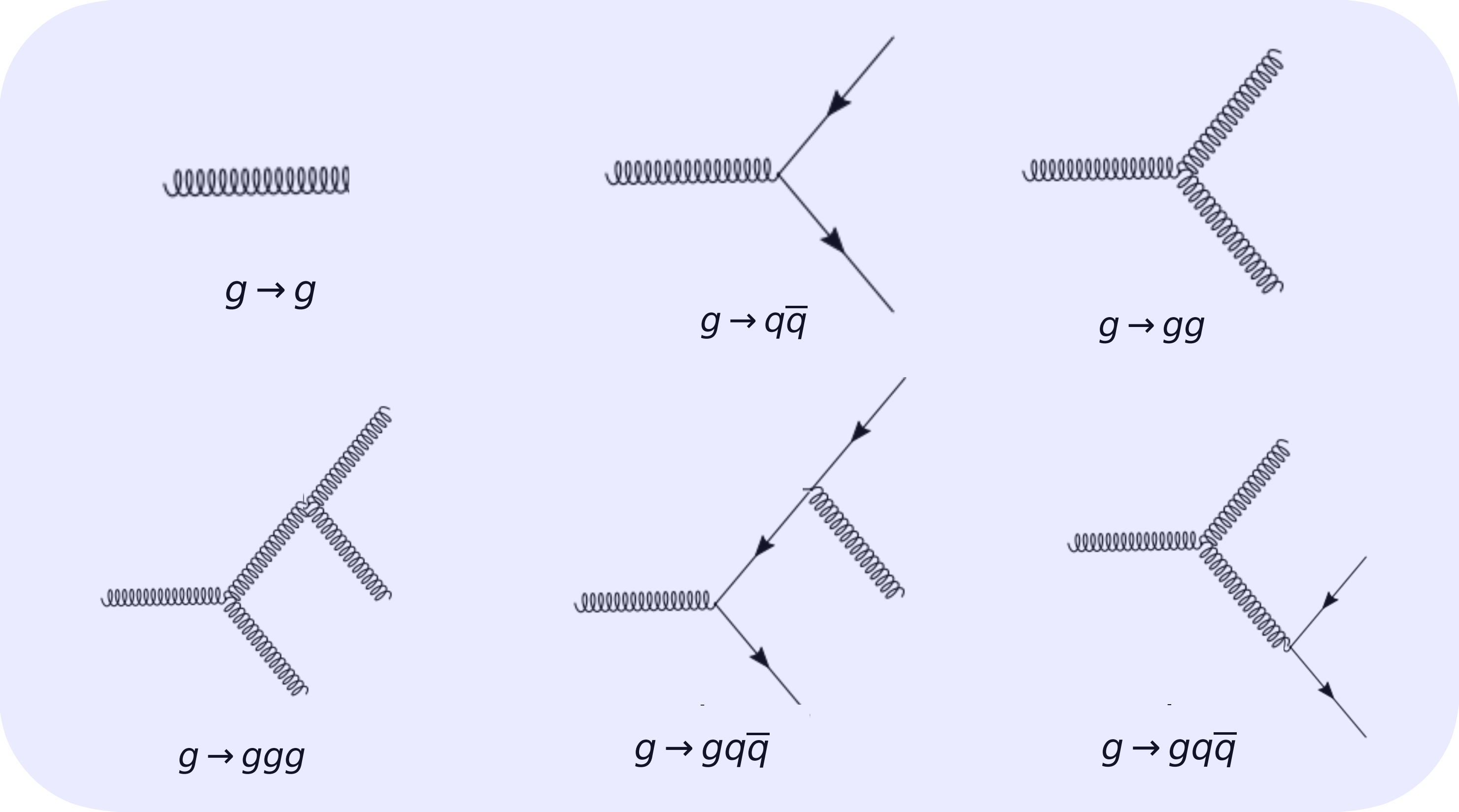
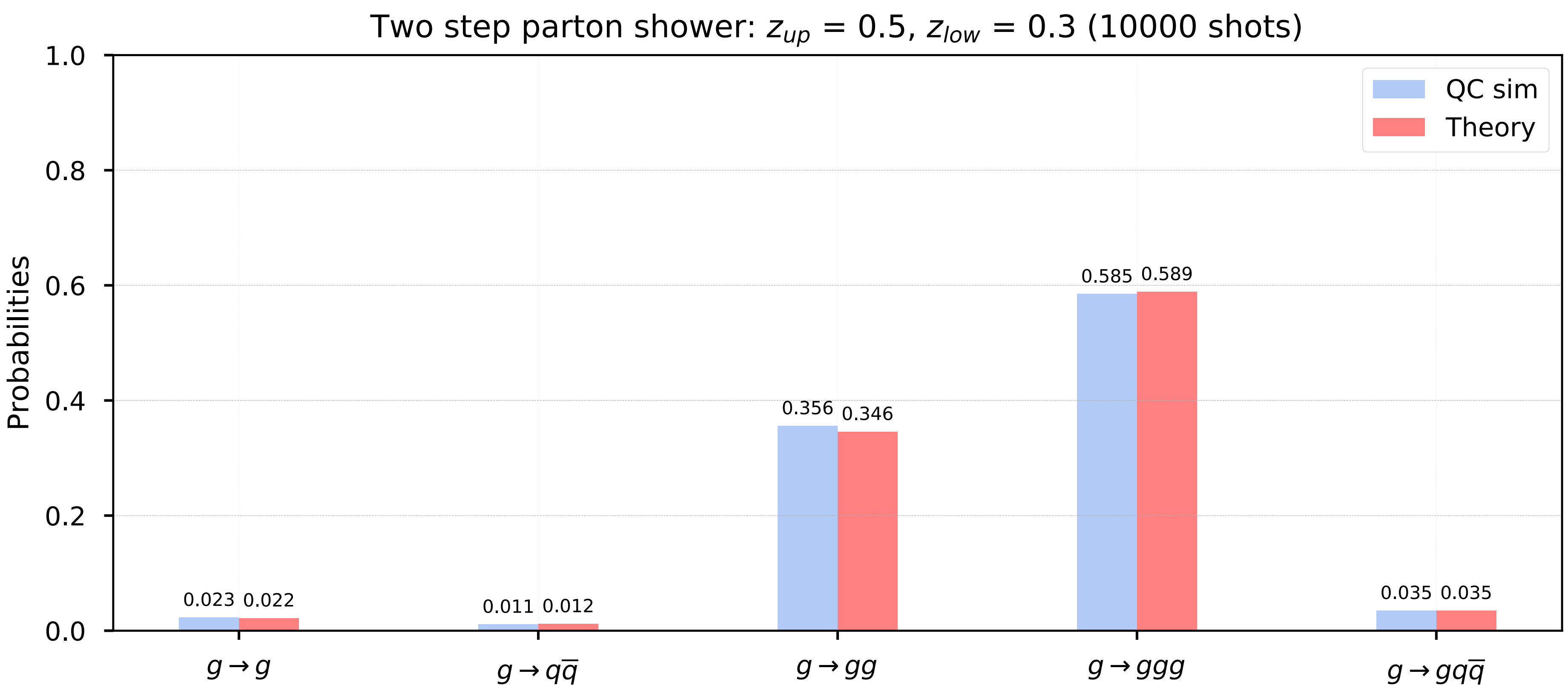
Results for parton shower algorithm

