Flavour physics at the precision frontier

Using b-hadron decays to probe the Standard Model and seek for New Physics

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High energy physics seminar

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UNIVERSITY OF CAMBRIDGE

Talk outline

Introduction

Theoretical framework

- $b \to s\ell^+\ell^-$ effective Hamiltonian
- form factors definition

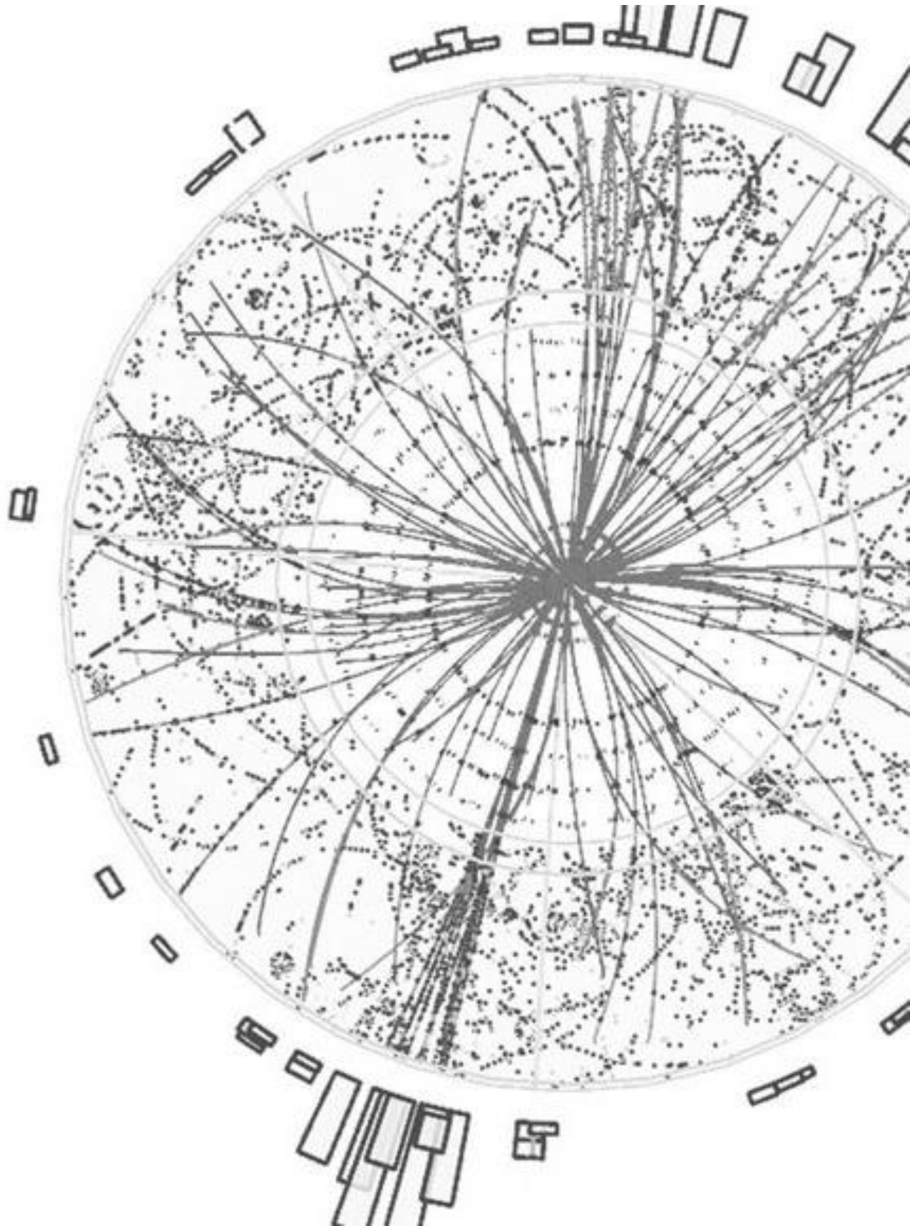
Local and non-local form factors

- calculation
- analysis and results

SM predictions

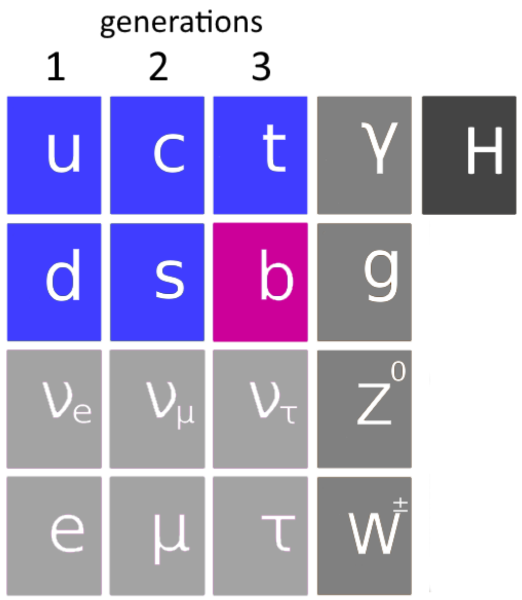
- comparison between SM predictions and data
- global fit to $b \to s\mu^+\mu^-$

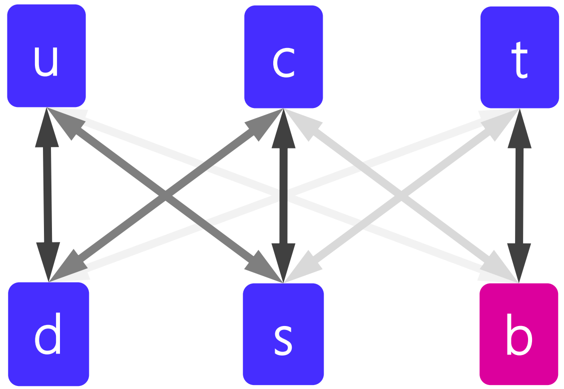
Summary and outlook



Introduction

The beauty of the Standard Model





SM: 6 quark flavours and 6 lepton flavours

Flavour physics investigates the properties, the transitions, and the spectrum of the different quark and lepton flavours

Transitions between different (flavours) mediated by $W^{\pm}$

Why is the *b* quark interesting?

- third generation quark
- heaviest fermion that forms bound states ($m_b \gg \Lambda_{\mathrm{QCD}}$)
- lighter than the t quark
  $\Longrightarrow$ decays in quarks of another generation
  $\Longrightarrow$ CKM suppressed decay

New Physics (NP) searches

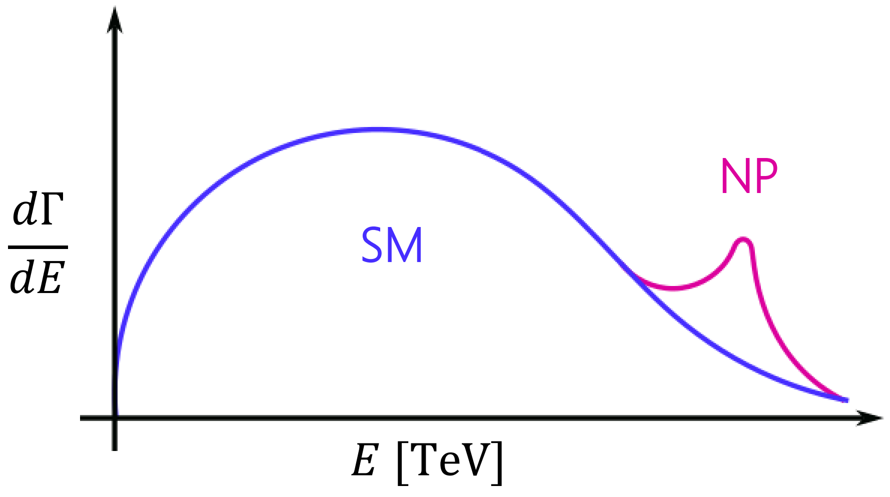
Direct searches

LHC has reached its maximum energy

No NP evidence so far (too heavy?)

