

Theory of Charm CP Violation

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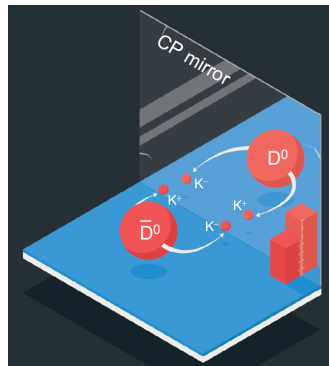
Imperial College London, February 2024

Unique gate to flavor structure of up-type quarks

$$a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) - a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-) \\ = (-0.161 \pm 0.028)\% .$$

[LHCb 1903.08726, HFLAV 2021]

The problem: Is it SM?



[CERN]

Direct CP Violation is an Interference Effect

$$a_{CP}^{\text{dir}}(f) \equiv \frac{|\mathcal{A}(D^0 \rightarrow f)|^2 - |\mathcal{A}(\bar{D}^0 \rightarrow f)|^2}{|\mathcal{A}(D^0 \rightarrow f)|^2 + |\mathcal{A}(\bar{D}^0 \rightarrow f)|^2} \approx 2 r_{\text{CKM}} r_{\text{QCD}} \sin \varphi_{\text{CKM}} \sin \delta_{\text{QCD}}$$

f = CP-eigenstate.

The decay amplitude:

$$\mathcal{A} = 1 + r_{\text{CKM}} r_{\text{QCD}} e^{i(\varphi_{\text{CKM}} + \delta_{\text{QCD}})}$$

- r_{CKM} : real ratio of CKM matrix elements.
- φ_{CKM} : weak phase.
- r_{QCD} : real ratio of hadronic matrix elements.
- δ_{QCD} : strong phase.

Where does the interference come from?

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+ \pi^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+ K^-, \dots \xrightarrow{\text{QCD}} \pi^+ \pi^-$$

$$D^0 \xrightarrow{V_{cd}^* V_{ud}} \pi^+ \pi^-, \dots \xrightarrow{\text{QCD}} K^+ K^-$$

$$D^0 \xrightarrow{V_{cs}^* V_{us}} K^+ K^-$$

Weak and strong factors

$$\frac{\mathcal{A}(D \rightarrow \pi\pi \rightarrow KK)}{\mathcal{A}(D \rightarrow KK)} = \left(r_{\text{CKM}} e^{i\varphi_{\text{CKM}}}\right) \left(r_{\text{QCD}} e^{i\delta_{\text{QCD}}}\right)$$

- r_{QCD} : ratio of hadronic amplitudes.
- $\delta_{\text{QCD}} = \mathcal{O}(1)$: strong phase.
- $r_{\text{CKM}} = 1$: ratio of CKM factors, $|V_{cd}^* V_{ud} / (V_{cs}^* V_{us})|$
- $\varphi_{\text{CKM}} \approx 6 \cdot 10^{-4}$: deviation from 2×2 unitarity.

Prediction

$$\Delta a_{CP}^{\text{dir}} \sim 10^{-3} \times r_{\text{QCD}}$$

- U -spin decomposition: $r_{\text{QCD}} = \mathcal{A}^{\Delta U=0} / \mathcal{A}^{\Delta U=1}$.

Can we overcome soft QCD in Charm?

Expansion parameters

- In kaon decays we have m/Λ .
- In B decays we have Λ/m .
- In charm...?

Need to revisit toolbox / find new strategies.

The three $\Delta I = 1/2$ rules for $P \rightarrow \pi\pi$

- Relevant ratio of strong isospin matrix elements:

$r_{QCD}^{\Delta I=1/2} \equiv A^{\Delta I=1/2} / A^{\Delta I=3/2}$	Kaon	Charm	Beauty
Data	22	2.5	1.5
"No QCD" limit	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
Enhancement	$\mathcal{O}(10)$	$\mathcal{O}(1)$	$\mathcal{O}(\alpha_s)$

[*D*: Franco Mishima Silvestrini 2012, *B*: Grinstein Pirtskhalava Stone Uttayarat 2014]

- Rescattering most important in *K* decays, less important but still significant in *D* decays, and small in *B* decays.

Can we tell a loop from a tree?



$$\Delta a_{CP}^{dir} \sim 10^{-3} \times r_{\text{QCD}}, \quad r_{\text{QCD}} = \mathcal{A}^{\Delta U=0} / \mathcal{A}^{\Delta U=1}$$

Assuming the SM, the data implies $r_{\text{QCD}}^{\text{Exp}} = \mathcal{O}(1)$.

What is $r_{\text{QCD}}^{\text{SM}} \equiv |P/T|$?

- Light Cone Sum Rules (LCSR): $r_{\text{QCD}}^{\text{SM}} \sim 0.1$.

[Petrov Khodjamirian 1706.07780, Chala Lenz Rusov Scholtz 1903.10490, Lenz Piscopo Rusov 2312.13245]

- Large non-pert. effects from low energy QCD: $r_{\text{QCD}}^{\text{SM}} = \mathcal{O}(1)$.
(Like in charm $\Delta I = 1/2$ rule.)

[Grossman Schacht 1903.10952, Brod Kagan Zupan 1111.5000, Schacht Soni 2110.07619]

- Using $\pi\pi/KK$ rescattering data: Under debate. [Franco Mishima Silvestrini 1203.3131, Bediaga Frederico Magalhaes 2203.04056, Pich Solomonidi Vale Silva 2305.11951]

The case for $r_{\text{QCD}} \sim 0.1$ just got a bit stronger

[Lenz Piscopo Rusov 2312.13245]

- Test of charm LCSR results with tree diagrams.
- Very good agreement with the data!
- However, heavy-quark and $1/N_c$ limit are identical for T .
- Therefore, ultimate test case:
Can we understand the charm $\Delta I = 1/2$ rule?
(need color-suppressed C and exchange diagrams E).