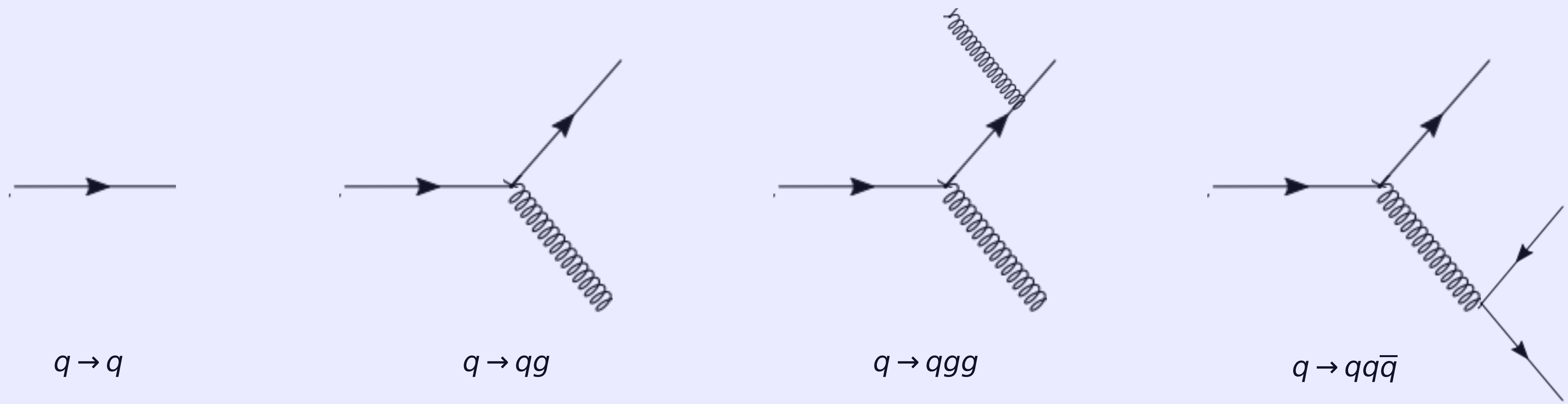
Run 10,000 shots on IBM Q 32-qubit Quantum Simulator