Next experiments will probably focus on **precision**

Direct NP discovery difficult in coming decades

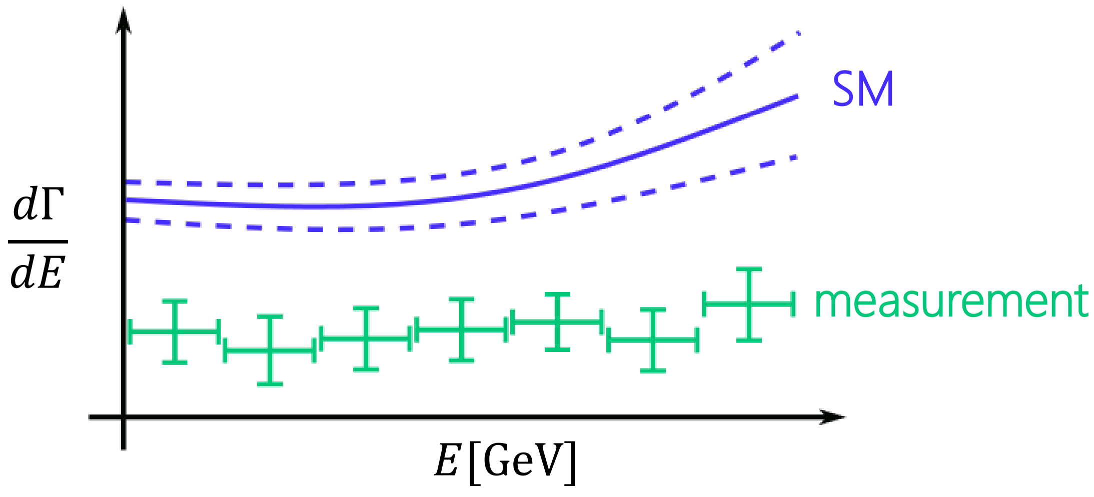


Indirect searches (with flavour)

Probe the SM at **higher energies** than direct searches

Compare precise measurements and calculations of flavour observables

$\Longrightarrow$ obtain constraints on NP (or new discovery?)



Flavour changing currents

Flavour changing charged currents (FCCC) occur at tree level (mediated by $W^{\pm}$) in the SM

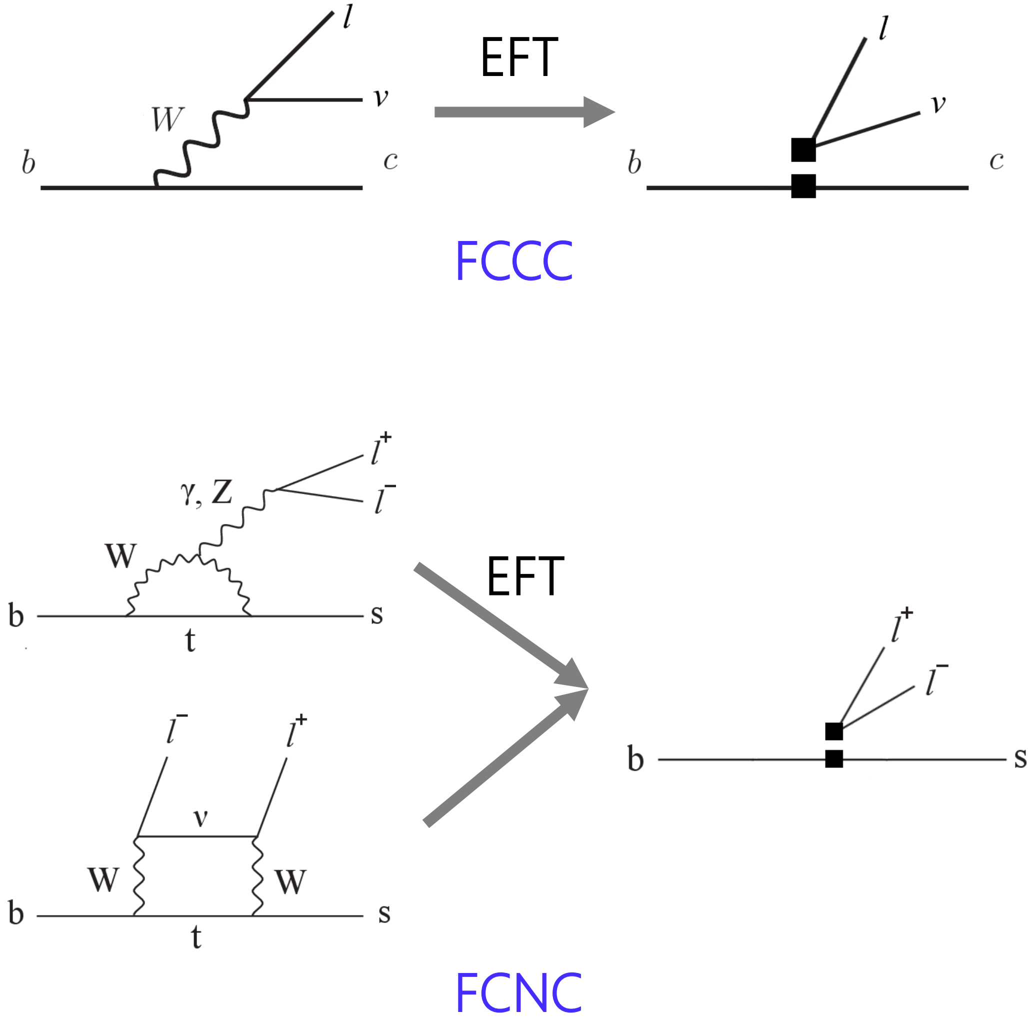
Flavour changing neutral currents (FCNC) absent at tree level in the SM. Focus on $b \to s\ell^+\ell^-$
FCNC are loop, GIM and CKM **suppressed in the SM**