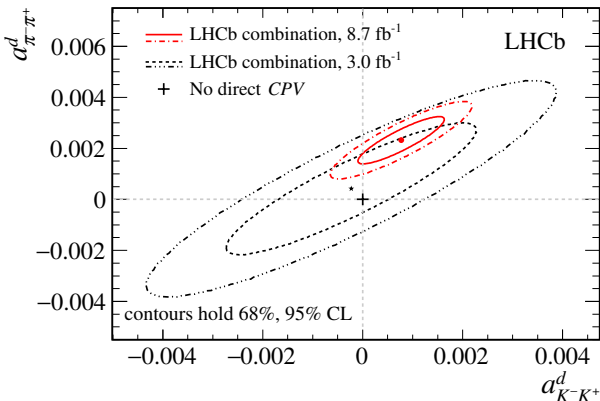
Outline

- Beyond ΔA_{CP} : A U-spin anomaly?
- Flavor Symmetry Methods at any Order.
- Determination of the $D \rightarrow \pi\pi$ penguin over tree ratio.

Beyond ΔA_{CP} : A U-spin anomaly?

Going beyond Δa_{CP}^{dir}

[LHCb, 2209.03179]



- First evidence of direct CPV in single decay, $D^0 \rightarrow \pi^+\pi^-$: 3.8σ .
- $a_{CP}^{dir}(D^0 \rightarrow K^+K^-) = (7.7 \pm 5.7) \cdot 10^{-4}$,
 $a_{CP}^{dir}(D^0 \rightarrow \pi^+\pi^-) = (23.2 \pm 6.1) \cdot 10^{-4}$.

Violation of a U-spin limit sum rule

- Separate measurement of both CP asymmetries allows for first time test of the U-spin expansion in CKM-suppressed amplitudes.

U-spin limit sum rule: **Broken at 2.7σ**

[LHCb, 2209.03179]

$$\Sigma a_{CP}^{dir} \equiv a_{CP}^{dir}(D^0 \rightarrow K^+ K^-) + a_{CP}^{dir}(D^0 \rightarrow \pi^+ \pi^-) \stackrel{\text{U-spin}}{=} 0$$

Improved U-spin limit sum rule: **Broken at 2.1σ**

[Schacht, 2207.08539]

$$-\frac{\Gamma(D^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-)} \frac{a_{CP}^{dir}(D^0 \rightarrow K^+ K^-)}{a_{CP}^{dir}(D^0 \rightarrow \pi^+ \pi^-)} = -0.93_{-0.41}^{+0.62} \neq +1.$$

- U-spin breaking is expected: Only approximate symmetry.
- Amount goes beyond generic expectations of $\sim 30\%$ at 1.9σ .

U-spin breaking in the CKM-subleading amplitude

[Schacht 2207.08539]

- To order $\mathcal{O}(\varepsilon^2)$:

$$\frac{1/2 \operatorname{Im}(\tilde{p}_1)}{\operatorname{Im}(\tilde{p}_0)} = \frac{\Sigma a_{CP}^{dir}}{\Delta a_{CP}^{dir}} + \frac{1}{2} R_{KK,\pi\pi}.$$

- We have no sensitivity yet to the corresponding real parts.
 - ➔ Need very precise measurements of time-dependent CP violation.

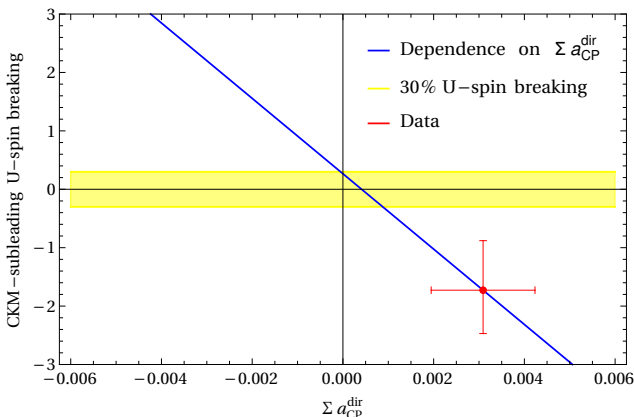
Assumption

Due to non-perturbative rescattering, the phases of \tilde{p}_0 and \tilde{p}_1 are $\mathcal{O}(1)$, resulting in

$$\left| \frac{\operatorname{Im}(\tilde{p}_1)}{\operatorname{Im}(\tilde{p}_0)} \right| \approx \frac{|\tilde{p}_1|}{|\tilde{p}_0|}.$$

CKM-subleading U-spin breaking: $\left(173^{+85}_{-74}\right) \%$

[Schacht 2207.08539]



- $\Sigma a_{CP}^{\text{dir}} \equiv a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-) + a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-)$.
- Large central value $\sim 2\sigma$ from $\mathcal{O}(30\%)$.

Model-Independent Predictions

- Large U -spin breaking indicates large $\Delta U = 1$ operator(s).
- It follows $\mathcal{O}(1)$ breaking of U -spin limit sum rule:

$$\frac{\Gamma(D^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow \pi^+ \pi^-)} = -\frac{a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-)}{a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-)} \quad \text{broken at } \mathcal{O}(1),$$

- Connected to wider class of decays via SU(3)-flavor symmetry.