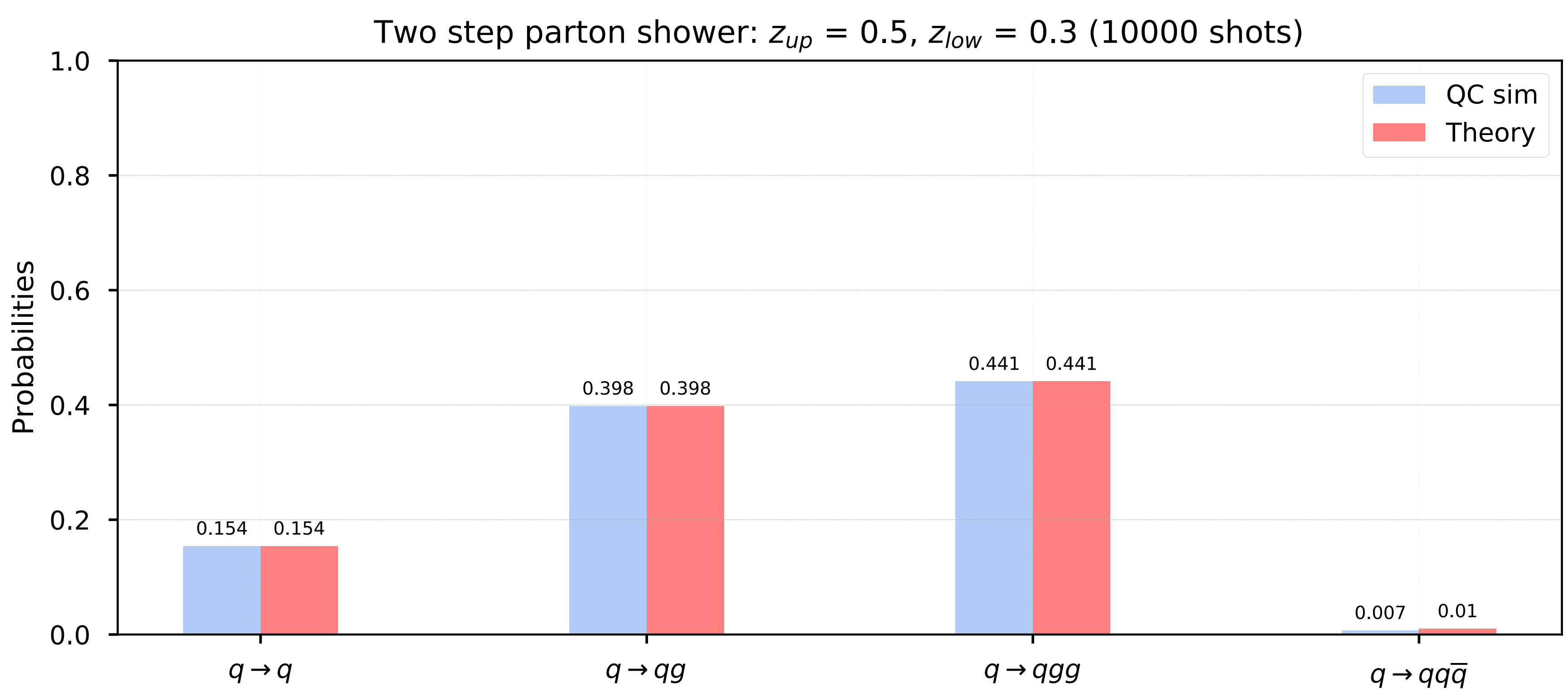




(a) Initial particle a gluon.

Results for parton shower algorithm





(b) Initial particle a quark.

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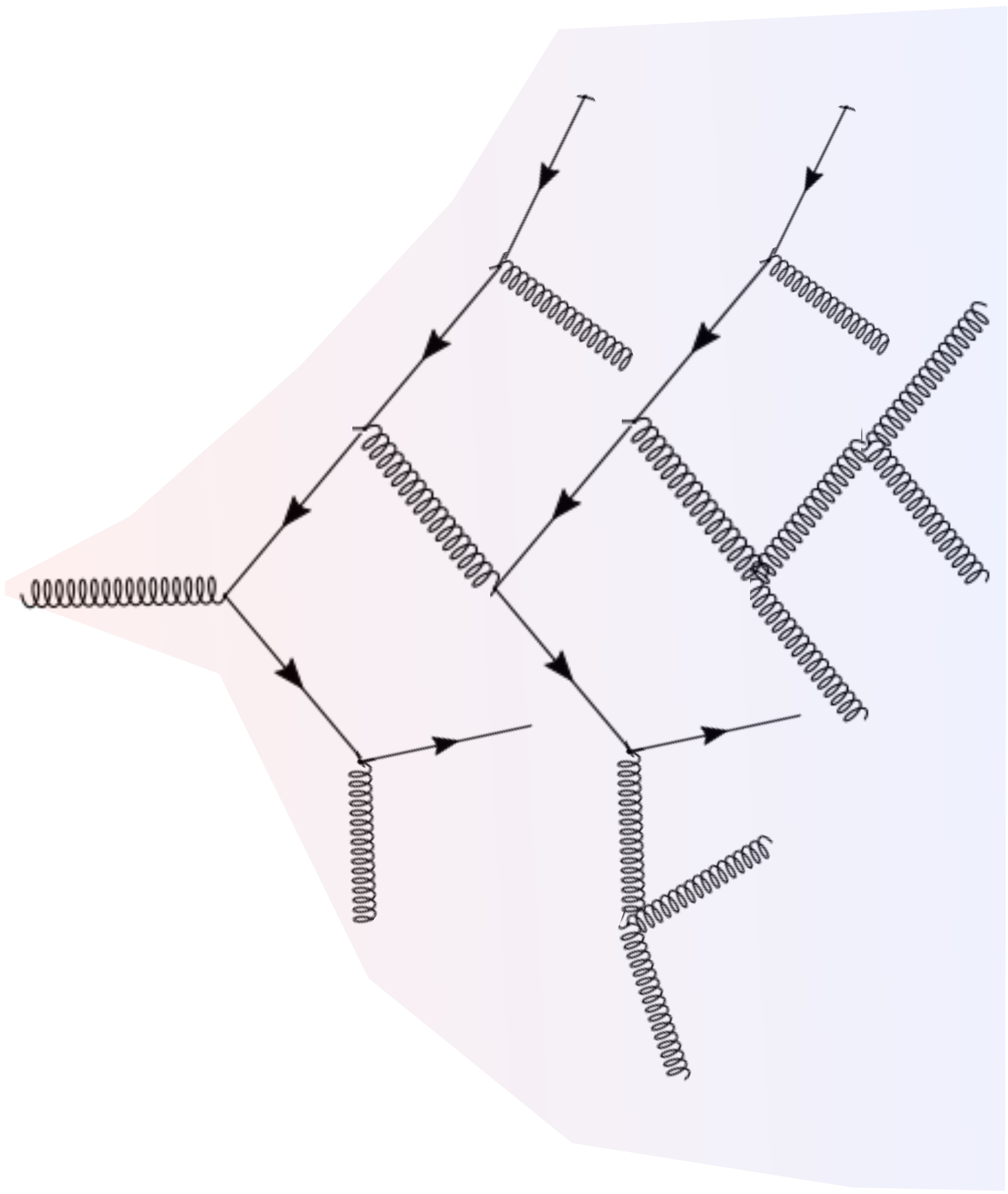
Summary of parton shower algorithm

- Algorithm builds on previous work by Bauer et. al. [1] by including a vector boson and boson splittings → significant changes in its implementation