FCNC **sensitive to new physics** contributions
Ideal for **indirect searches**

Integrate out DOF heavier than the b

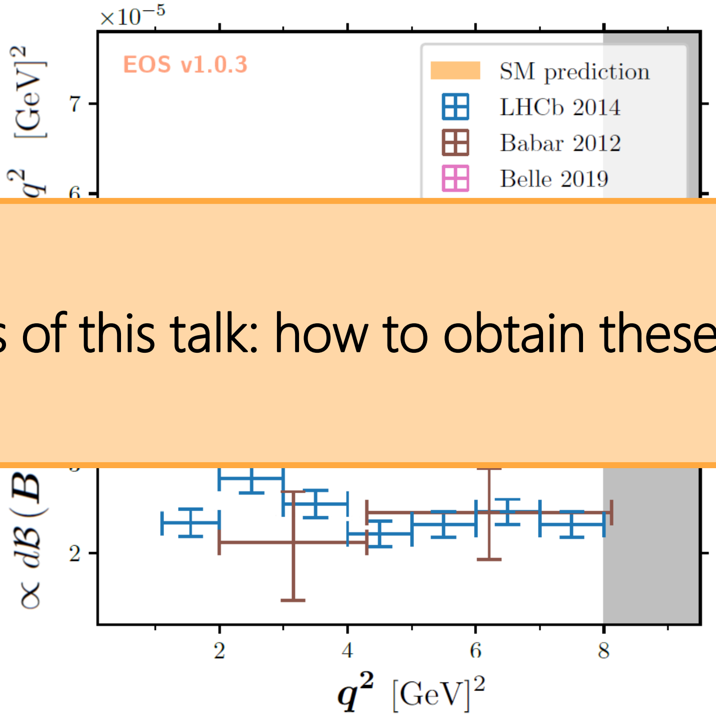
$\Downarrow$

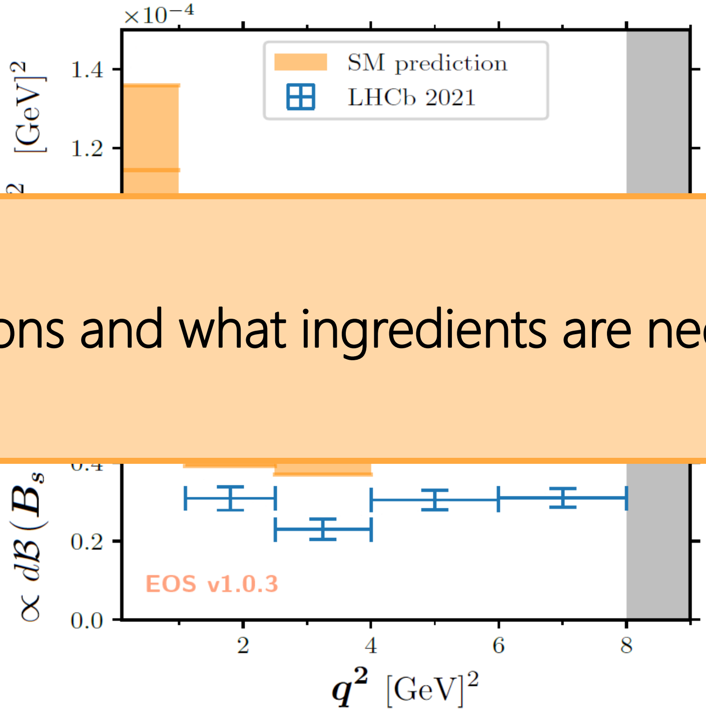
Weak effective field theory



Indirect searches with $b \to s\mu^+\mu^-$

Test the SM and constrain NP with $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays





Focus of this talk: how to obtain these SM predictions and what ingredients are needed

Agreement between theory and experiment for LFU ratios R_K and R_{K^*},
but **tension (or anomalies) remains for** $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ observables
$\Longrightarrow$ need to understand this tension

Importance of theory predictions

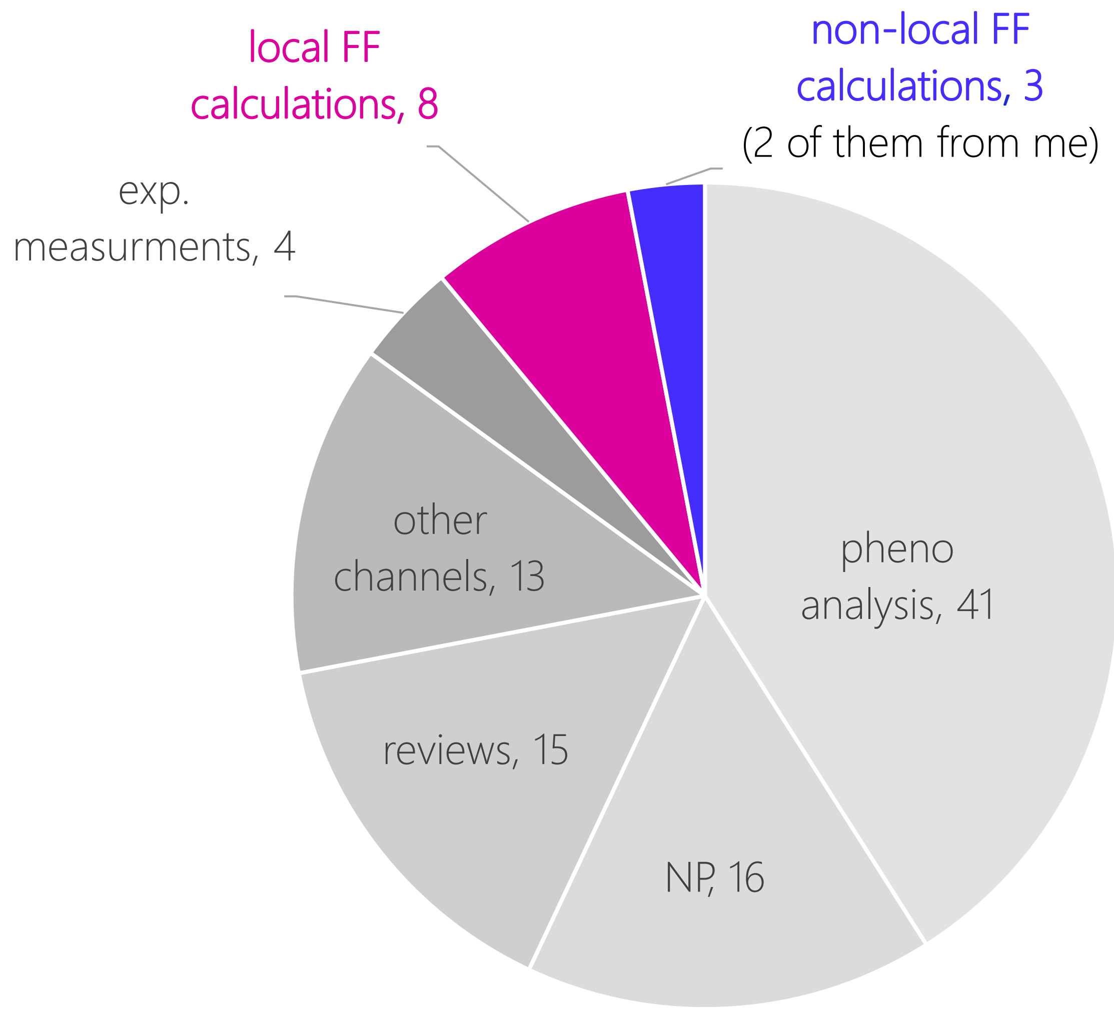
1. Tensions in $b \to s\ell^+\ell^-$:
   NP or misestimated QCD effects?
2. **Constrain physics beyond the SM**
   (SMEFT Wilson coefficients)

Very active field of research

$\Updownarrow$

Tremendous experimental efforts
LHCb, CMS, ATLAS, Belle (II)

Need more theory calculations
to fully exploit experimental work



Example: distribution of first 100 citations of [NG/van Dyk/Virto 2020]

Theoretical framework

$b \to s\ell^+\ell^-$ effective Hamiltonian

Transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)O_i(\mu) \qquad \mu = m_b$$

Main contributions to $B \to K^{(*)}\mu^+\mu^-$ in the SM given by operators O_7, O_9, O_{10}

$$O_7 = \frac{e}{16\pi^2} m_b(\bar{s}_L\sigma^{\mu\nu}b_R)F_{\mu\nu} \qquad O_9 = \frac{e^2}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\sum_\ell(\bar{\ell}\gamma_\mu\ell) \qquad O_{10} = \frac{e^2}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\sum_\ell(\bar{\ell}\gamma_\mu\gamma_5\ell)$$

Additional contributions given by operators O_1, O_2

$$O_2 = (\bar{s}_L T^a\gamma^\mu c_L)(\bar{c}_L T^a\gamma^\mu b_L) \qquad O_2 = (\bar{s}_L\gamma^\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

Calculate decay amplitudes precisely to probe the SM

$$\langle K^{(*)}\ell^+\ell^-|O_{\mathrm{eff}}|B\rangle = \langle \ell\ell|\mathcal{O}_{\mathrm{lep}}|0\rangle\langle K^{(*)}|O_{\mathrm{had}}|B\rangle + \text{non-fact.}$$

Analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays

$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

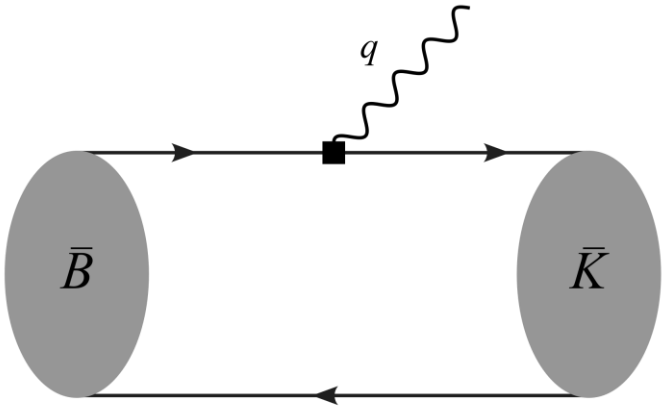
Calculate decay amplitudes precisely to probe the SM

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

Easily obtain the (differential) branching ratio and angular observables from the amplitude

$$\frac{d\mathcal{B}}{dq^2} = \frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dq^2} \propto |\mathcal{A}|^2$$

q^2 is the momentum transfer squared



$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

Calculate decay amplitudes precisely to probe the SM

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\underbrace{\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)} \mathcal{F}_\mu - \underbrace{\frac{L_V^\mu}{q^2}}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