Expect
$$\frac{\Gamma(D^+ \rightarrow \bar{K}^0 K^+)}{\Gamma(D_s^+ \rightarrow K^0 \pi^+)} = -\frac{a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^0 \pi^+)}{a_{CP}^{\text{dir}}(D^+ \rightarrow \bar{K}^0 K^+)} \quad \text{also broken at } \mathcal{O}(1).$$

- Improved versions of these sum rules: [\[Müller Nierste Schacht 1506.04121\]](#)

$$a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^-), \quad a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \pi^-), \quad a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^0 \pi^0), \quad \text{and} \\ a_{CP}^{\text{dir}}(D^+ \rightarrow K_S K^+), \quad a_{CP}^{\text{dir}}(D_s^+ \rightarrow K_S \pi^+), \quad a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \pi^0).$$

These should also be broken at $\mathcal{O}(1)$.

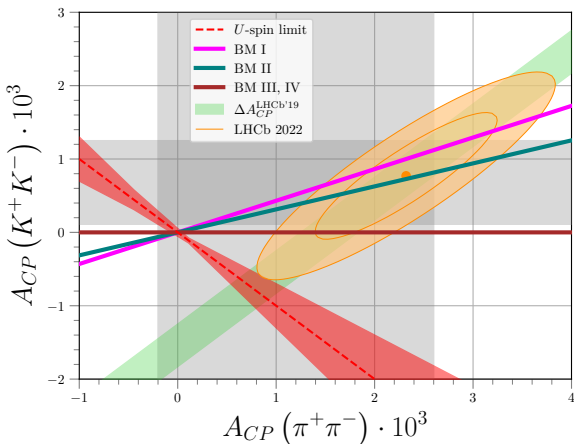
Explanations beyond the SM: “ $\Delta U = 1$ models”

- NP models with $\Delta U = 1$ operators can explain breaking of U-spin limit sum rules. [Hiller Jung Schacht 1211.3734]
- Additional operators with flavor content $\bar{s}c\bar{u}s$ and/or $\bar{d}c\bar{u}d$ with non-universal coefficients.
- Example: Z' models where fermion charges depend on generation.
[Bause Gisbert Golz Hiller 2004.01206],
[Bause Gisbert Hiller Höhne Litim Steudtner 2210.16330]

$$\begin{aligned} \mathcal{L}_{Z'} \supset & \left(g_L^{uc} \bar{u}_L \gamma^\mu c_L Z'_\mu + g_R^{uc} \bar{u}_R \gamma^\mu c_R Z'_\mu + \text{h.c.} \right) \\ & + g_L^d \bar{d}_L \gamma^\mu d_L Z'_\mu + g_R^d \bar{d}_R \gamma^\mu d_R Z'_\mu \\ & + g_L^s \bar{s}_L \gamma^\mu s_L Z'_\mu + g_R^s \bar{s}_R \gamma^\mu s_R Z'_\mu \\ & + g_L^{ll} \bar{l}_L \gamma^\mu l_L Z'_\mu + g_R^{ll} \bar{l}_R \gamma^\mu l_R Z'_\mu \end{aligned}$$

Z' model predictions

[Bause Gisbert Hiller Hhne Litim Steudtner 2210.16330]



- Viable models with leptophobic Z' below $\mathcal{O}(20 \text{ GeV})$.
- Pattern of CP violation in $D \rightarrow \pi\pi$, including $a_{CP}^{dir}(D^+ \rightarrow \pi^+\pi^0) \neq 0$.

But is U-spin actually a good symmetry?

But is U-spin actually a good symmetry?

Spectroscopy: Eightfold way.

[Gell-Mann, Ne'eman 1961]

- $SU(3)_F$ limit agrees with baryon octet mass splitting to $\sim 10\%$

[Greiner Müller 1989]

Does it work for rates, too?

- Estimate for breaking on amplitude level: $f_K/f_\pi - 1 \sim 0.2$.
- Two often-cited examples of seemingly $O(1)$ U-spin breaking:

$$\left. \frac{\mathcal{B}(D^0 \rightarrow K^+ K^-)}{\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)} \right|_{\text{exp}} \sim 3, \quad \left. \frac{\mathcal{B}(D^0 \rightarrow K_S K_S)}{\mathcal{B}(D^0 \rightarrow K^+ K^-)} \right|_{\text{exp}} \sim 0.03.$$

- Strict $SU(3)_F$ limit (including phase space):

$$\frac{\mathcal{B}(D^0 \rightarrow K^+ K^-)}{\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)} = 1, \quad \frac{\mathcal{B}(D^0 \rightarrow K_S K_S)}{\mathcal{B}(D^0 \rightarrow K^+ K^-)} = 0,$$

Yes. (But we keep testing it at every opportunity.)

[detailed review in Schacht 2207.08539]

A closer look

- Amplitude-level $SU(3)_F$ breaking of $\varepsilon \sim 30\%$ suffices in order to explain the data. [Savage 1991]

$$\frac{(1 + \varepsilon)^2}{(1 - \varepsilon)^2} \sim 3.$$

- Amplitude-level $SU(3)_F$ -breaking in $D^0 \rightarrow K_S K_S$:

$$\varepsilon' \sim \sqrt{\frac{\mathcal{B}(D^0 \rightarrow K^0 \bar{K}^0)}{\mathcal{B}(D^0 \rightarrow K^+ K^-)}} = \sqrt{\frac{2\mathcal{B}(D^0 \rightarrow K_S K_S)}{\mathcal{B}(D^0 \rightarrow K^+ K^-)}} \sim 0.26,$$

- Observations agree with global fits.