- Can simulate both gluon and quark splittings, thus provides the foundations for developing a general parton shower algorithm

- With advancements in quantum technologies, algorithm can be extended to include all flavours of quarks without adding disproportionate computational complexity



[1] : arXiv:1904.03196

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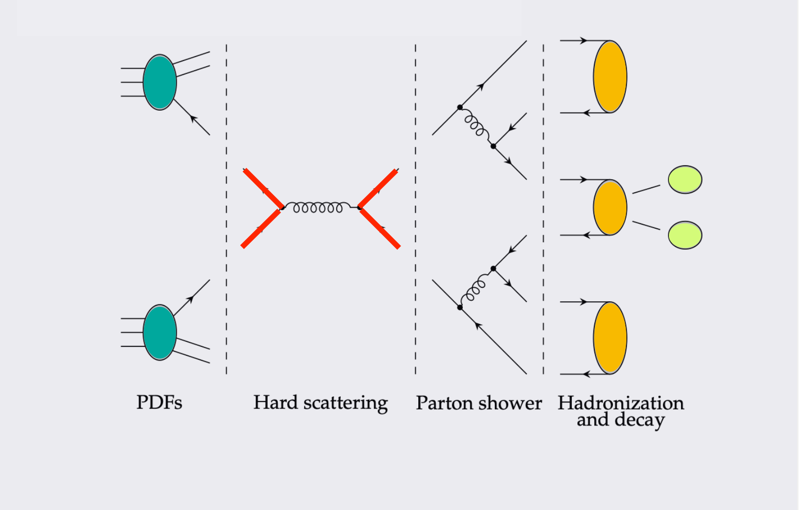
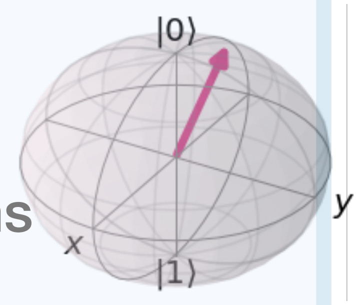
Summary of arXiv:2010.00046

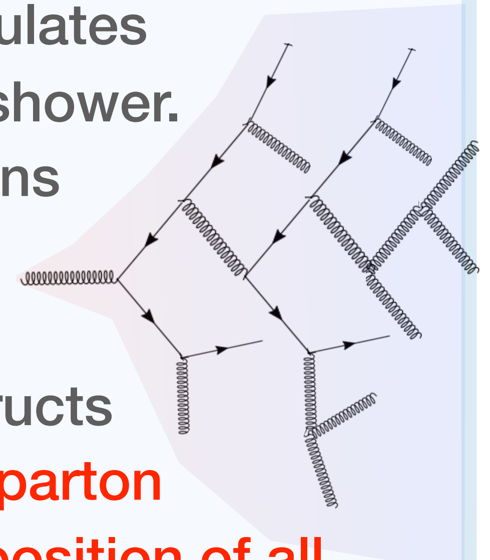
- Modeling complexity of collisions at LHC relies on theoretical calculations of multi-particle final states.
- Working with quantum objects and quantum phenomena; can quantum computers help?
- Propose general and extendable quantum algorithms to calculate the hard interaction using helicity amplitudes and a 2-step parton shower



Helicity amplitude algorithm exploits **equivalence of spinors and qubits**, encodes operators as unitary operations in a quantum circuit. Using Hadamard gates to introduce a superposition between helicity qubits, it enables **simultaneous calculation of the + and − helicity states of each particle AND the s- and t-channel amplitudes for a 2→2 process**



Parton shower algorithm calculates collinear emission for 2-step shower. While classical implementations must explicitly keep track of individual shower histories, our quantum algorithm constructs a **wavefunction for the whole parton shower process with a superposition of all shower histories**



First step towards a quantum computing algorithm to model the full collision event at LHC and demonstrate an excellent example of using quantum computers to model intrinsic quantum behaviour of the system

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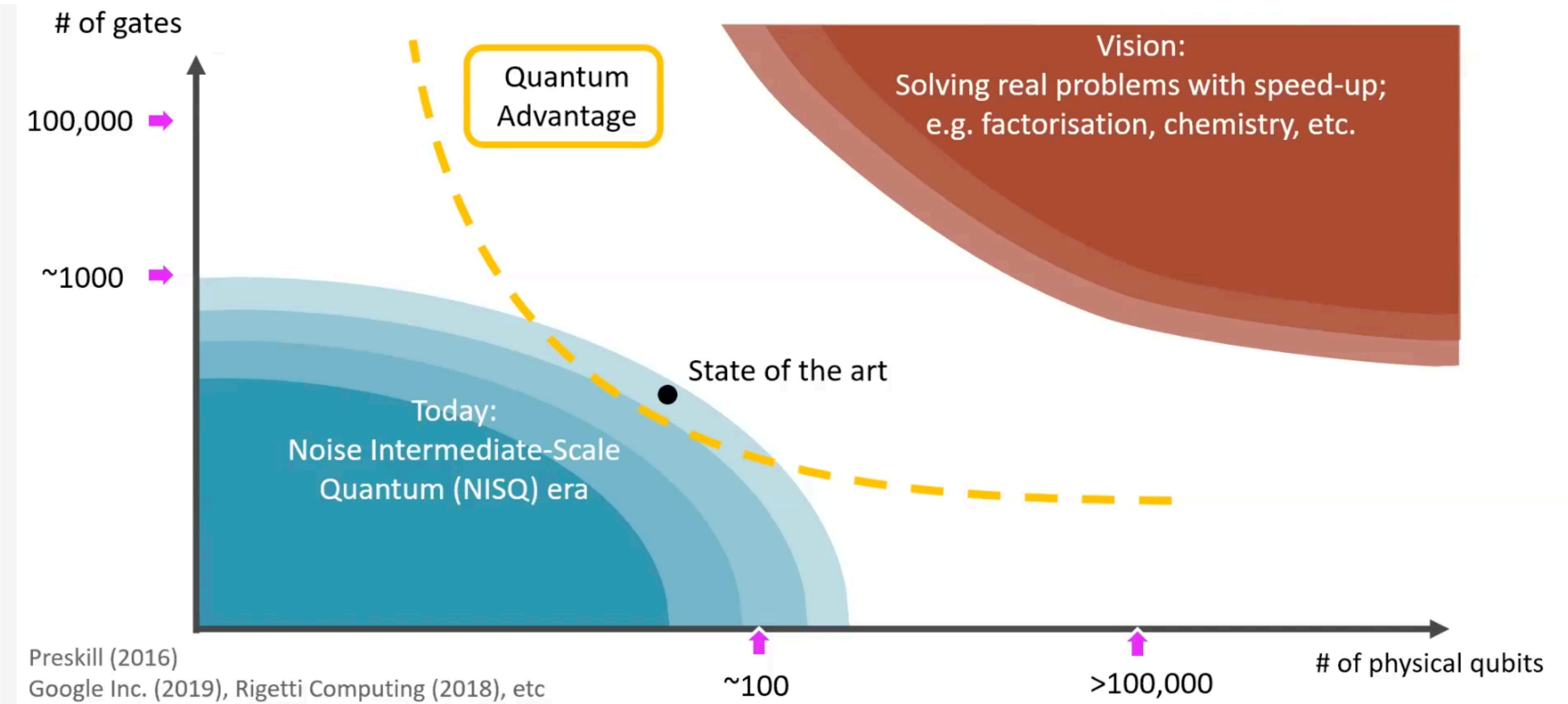
Future outlook