Wilson coefficients, leptonic matrix elements (and constants $\boldsymbol{\alpha}$, $\boldsymbol{V_{CKM}}$...)

perturbative objects, small uncertainties

$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

Calculate decay amplitudes precisely to probe the SM

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

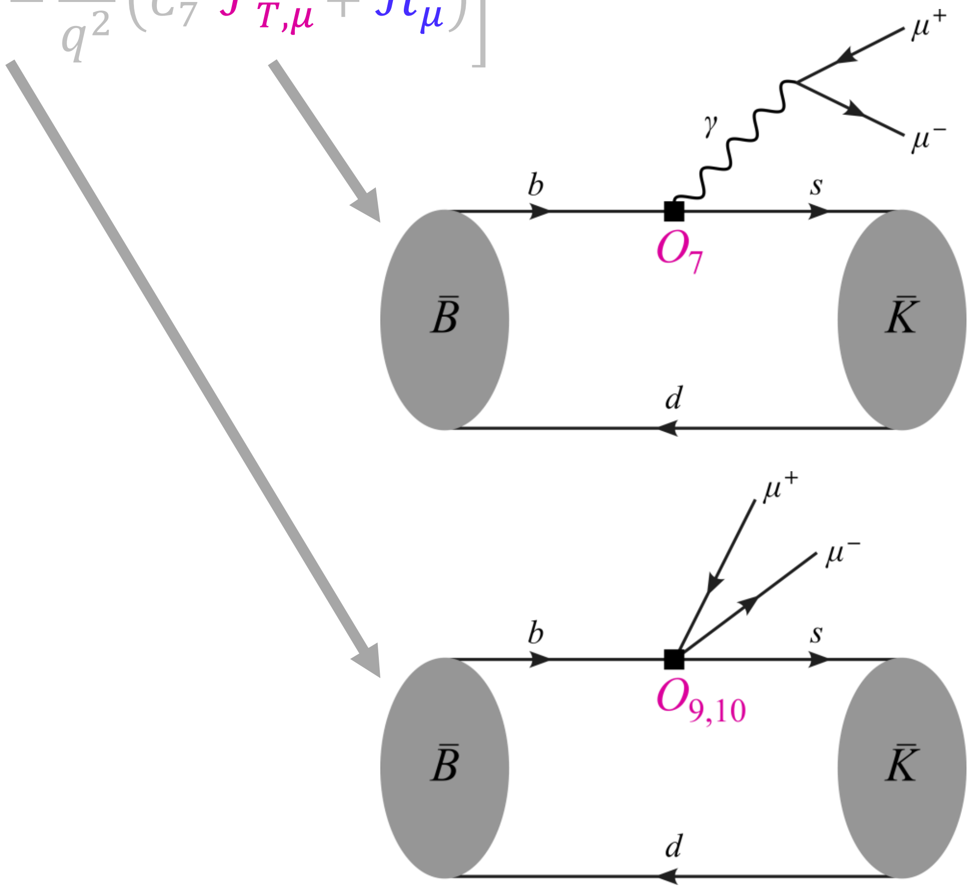
Local hadronic matrix elements

$$\mathcal{F}_\mu = \langle K^{(*)} | O_{7,9,10}^{\text{had}} | B \rangle \qquad O_{7,9,10}^{\text{had}} = (\bar{s}\,\Gamma\, b)$$

leading hadronic contributions

non-perturbative QCD objects
$\Longrightarrow$ calculate with lattice QCD (or LCSR)

moderate uncertainties (3% – 15%)



$B \to K^{(*)}\ell^+\ell^-$ decay amplitude

Calculate decay amplitudes precisely to probe the SM

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\,\mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

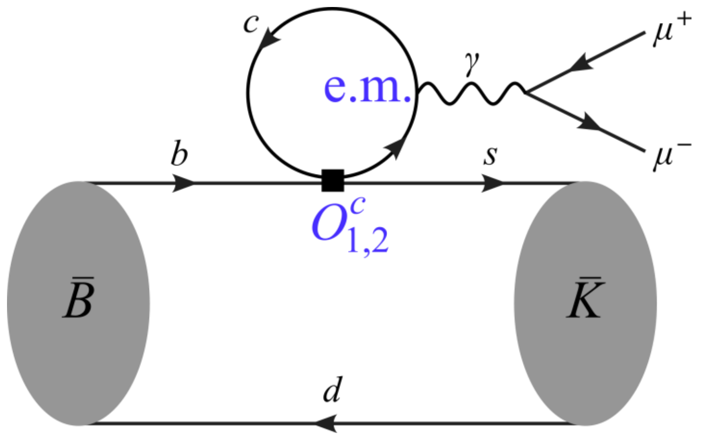
Non-local hadronic matrix elements

$$\mathcal{H}_\mu = i\int d^4x\, e^{iq\cdot x}\langle K^{(*)}|T\{j_\mu^{\mathrm{em}}(x), O_{1,2}^c(0)\}|B\rangle$$

subleading (?) hadronic contributions

non-perturbative QCD objects
$\Longrightarrow$ very hard to calculate

large uncertainties



Form factors definitions

Form factors (FFs) **parametrize hadronic matrix elements**

FFs are functions of the momentum transfer squared $\boldsymbol{q^2}$

Local FFs

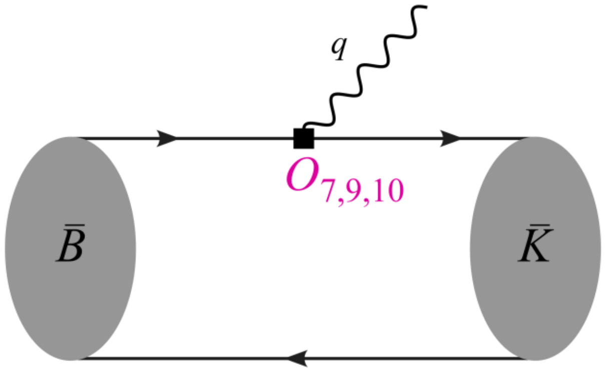
$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\,\mathcal{F}_\lambda(q^2)$$

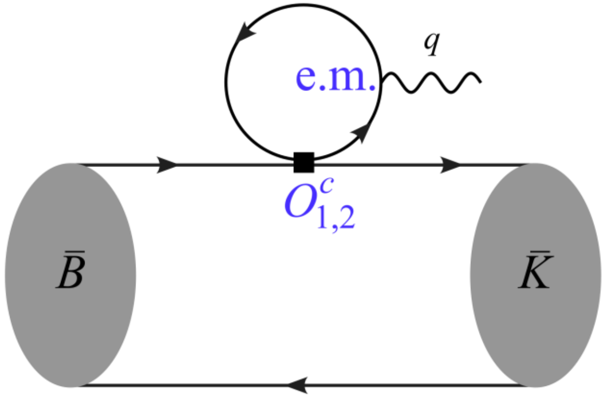
decomposition follows from Lorentz invariance

Non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\mathcal{H}_\lambda(q^2)$$

analogous to local FFs



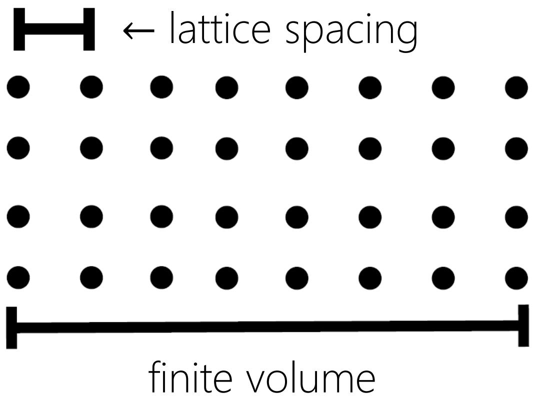


Local form factors

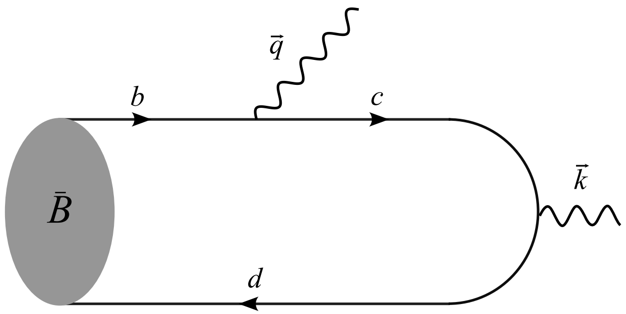
Methods to compute FFs

Non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   more efficient usually at **high** q^2