[Hiller Jung Schacht 1211.3734, Müller Nierste Schacht 1503.06759]

The picture holds at higher order, too.

[Brod Grossman Kagan Zupan 1203.6659]

Ratio of branching ratios:

$$R_{DPP} \equiv \frac{|\mathcal{A}(D^0 \rightarrow K^+ K^-)/(V_{cs} V_{us})| + |\mathcal{A}(D^0 \rightarrow \pi^+ \pi^-)/(V_{cd} V_{ud})|}{|\mathcal{A}(D^0 \rightarrow K^+ \pi^-)/(V_{cd} V_{us})| + |\mathcal{A}(D^0 \rightarrow K^- \pi^+)/(V_{cs} V_{ud})|} - 1$$

U-spin prediction

$$R_{DPP}^{\text{th}} = \mathcal{O}(\varepsilon^2).$$

Data

$$R_{DPP}^{\text{exp}} = 0.046 \pm 0.008,$$

- If U -spin breaking were $\mathcal{O}(1)$, we would have $R_{DPP}^{\text{exp}} = \mathcal{O}(1)$.
- Instead, perfectly consistent with $\mathcal{O}(\varepsilon^2)$.

Flavor Symmetry Methods at any Order

SU(3)-flavor

- SU(3): **Approximate** symmetry for the light quarks u, d, s .
- Very useful, but **$O(30\%)$ breaking** from corrections.
- Going to **higher order**: complicated.

$$\begin{aligned}
 (\mathbf{15}) \otimes (\mathbf{8}) &= (\mathbf{42}) \oplus (\mathbf{24}) \oplus (\mathbf{15}_1) \oplus (\mathbf{15}_2) \oplus (\mathbf{15}') \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3}) \\
 (\bar{\mathbf{6}}) \otimes (\mathbf{8}) &= (\mathbf{24}) \oplus (\mathbf{15}) \oplus (\bar{\mathbf{6}}) \oplus (\mathbf{3})
 \end{aligned}$$

Decay d	B_1^{31}	B_1^{32}	B_8^{31}	B_8^{32}	B_8^{61}	B_8^{62}	B_8^{151}	...
$D^0 \rightarrow K^+ K^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{10}$	$-\frac{1}{10\sqrt{2}}$	$-\frac{7}{10\sqrt{122}}$...
$D^0 \rightarrow \pi^+ \pi^-$	$\frac{1}{4\sqrt{10}}$	$\frac{1}{8}$	$\frac{1}{10\sqrt{2}}$	$\frac{1}{4\sqrt{5}}$	$-\frac{1}{10}$	$\frac{1}{10\sqrt{2}}$	$-\frac{11}{10\sqrt{122}}$...
$D^0 \rightarrow \bar{K}^0 K^0$	$-\frac{1}{4\sqrt{10}}$	$-\frac{1}{8}$	$\frac{1}{5\sqrt{2}}$	$\frac{1}{2\sqrt{5}}$	0	0	$-\frac{5}{\sqrt{122}}$...
$D^0 \rightarrow \pi^0 \pi^0$	$-\frac{1}{8\sqrt{5}}$	$-\frac{1}{8\sqrt{2}}$	$-\frac{1}{20}$	$-\frac{1}{4\sqrt{10}}$	$\frac{1}{10\sqrt{2}}$	$-\frac{1}{20}$	$\frac{11}{20\sqrt{61}}$...
...

Solving the Problem of Higher Order U-spin

[Gavrilova Grossman Schacht, 2205.12975]

Theorems enabling calculations to arbitrary order.

- We are able to determine **a priori** up to which order sum rules exist.
- We do not need explicit Clebsches. Big **complexity reduction**.
- Hope: Opens the door for **precision in hadronic decays**.
- **Close a gap** between theory and experiment, especially regarding multi-body decays.

Take advantage of precision data on nonleptonic decays.

Systematics of U-spin breaking

- U-spin breaking from mass difference of strange and down quarks:

$$\varepsilon = \frac{m_s - m_d}{\Lambda_{\text{QCD}}} \sim 0.3.$$

- Parametrized by triplet-operator H_ε :

$$\mathcal{H}_{\text{eff}} = \sum_{m,b} f_{u,m} \left(H_m^u \otimes H_\varepsilon^{\otimes b} \right), \quad H_\varepsilon^{\otimes b} \equiv \underbrace{H_\varepsilon \otimes \cdots \otimes H_\varepsilon}_b.$$

- Any system can be constructed from tensor products of doublets.
- Moving irreps ("crossing sym.") does not affect structure of sum rules.
- Without loss of generality, consider doublet-only system with

$$0 \rightarrow \left(\frac{1}{2} \right)^{\otimes n} \quad \text{and singlet Hamiltonian.}$$

Properties of U-spin pairs

[Gavrilova Grossman Schacht, 2205.12975]

- Amplitude:

$$A_j = \underbrace{(-, -, +, -, +, \dots, +)}_n = \sum_{\alpha} C_{j\alpha} X_{\alpha}.$$

- U-spin conjugated** amplitude (complete interchange $s \leftrightarrow d$):

$$\bar{A}_j = \underbrace{(+, +, -, +, -, \dots, -)}_n = (-1)^p \sum_{\alpha} (-1)^b C_{j\alpha} X_{\alpha}.$$

- Notation: Abbreviate m -quantum number: $\pm 1/2 \mapsto \pm$.