Slide credit: Steven Touzard's talk given at CQT



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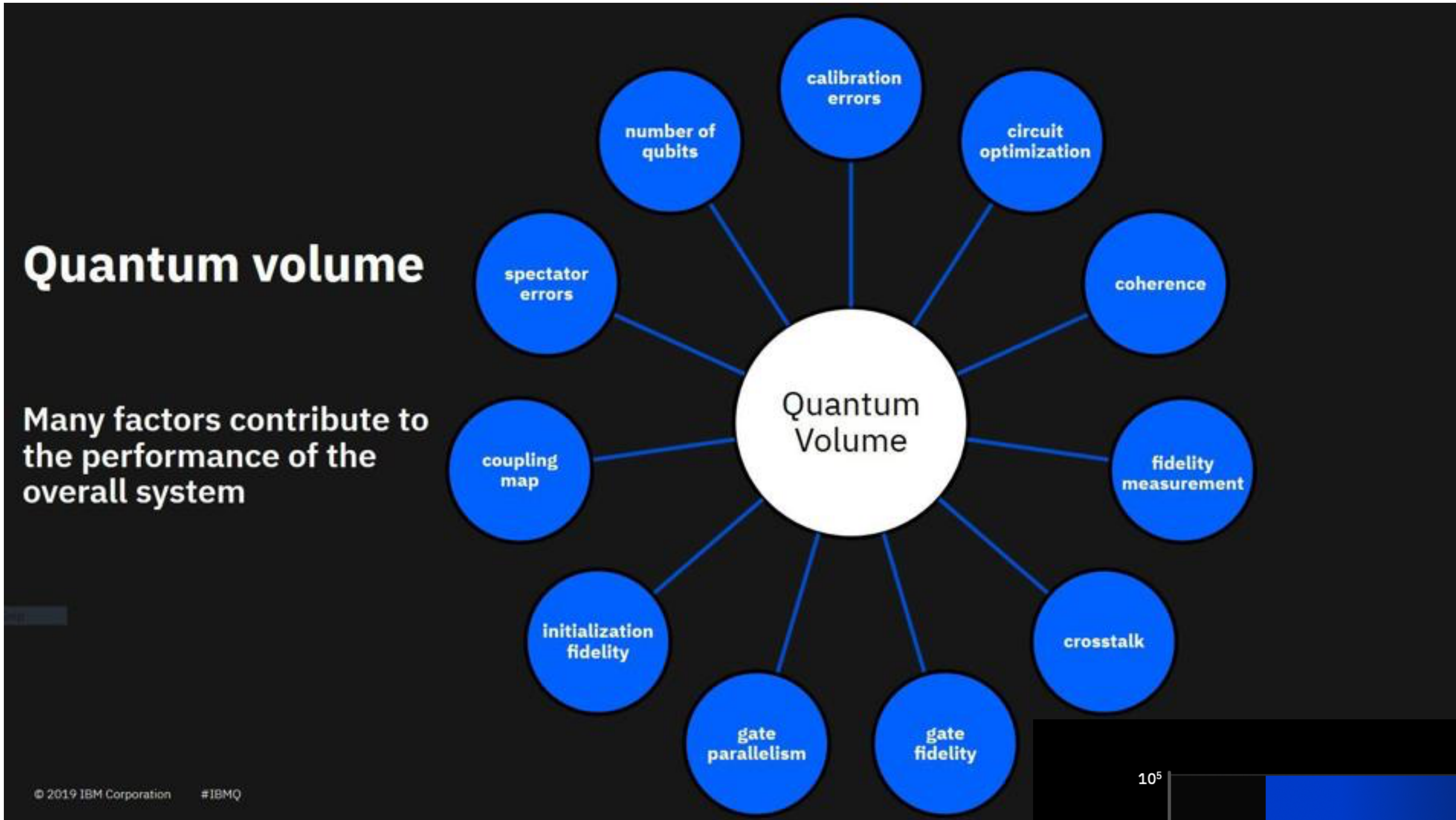