2. Light-cone sum rules (LCSRs)
   only applicable at **low** q^2

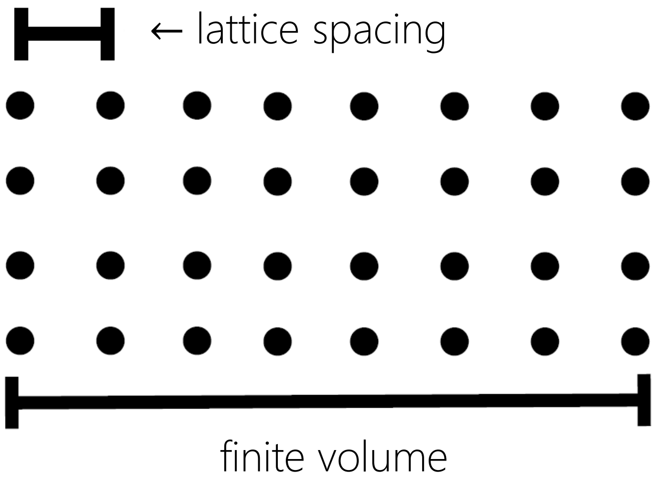


Complementary approaches to calculate FFs

Lattice QCD in a nutshell

LQCD = evaluating path integrals numerically

$$\text{matrix element} = \int \prod_i \mathrm{d}\phi_i \,(\text{correlator})$$



To perform the calculation approximations are needed

1. nonzero lattice spacing
2. finite volume
3. Euclidian space time

Pros

first principles calculations

reducible systematic uncertainties

Cons

nonlocal matrix elements and unstable states, are still work in progress

computationally very expensive

LCSRs in a nutshell

LCSRs are a method to calculate hadronic matrix elements

Method based on:

dispersion relation | quark-hadron duality assumption | Operator Product Expansion

Pros

compute hadronic matrix elements not accessible yet with LQCD

complementary w.r.t. LQCD

relatively faster

Cons

need universal non-perturbative inputs (QCD condensates or distribution amplitudes)

non-reducible (but quantifiable) systematic uncertainties

Local form factors predictions

Available theory calculations for local FFs $\mathcal{F}_\lambda$

$B \to K$:

- LQCD calculations at **high** q^2
  [HPQCD 2013] [FNAL/MILC 2015]
  and in the **whole** semileptonic region
  [HPQCD 2023]
- LCSR at **low** q^2
  [Khodjamirian/Rusov 2017] [NG/Kokulu/van Dyk 2018]

$B \to K^*$ and $B_s \to \phi$:

- LQCD calculations at **high** q^2
  [Horgan et al. 2015]
- LCSR calculation at **low** q^2
  [Bharucha et al. 2015] [NG/Kokulu/van Dyk 2018]

$B \to K$ **FFs excellent** status (need independent calculation at low q^2)

More LQCD results needed for vector states (for high precision K^* width cannot be neglected)

How to **combine** different calculations and obtain result **whole** semileptonic region?

Map for local FFs

Obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by **combining all LQCD** (and **LCSRs**) results

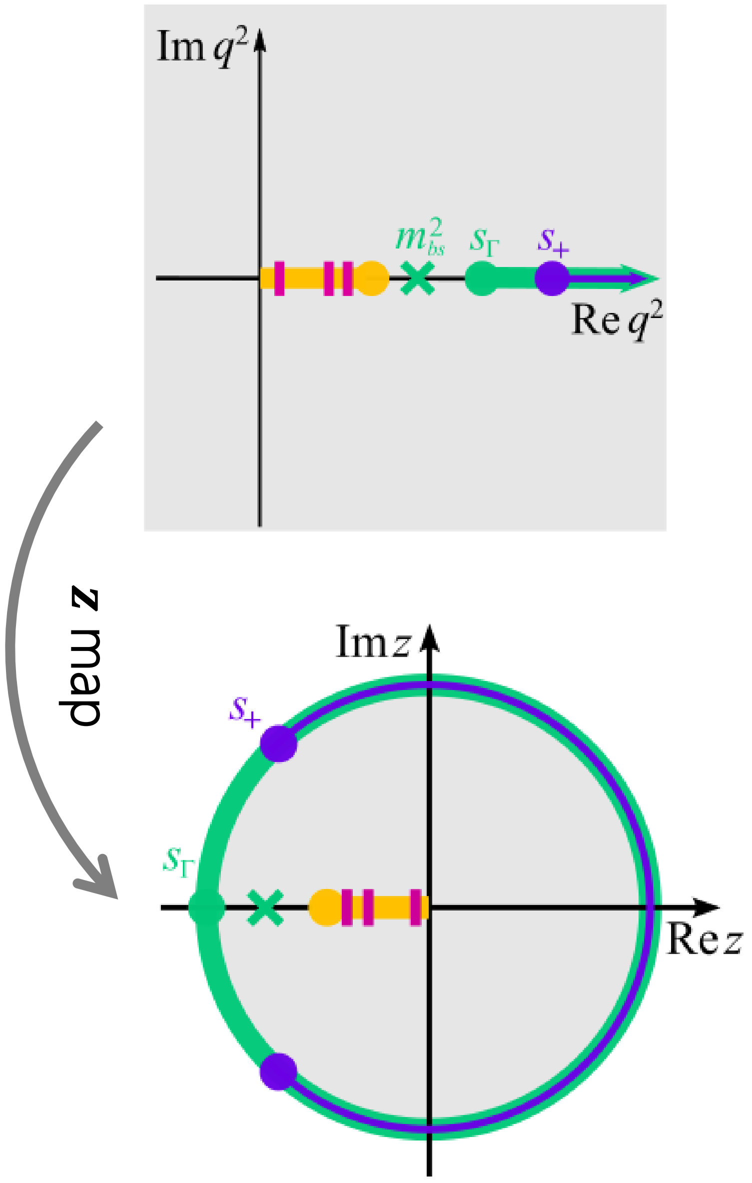
$\mathcal{F}_\lambda$ analytic functions of q^2 except for isolated $s\bar{b}$ poles and a branch cut for $q^2 > s_\Gamma = (m_{B_s} + (2)\, m_\pi)^2$

Branch cut differs from the pair production threshold:

$s_\Gamma \neq s_+ = (m_B + m_{K^{(*)}})^2$ contrary to, e.g., $B \to \pi$

Define the map

$$z(q^2) = \frac{\sqrt{s_\Gamma - q^2} - \sqrt{s_\Gamma}}{\sqrt{s_\Gamma - q^2} + \sqrt{s_\Gamma}}$$



Parametrization for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ **expand $\mathcal{F}_\lambda$ in a Taylor series in z**

We propose a **new parametrization** [Gopal/NG 2024]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} a_k\, z^k \qquad\qquad \sum_{k=0}^{\infty} |a_k|^2 < 1$$

$\mathcal{P}(z)$ and $\phi(z)$ are known functions, **fit c_k** coefficients to LQCD (and LCSR) results

First parametrization that is simultaneously:

- valid for $s_\Gamma \neq s_+$
- **unitarity bounded**

Supersede BGL (approximates $s_\Gamma = s_+$) $\Longrightarrow$ **non-quantifiable systematic uncertainties**