X_{α} : Reduced matrix element. $C_{j\alpha}$: Clebsches.

- Define **(anti)-symmetric combinations** of U -spin pairs:

$$a_j \equiv \underbrace{A_j - (-1)^p \bar{A}_j}_{\text{odd in } b}, \quad s_j \equiv \underbrace{A_j + (-1)^p \bar{A}_j}_{\text{even in } b}.$$

Results: Sum Rules at any order of U-spin breaking

[Gavrilova Grossman Schacht, 2205.12975]

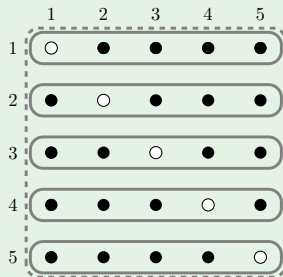
All sum rules at any order b can be written as:

$$\Sigma_j a_j = 0,$$

$$\Sigma_j s_j = 0.$$

Example: $n = 6$ doublets. Lattice dimension $d = n/2 - 1 = 2$.

- Each node \Leftrightarrow U-spin pair.
- Each node (points):
 a -type sum rule valid to $b = 0$.
- Sums of nodes in lines:
 s -type sum rules valid to $b = 1$.
- Sum of all nodes in plane:
 a -type sum rule valid up to $b = 2$.



What next? Generalization to $SU(3)_F$, implications for observables.

Determination of the $D \rightarrow \pi\pi$ penguin over tree ratio

A caveat for the interpretation of the data

“The data implies $|P/T| = \mathcal{O}(1)$ ”

This statement actually relies on an underlying, commonly made assumption:

- The relative strong phase between P and T is assumed $\mathcal{O}(1)$ (from rescattering).

Reminder: CP violation is an interference effect.

$$A = (-V_{cd}^* V_{ud}) \times T - \left(\frac{V_{cb}^* V_{ub}}{2} \right) \times P.$$

Direct CP asymmetry:

$$a_{CP}^f \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \text{CKM} \times \left| \frac{P}{T} \right| \times \sin \left(\arg \left(\frac{P}{T} \right) \right)$$

How to determine the strong phase from the data

- Strong phases can be obtained from measurements of time-dependent CP violation or quantum-correlated decays. [Xing hep-ph/9606422, Gronau Grossman Rosner

hep-ph/0103110, Bevan Inguglia Meadows 1106.5075, Bevan Meadows 1310.0050, Grossman Kagan Zupan 1204.3557, Xing 1903.09566,

Grossman Schacht 1903.10952, Schacht 2207.08539]

$$A_{CP}(f, t) \approx a_{CP}^f + \Delta Y^f \frac{t}{\tau_{D^0}}$$

- Analogous strong phase in multi-body decays can be determined from fit to CP violating time-integrated Dalitz plot.

[Dery Grossman Schacht Soffer 2101.02560]

- No measurements of phases yet, despite big experimental advances. ΔY^f have large errors, and sensitivity to phase is subleading.

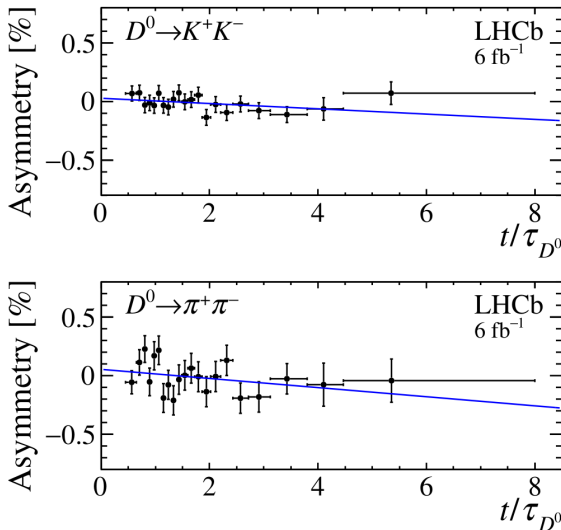
$$\Delta Y^{K^+K^-} = (-0.3 \pm 1.3 \pm 0.3) \cdot 10^{-4},$$

$$\Delta Y^{\pi^+\pi^-} = (-3.6 \pm 2.4 \pm 0.4) \cdot 10^{-4}.$$

[LHCb combination 2105.09889]

$$A_{CP}(f, t) \approx a_{CP}^f + \Delta Y^f \times t/\tau_{D^0}$$

[LHCb 2105.09889]



Extraction of Penguin over tree in three-body decays

[Dery Grossman Schacht Soffer 2101.02560]

$$\mathcal{A}(D^0 \rightarrow \pi^+ \rho^-) = -\lambda T^{P_1 V_2} - V_{cb}^* V_{ub} R^{P_1 V_2}$$

$$\mathcal{A}(D^0 \rightarrow \pi^- \rho^+) = -\lambda T^{P_2 V_1} - V_{cb}^* V_{ub} R^{P_2 V_1}$$

- Time-integrated CP asym. of **2-body decays** give only combinations

$$|\widetilde{R}^{P_1 V_2}| \sin(\delta_{P_1 V_2}) \quad \text{and} \quad |\widetilde{R}^{P_2 V_1}| \sin(\delta_{P_2 V_1}),$$

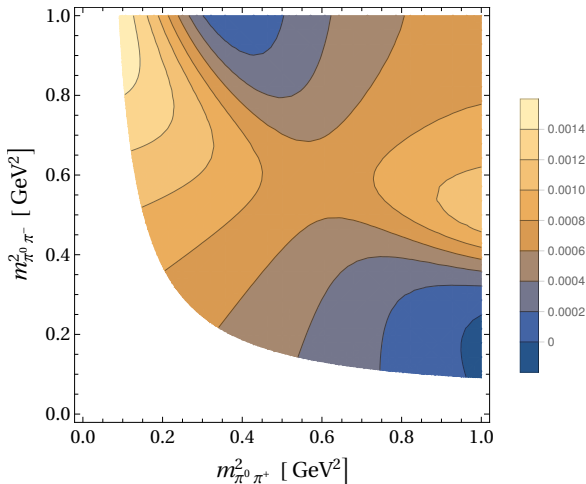
but **not magnitudes and phases separately**.