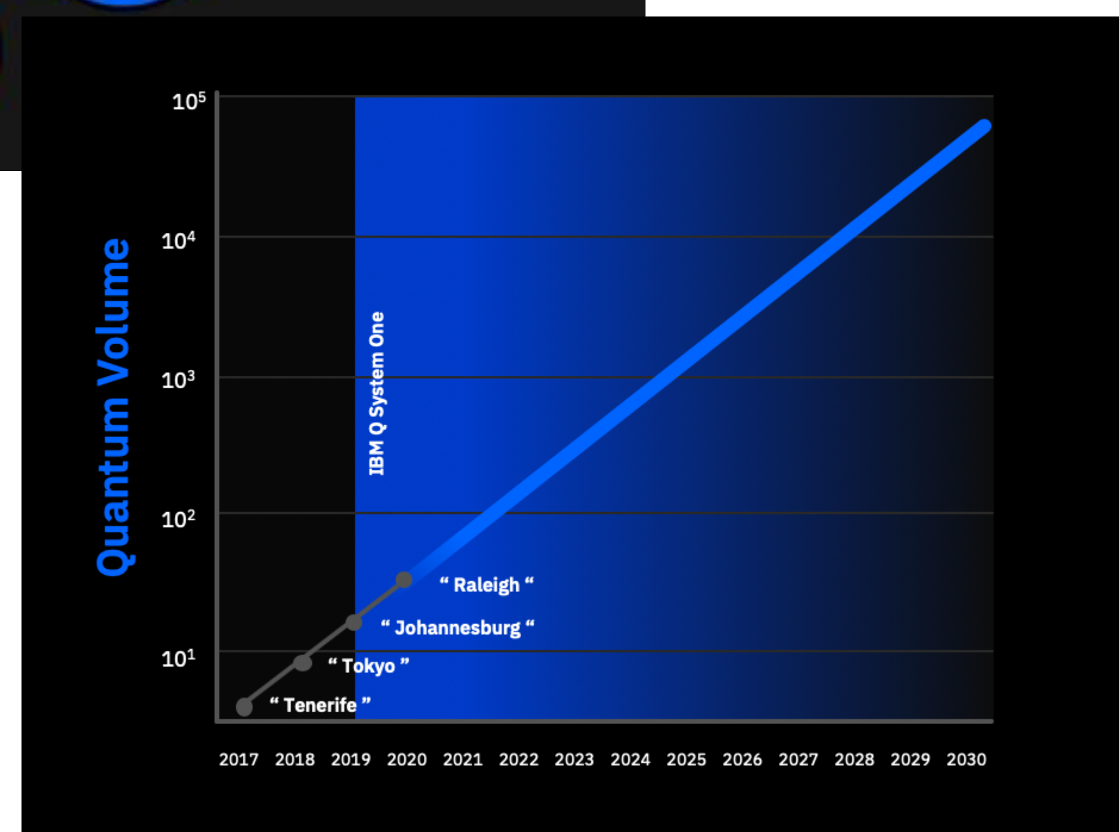
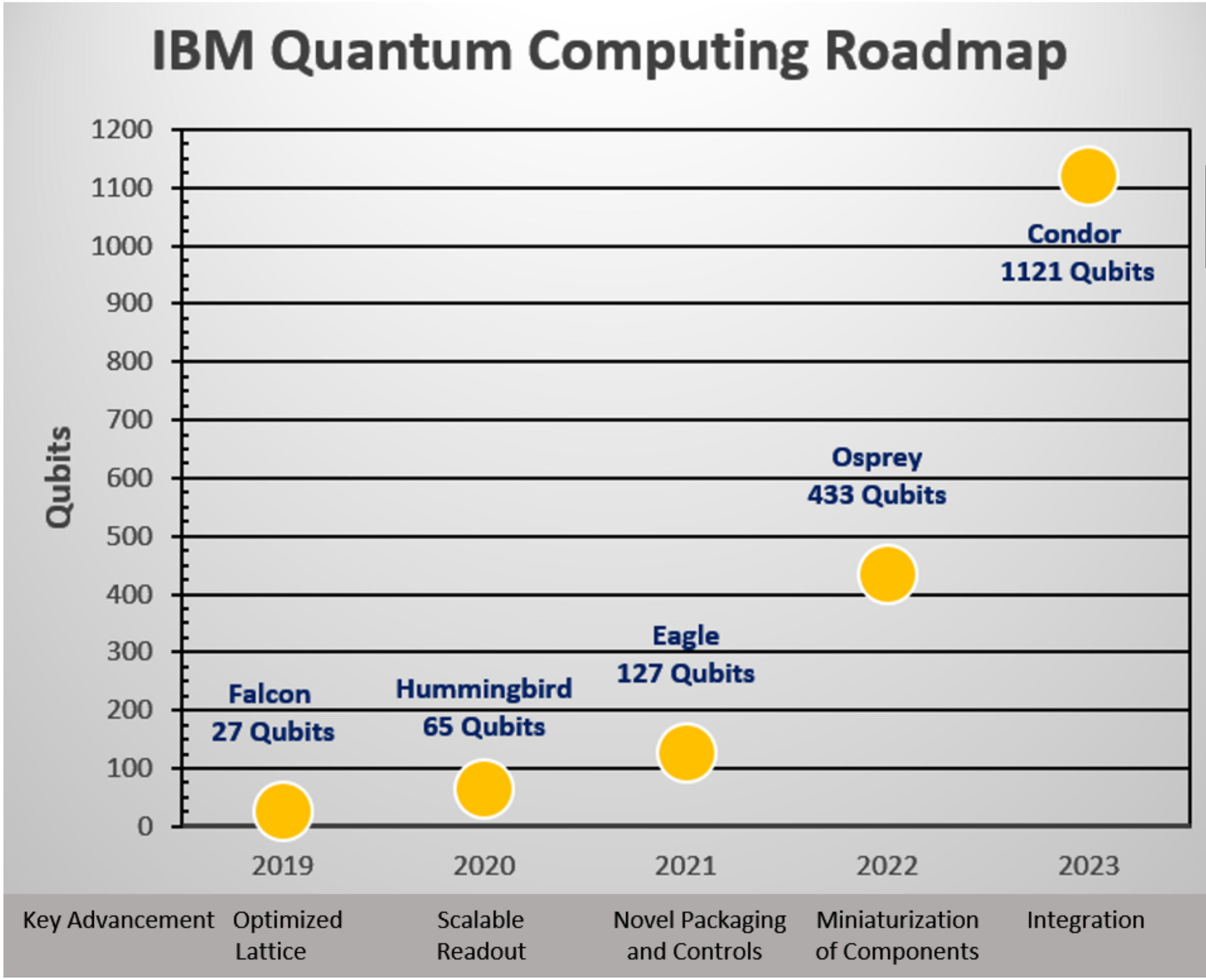
Performance of quantum computers