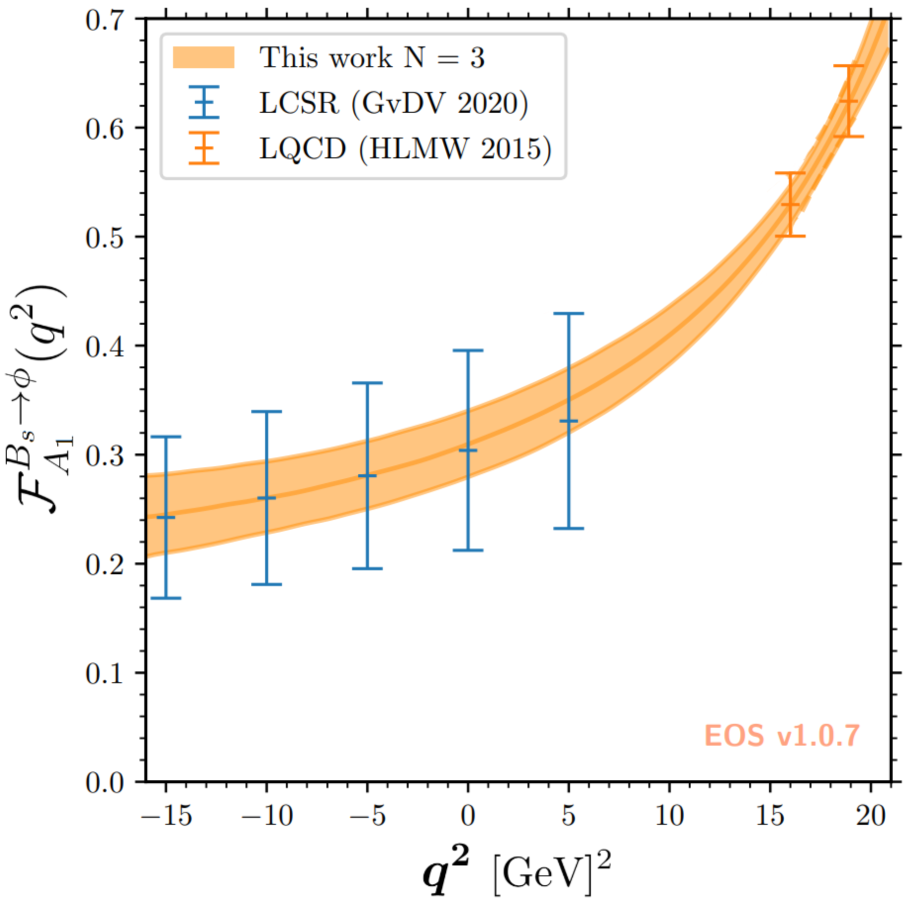
Local form factors predictions

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

Fit available inputs to

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{3} a_k\, z^k \qquad \sum_{k=0}^{3} |a_k|^2 < 1$$

Obtain numerical results for $B \to K^{(*)}$ and $B_s \to \phi$ in the whole semileptonic region



Agreement between LQCD and LCSRs

Fit done in [NG/Reboud/van Dyk/Virto 2023]

Update with new parametrization

Non-local form factors

Methods to calculate non-local FFs

Non-perturbative techniques are needed to compute non-local FFs $\mathcal{H}_\lambda(q^2)$

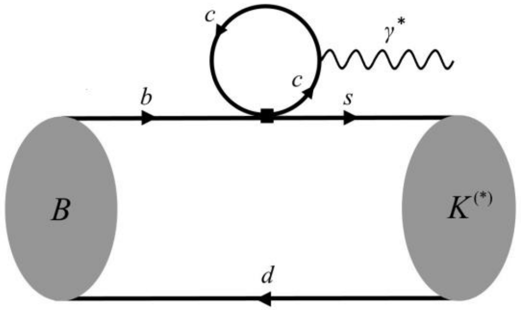
- lattice QCD $\Longrightarrow$ work in progress
- QCD factorization:
  factorize hard and soft contributions
  $\Longrightarrow$ double expansion in $1/m_b$ and $1/E_{K^{(*)}}$
  valid for $q^2 < 7\ \mathrm{GeV}^2$
  How to calculate power corrections? How extend to Λ_b decays?
  Is the perturbative treatment of the charm loop reliable close to threshold?
- light-cone operator product expansion (LCOPE) $\Longrightarrow$ see next slide

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

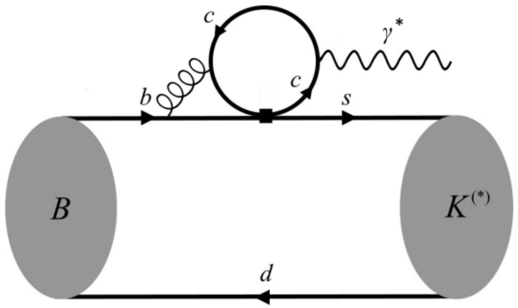
1. Calculate non-local FFs $\mathcal{H}_\lambda$ using a LCOPE at negative q^2

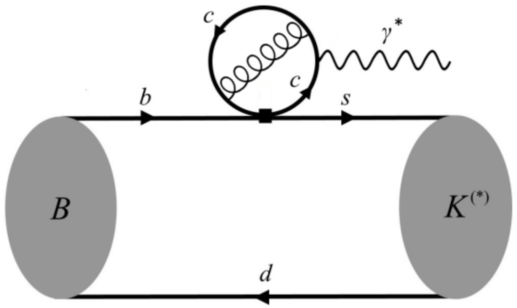
$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

leading power (LO in α_s)



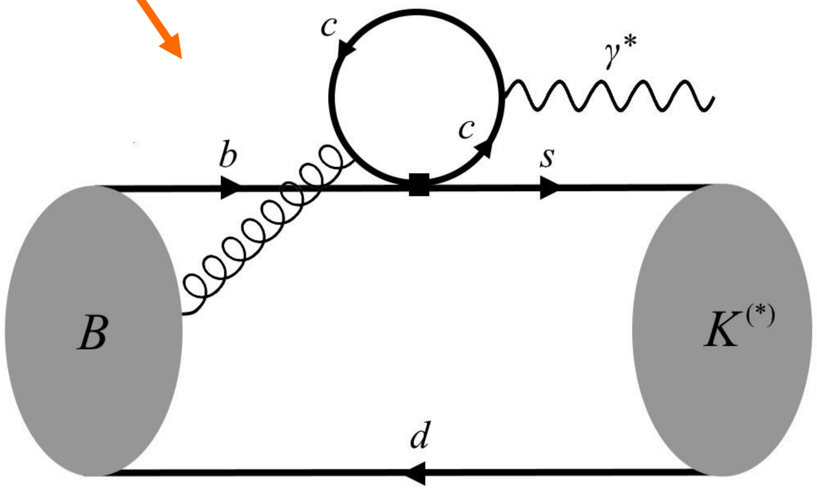
+ hard gluons (α_s) corrections





[Bell/Huber 2014] [Asatrian/Greub/Virto 2019]

soft gluon correction
non-perturbative
$\Longrightarrow$ not α_s suppressed



[Khodjamirian et al. 2010]
[NG/van Dyk/Virto 2020]

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

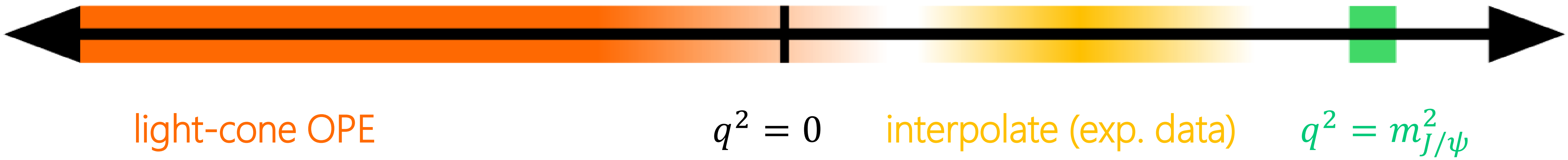
1. Calculate non-local FFs $\mathcal{H}_\lambda$ using a LCOPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

2. Extract $\mathcal{H}_\lambda$ at $q^2 = m_{J/\psi}^2$ from $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements (decay amplitudes independent of the local FFs)

3. **New approach: interpolate** these two results to obtain theoretical predictions in the low q^2 $(0 < q^2 < 8\ \mathrm{GeV}^2)$ region $\Longrightarrow$ compare with experimental data

Need a parametrization to interpolate $\mathcal{H}_\lambda$: which is the optimal parametrization?