- Three body decay** changes 2 things:
 - ▶ We have additional kinematic dependences.
 - ▶ Only in a three-body decay we have **interference** between $D^0 \rightarrow \pi^+(\rho^- \rightarrow \pi^- \pi^0)$ and $D^0 \rightarrow \pi^-(\rho^+ \rightarrow \pi^+ \pi^0)$.

➡ **Extraction of all parameters** from **time-integrated** CP meas.

Local $a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+\pi^-\pi^0)$ in overlap region of ρ^\pm

[Dery Grossman Schacht Soffer 2101.02560]



Numerical example: $\widetilde{R}^{P_1 V_2} = \exp(i\pi/2)$, $\widetilde{R}^{P_2 V_1} = \frac{1}{4} \exp(i\pi/3)$

Back to $D \rightarrow \pi\pi$: Isospin to the rescue

[Gavrilova Grossman Schacht, 2312.10140]

- Assuming SM, isospin allows determination of strong P/T phase from direct CP asymmetries and branching ratios only.
- Also enables extraction of magnitude of P/T .
- $D \rightarrow \pi\pi$ has same group-theory structure as $B \rightarrow \pi\pi$ [Gronau London 1990], however, different approximations are used in the two systems.
- Before our work: Assume $\arg \sim \mathcal{O}(1)$ (rescattering) to extract $|P/T|$.
- Our work: Extract $|P/T|$ from data without assumption about phase.

Isospin Decomposition

$$A^{+-} = -\lambda_d \left(\sqrt{2} (t_{1/2} + t_{3/2}) \right) - \frac{\lambda_b}{2} \left(\sqrt{2} p_{1/2} \right),$$

$$A^{00} = -\lambda_d (2t_{3/2} - t_{1/2}) - \frac{\lambda_b}{2} (-p_{1/2}),$$

$$A^{+0} = -\lambda_d (3 t_{3/2}).$$

Assumptions, expected to hold at $O(1\%)$.

- Isospin limit, neglect isospin breaking effects.
- Neglect electromagnetic corrections and electroweak penguin contributions. Subleading due to smallness of Wilson coefficients.

Consequences:

- No $\Delta I = 5/2$ matrix elements.
- No $\Delta I = 3/2$ contribution to CKM-subleading amplitude.

Observables and Theory Parameters

- 5 Observables:

$$\mathcal{B}^{+-}, \quad \mathcal{B}^{00}, \quad \mathcal{B}^{+0}, \quad a_{CP}^{+-}, \quad a_{CP}^{00}.$$

- 5 Parameters:

$$|t_{3/2}|, \quad |t_{1/2}|, \quad |p_{1/2}|, \quad \arg(t_{1/2}), \quad \arg(p_{1/2}).$$

- Isospin limit: system completely solvable, up to discrete ambiguities.
- Combinations:

$$r_t = \left| \frac{t_{1/2}}{t_{3/2}} \right|, \quad \delta_t = \arg\left(\frac{t_{1/2}}{t_{3/2}}\right), \quad \delta_d \equiv \arg\left(\frac{A_d^{+-}}{A_d^{00}}\right).$$

$$r^f = \left| \frac{A_b^f}{A_d^f} \right|, \quad \delta^f = \arg\left(\frac{A_b^f}{A_d^f}\right), \quad f = +-, 00.$$

P/T in terms of operator matrix elements

- Ambiguity to define two interfering amplitudes (using CKM unitarity).
- Therefore, ratio of two interfering amplitudes P/T not clearly defined.
- Unambiguous definition with operator matrix elements:

[Khodjamirian Petrov 1706.07780]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left(\sum_{q=d,s} \lambda_q (C_1 \mathcal{Q}_1^q + C_2 \mathcal{Q}_2^q) \right) \equiv \sum_{q=d,s} \lambda_q \mathcal{O}^q,$$

$$\langle \mathcal{O}^q \rangle^f \equiv \langle f | \mathcal{O}^q | D^0 \rangle,$$

$$A(D^0 \rightarrow f) = \lambda_d \langle \mathcal{O}^d \rangle^f + \lambda_s \langle \mathcal{O}^s \rangle^f = -\lambda_d (\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f) - \frac{\lambda_b}{2} (2\langle \mathcal{O}^s \rangle^f),$$

Unambiguous definition of P/T ratio as:

$$r^f \equiv \left| \frac{2\langle \mathcal{O}^s \rangle^f}{\langle \mathcal{O}^s \rangle^f - \langle \mathcal{O}^d \rangle^f} \right|,$$

Theory Limits

$$r^f \equiv \left| \frac{2\langle O^s \rangle^f}{\langle O^s \rangle^f - \langle O^d \rangle^f} \right|$$

- “Rescattering” \equiv matrix element of operator $O^s \neq 0$.
- Naive interpretation: $s\bar{s}$ pair rescatters into $d\bar{d}$, hadronization to pions.