Future outlook



> 1000 qubits by 2023

Intermediate, near term goal: 1,121-qubit system by the end of 2023

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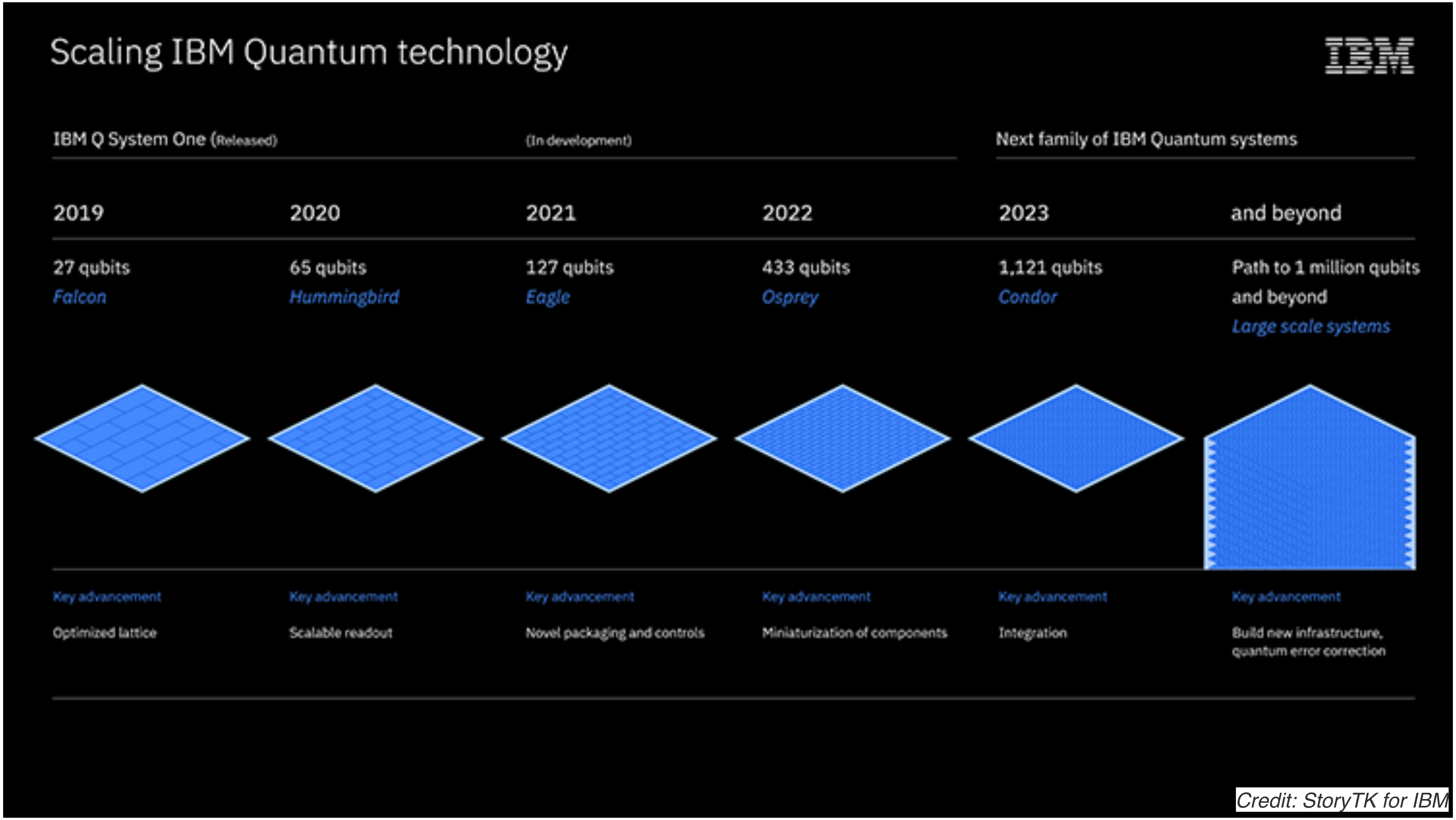
Future outlook