Map for non-local FFs

Similar situation with respect to $\mathcal{F}_\lambda$

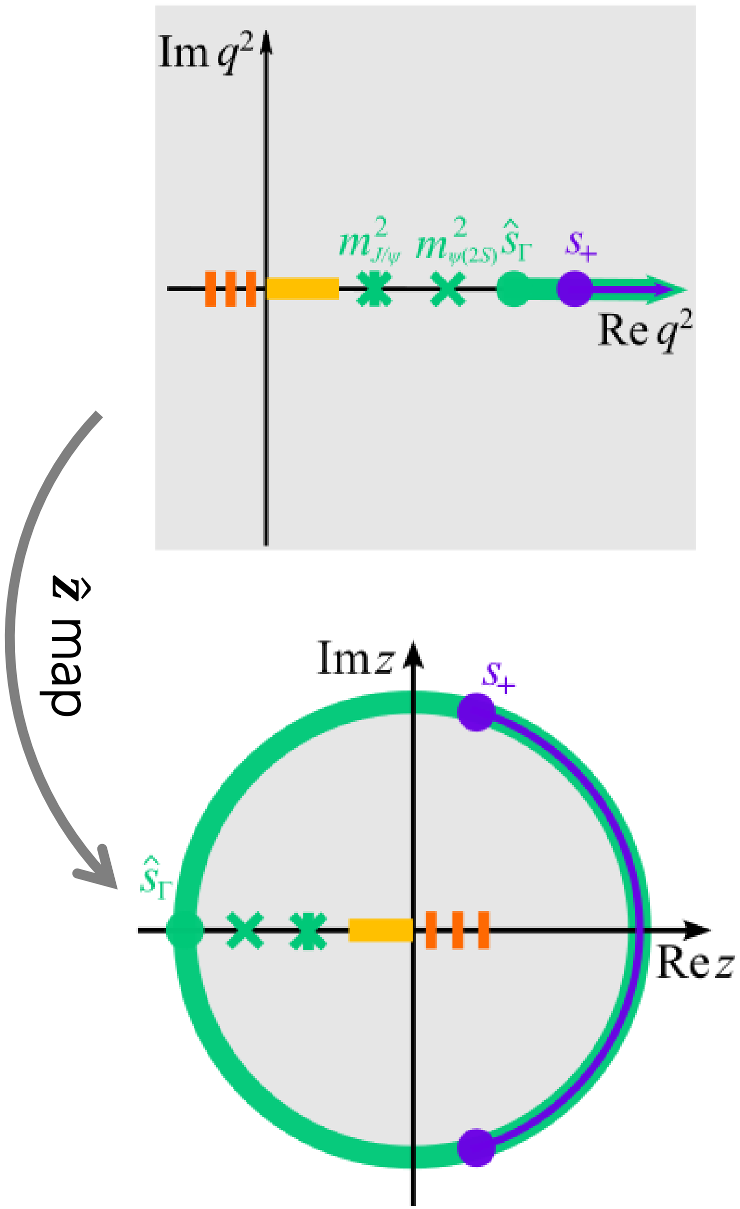
$\mathcal{H}_\lambda$ analytic functions of q^2 except for isolated $c\bar{c}$ poles (J/ψ and $\psi(2S)$) and a branch cut for $q^2 > \hat{s}_\Gamma = 4m_D^2$

Branch cut differs from the pair production threshold:

$\hat{s}_\Gamma \neq s_+ = \left(m_B + m_{K^{(*)}}\right)^2$

Define the map

$$\hat{z}(q^2) = \frac{\sqrt{\hat{s}_\Gamma - q^2} - \sqrt{\hat{s}_\Gamma}}{\sqrt{\hat{s}_\Gamma - q^2} + \sqrt{\hat{s}_\Gamma}}$$



Anomalous cuts will be discussed later! (neglected for the moment being)

Non-local form factors predictions

$$\mathcal{A}(B \to K^{(*)}\ell\ell) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\,\mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

Obtain numerical results for the non-local FFs $\mathcal{H}_\lambda$

$$\mathcal{H}_\lambda(\hat{z}) \propto \sum_{k=0}^{5} b_k p_k(\hat{z}) \qquad \sum_{k=0}^{\infty} |b_k|^2 < 1$$

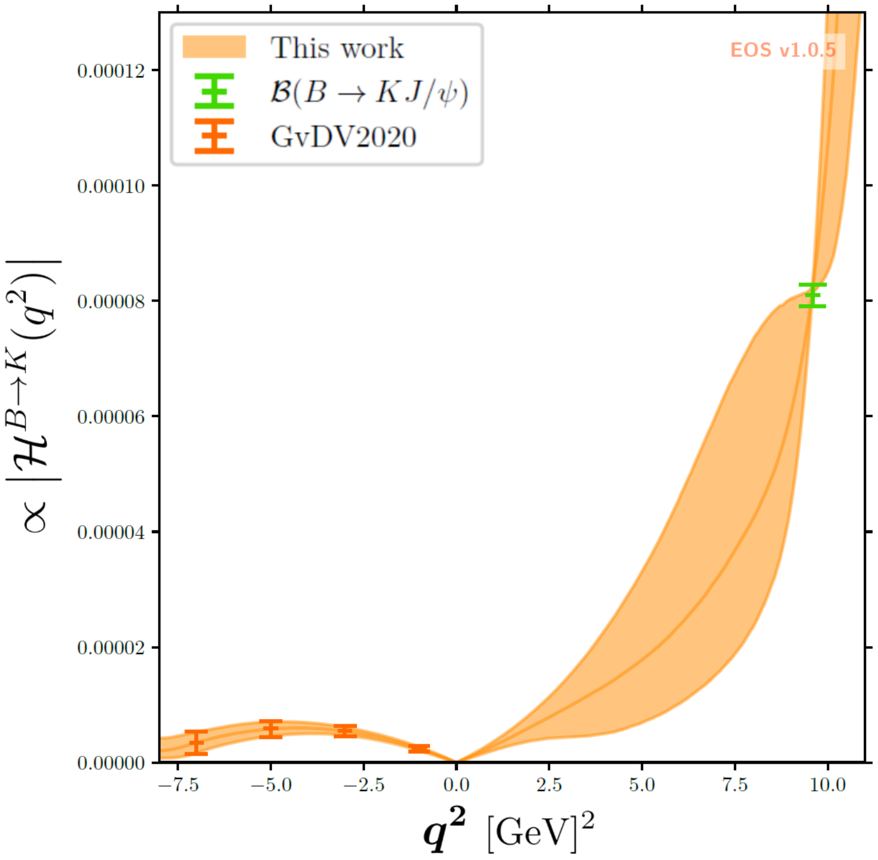
Fit the $\hat{z}$ parametrization

- light-cone OPE calculation at negative q^2

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

- $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements at $q^2 = m_{J/\psi}^2$
- unitarity bound (derived for the first time)

Need to update including anomalous cuts!