No-rescattering limit

$$\langle O^s \rangle_{\text{no rescatt.}} = 0, \quad \text{i.e.} \quad r_{\text{no rescatt.}}^f = 0.$$

Limit of large rescattering

$$|\langle O^s \rangle| \sim |\langle O^d \rangle|, \quad \text{i.e.} \quad r^f \gtrsim 1, \quad \text{depending on relative phase.}$$

Strict large N_c limit

[Buras Gerard Bardeen 1401.1385]

$$\left| \frac{t_{1/2}}{t_{3/2}} \right|^{N_c \rightarrow \infty} = 2.$$

In $K \rightarrow \pi\pi$ this is much larger ($\Delta I = 1/2$ rule).

Determination of Tree Parameters

Assumption

For branching ratios, we neglect contributions which are suppressed by $\mathcal{O}(|\lambda_b/\lambda_d|) \sim \lambda^4$, where $\lambda \approx 0.23$ is the Wolfenstein parameter, such that

$$\mathcal{B}^f = \mathcal{P}^f \cdot |\lambda_d|^2 |A_d^f|^2.$$

We can then determine $|t_{3/2}|$, $|t_{1/2}|$, $\cos \delta_t$ from \mathcal{B}^{+-} , \mathcal{B}^{00} , \mathcal{B}^{+0} .

$$r_t \equiv \frac{|t_{1/2}|}{|t_{3/2}|} = \sqrt{3 \frac{\mathcal{B}^{+-}}{\mathcal{B}^{+0}} \frac{\mathcal{P}^{+0}}{\mathcal{P}^{+-}} + 3 \frac{\mathcal{B}^{00}}{\mathcal{B}^{+0}} \frac{\mathcal{P}^{+0}}{\mathcal{P}^{00}} - 2},$$

$$\cos \delta_t \equiv \cos \left(\arg \left(\frac{t_{1/2}}{t_{3/2}} \right) \right) = \frac{1}{2r_t} \left(\frac{9}{2} \frac{\mathcal{B}^{+-}}{\mathcal{B}^{+0}} \frac{\mathcal{P}^{+0}}{\mathcal{P}^{+-}} - r_t^2 - 1 \right).$$

$\Rightarrow \cos \delta_d$ and $|\sin \delta_d|$.

Determination of P/T

Assumption

We calculate direct CP asymmetries at leading order in $|\lambda_b/\lambda_d|$

$$a_{CP}^f = \text{Im} \left(\frac{\lambda_b}{-\lambda_d} \right) r^f \sin(\delta^f).$$

Relation between CKM-subleading amplitudes:

$$\frac{1}{\sqrt{2}} A_b^{+-} = -A_b^{00}.$$

These are actually two relations:

$$\left| \frac{A_b^{+-}}{A_b^{00}} \right| = \sqrt{2}, \quad \arg \left(\frac{A_b^{+-}}{A_b^{00}} \right) = \pi.$$

- ➡ 2 isospin limit relations which we can solve for $\sin \delta^{+-}$ and $\sin \delta^{00}$.
- ➡ Once we have these, get r^{00} and r^{+-} from a_{CP}^f

Closed-form solution

[Gavrilova, Grossman, Schacht 2312.10140]

$$\sin \delta^{00} = \frac{-\text{sign}(a_{CP}^{00})}{\sqrt{1 + \frac{1}{\sin^2 \delta_d} \left(\frac{a_{CP}^{+-}}{a_{CP}^{00}} \sqrt{\frac{1}{2} \frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}} \frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}} + \cos \delta_d \right)^2}},$$

$$\sin \delta^{+-} = \frac{-\text{sign}(a_{CP}^{+-})}{\sqrt{1 + \frac{1}{\sin^2 \delta_d} \left(\frac{a_{CP}^{00}}{a_{CP}^{+-}} \sqrt{2 \frac{\mathcal{P}^{+-}}{\mathcal{B}^{+-}} \frac{\mathcal{B}^{00}}{\mathcal{P}^{00}}} + \cos \delta_d \right)^2}},$$

$$r^{00} = \frac{1}{|\text{Im}(-\lambda_b/\lambda_d)|} \sqrt{(a_{CP}^{00})^2 + \frac{(a_{CP}^{+-} \sqrt{\mathcal{B}^{+-} \mathcal{P}^{00}} + a_{CP}^{00} \sqrt{2 \mathcal{B}^{00} \mathcal{P}^{+-}} \cos \delta_d)^2}{2 \mathcal{B}^{00} \mathcal{P}^{+-} \sin^2 \delta_d}},$$

$$r^{+-} = \frac{1}{|\text{Im}(-\lambda_d/\lambda_d)|} \sqrt{(a_{CP}^{+-})^2 + \frac{(a_{CP}^{00} \sqrt{2 \mathcal{B}^{00} \mathcal{P}^{+-}} + a_{CP}^{+-} \sqrt{\mathcal{B}^{+-} \mathcal{P}^{00}} \cos \delta_d)^2}{\mathcal{B}^{+-} \mathcal{P}^{00} \sin^2 \delta_d}}.$$

Knowledge of $D \rightarrow \pi^+\pi^-$ translates into $D \rightarrow \pi^0\pi^0$

[Gavrilova, Grossman, Schacht 2312.10140]

$$\frac{\sin \delta^{+-}}{\sin \delta^{00}} = \frac{a_{CP}^{+-}}{a_{CP}^{00}} \sqrt{\frac{1}{2} \frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}} \frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}},$$

$$\frac{r^{00}}{r^{+-}} = \sqrt{\frac{1}{2} \frac{\mathcal{B}^{+-}}{\mathcal{P}^{+-}} \frac{\mathcal{P}^{00}}{\mathcal{B}^{00}}}.$$

- Although we have essentially no information about $\sin \delta^{00}$ we can obtain non-trivial information about r^{00} , due to the correlation to r^{+-} from isospin.