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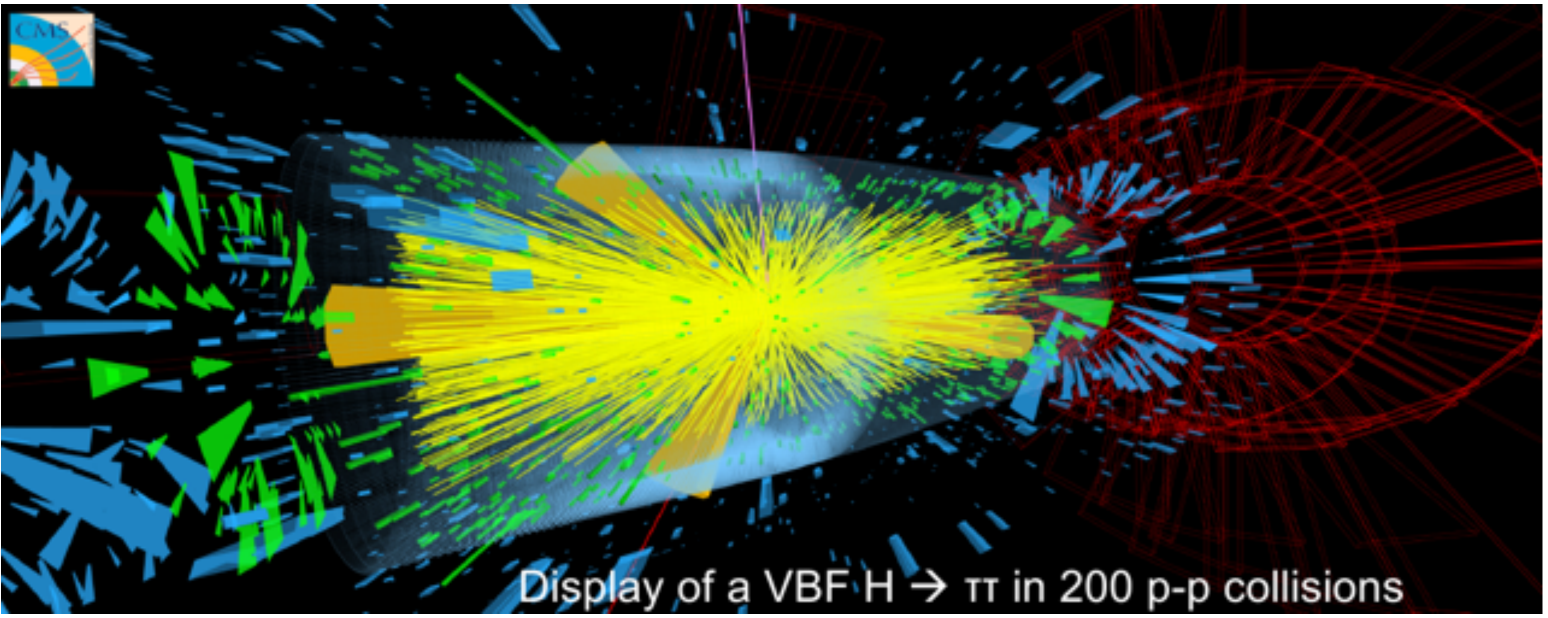
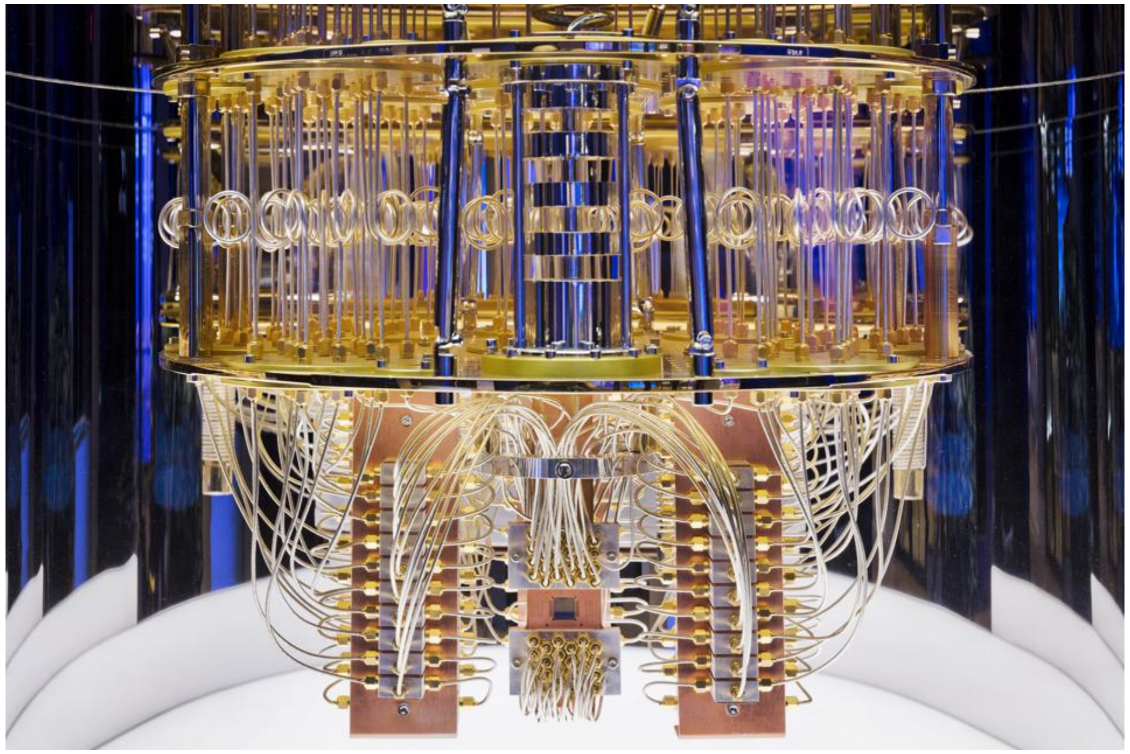
Conclusions

- Quantum computing an emergent and rapidly developing field with potential applications in variety of different areas

- Solutions to some of the most challenging problems in HEP may well be at the intersection of these two fields

- Current machines are excellent test beds for demonstrating proof-of-principle studies to make way for quantum revolution





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