Numerical Results: Systematic Uncertainties

- Our results relate P/T to \mathcal{B}^f and a_{CP}^f .
- In experimental extraction of a_{CP}^f additional assumptions are made.
- For extraction of a_{CP}^{+-} LHCb assumes a universal ΔY .
We use same ΔY for the extraction of a_{CP}^{00} .
- A universal ΔY is motivated by U-spin [Kagan Silvestrini 2001.07207]
It follows overall systematic theory uncertainty of $\mathcal{O}(\varepsilon^2) \sim 10\%$.
- In order to reach theory uncertainty of $\mathcal{O}(1\%)$ we need extractions of a_{CP}^f without universality assumption: future data on ΔY^{+-} and ΔY^{00} .
- Our numerics with current data include universality assumption, implying an additional overall theory uncertainty of $\mathcal{O}(10\%)$.

Numerical Results

[Gavrilova, Grossman, Schacht 2312.10140]

Direct CP Asymmetries		
a_{CP}^{+0}	$+0.004 \pm 0.008$	[79–82]
a_{CP}^{00}	-0.0002 ± 0.0064	^a [79, 83, 84]
a_{CP}^{+-}	0.00232 ± 0.00061	[2]
Branching Ratios		
$\mathcal{B}(D^0 \rightarrow \pi^+\pi^0)$	$(1.247 \pm 0.033) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \rightarrow \pi^+\pi^-)$	$(1.454 \pm 0.024) \cdot 10^{-3}$	[85]
$\mathcal{B}(D^0 \rightarrow \pi^0\pi^0)$	$(8.26 \pm 0.25) \cdot 10^{-4}$	[85]
Further Numerical Inputs		
$\text{Im}(\lambda_b/(-\lambda_d))$	$(-6.1 \pm 0.3) \cdot 10^{-4}$	[85]

TABLE I. Experimental input data. We use the decay times and masses from Ref. [85]. ^aOur extraction from $A_{CP}(D^0 \rightarrow \pi^0\pi^0) = -0.0003 \pm 0.0064$ [79] and $\Delta Y = (-1.0 \pm 1.1 \pm 0.3) \cdot 10^{-4}$ [52].

a_{CP}^{+-}	$(2.32 \pm 0.07) \cdot 10^{-3}$
a_{CP}^{00}	$(-2 \pm 9) \cdot 10^{-4}$

TABLE II. Future data scenario employing the current central values and using prospects for the errors from Table 6.5 of Ref. [86] (300 fb^{-1}) and Table 122 of Ref. [87] (50 ab^{-1}) for $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow \pi^0\pi^0$, respectively. All other input data is left as specified in Table I.

Parameter	Current data	Future data scenario
r_t	3.43 ± 0.06	3.43 ± 0.06
$\cos \delta_t$	0.06 ± 0.02	0.06 ± 0.02
$\cos \delta_d$	-0.68 ± 0.01	-0.68 ± 0.01
$ \sin \delta^{00} $	0_{-0}^{+1}	$0.06_{-0.06}^{+0.20}$
$ \sin \delta^{+-} $	$0.7_{-0.5}^{+0.3}$	$0.69_{-0.16}^{+0.21}$
r^{00}	$5.2_{-2.4}^{+13.3}$	$5.2_{-1.2}^{+1.6}$
r^{+-}	$5.5_{-2.7}^{+14.2}$	$5.5_{-1.3}^{+1.8}$

TABLE III. Numerical results for current and hypothetical future data. In the future data scenario, the results for r_t , $\cos \delta_t$ and $\cos \delta_d$ are identical to the ones with current data, as these depend only on the branching ratio data which is not modified in the future data scenario compared to current data. Furthermore, in the future data scenario $\sin \delta^{+-} < 0$. The overall additional relative systematic uncertainty of $\mathcal{O}(10\%)$ due to the universality assumption of ΔY for the extraction of the direct CP asymmetries comes on top of the errors shown here, see text for details.

- $|P/T|$ is large. Future data will significantly reduce errors.

Benchmark Scenarios [Gavrilova, Grossman, Schacht 2312.10140]

Hypothesis	Current data
$r^{+-} = 1.0$	2.7σ
$r^{+-} = 0.1$	3.7σ
$r^{00} = 1.0$	2.6σ
$r^{00} = 0.1$	3.7σ
$p_{1/2} = 0$	3.8σ

TABLE IV. Test of benchmark hypotheses and significance of their rejection for current data. In the considered future data scenario all hypotheses listed here are rejected at $> 5\sigma$. In order to account for the overall $\mathcal{O}(10\%)$ relative systematic uncertainty due to the assumption of a universal ΔY for the extraction of the direct CP asymmetries, we multiply the hypotheses for r^f by a factor 1.10, resulting in a more conservative (lower) significance of rejection, see text for details.

- Large rescattering or BSM?

Conclusions

- This is **just the beginning** of the exploration of charm CPV.
- Charm is a **unique gate** to flavor structure of **up-type** quarks.
- Can we tell a loop from a tree?



- No matter what, we will learn sth new: **QCD** or **New Physics**.



BACK-UP

Charm: Non-perturbative Diagrams