[NG/Reboud/van Dyk/Virto 2022]

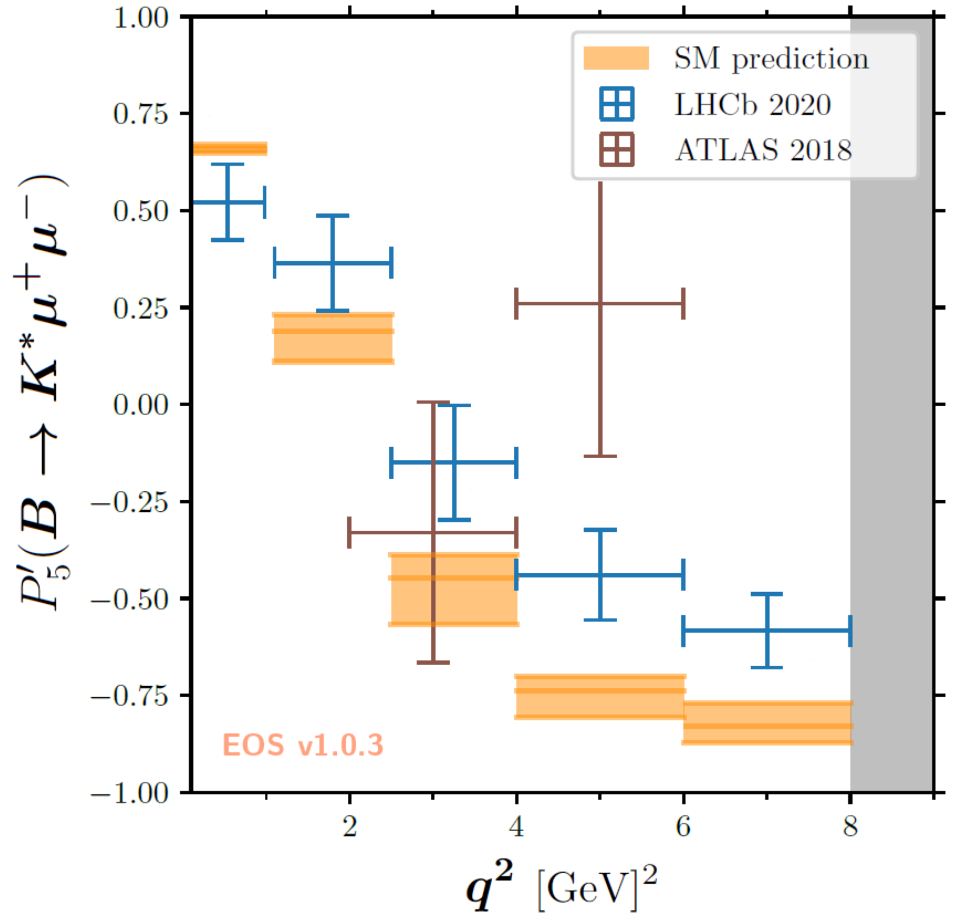
SM predictions and confrontation with data

SM predictions vs. data

Predict observables using our $\mathcal{F}_\lambda$ and $\mathcal{H}_\lambda$ results:

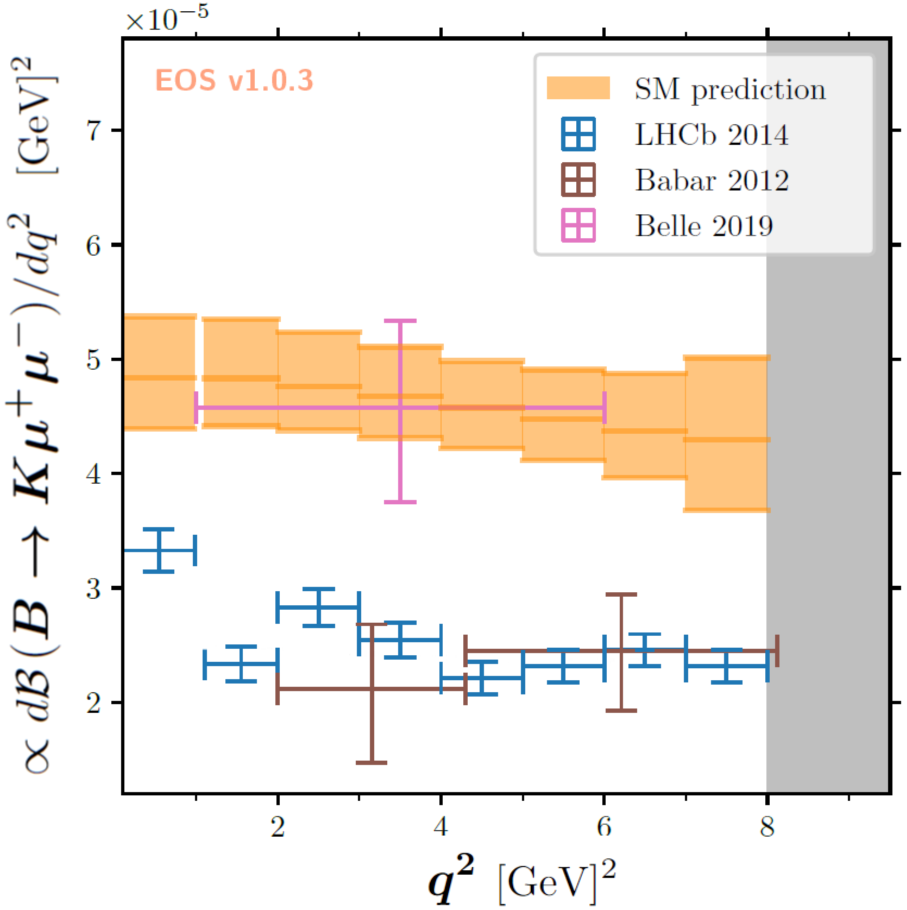
BRs and angular observables
for $B \to K^{(*)}\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$

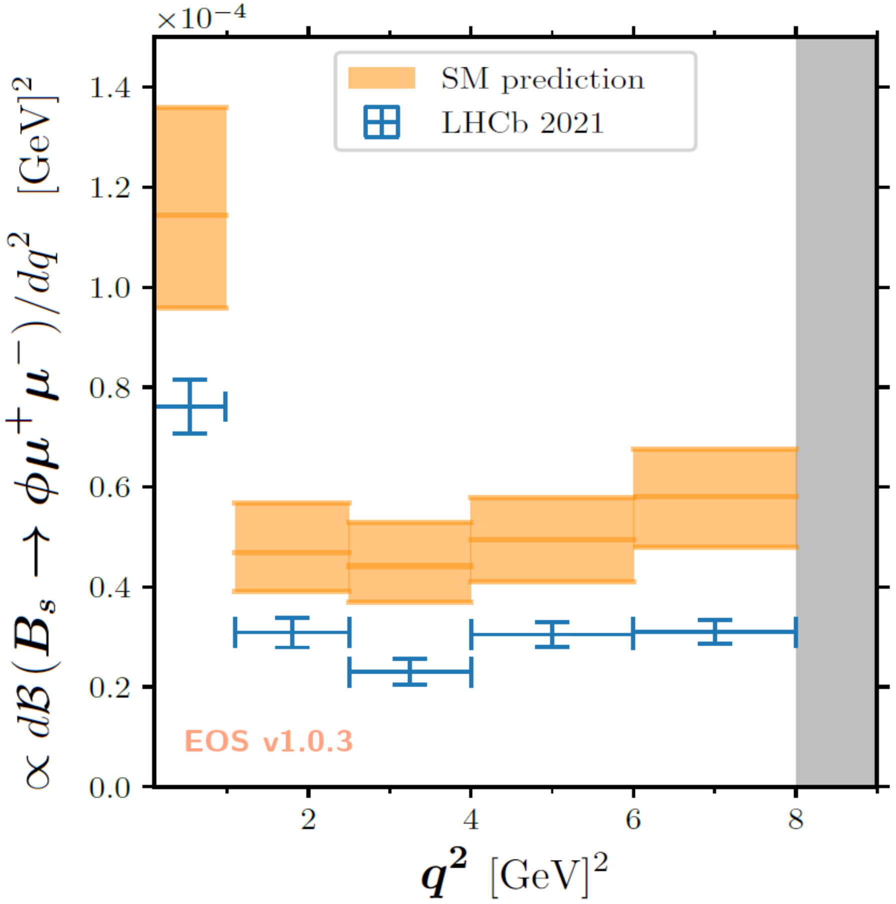
- theory uncertainties mostly due to $\mathcal{F}_\lambda$
- progress in $\mathcal{H}_\lambda$ calculations urgently needed
- more measurements on the way



[NG/Reboud/van Dyk/Virto 2022]

SM predictions vs. data





[NG/Reboud/van Dyk/Virto 2022]

Coherent tensions between SM predictions and data

Global fit to $b \to s\mu^+\mu^-$ (setup)

Use our predictions for the local and non-local FFs as priors

Fit the Wilson coefficients C_9^{NP} and C_{10}^{NP} to the available experimental measurements in $b \to s\mu^+\mu^-$ transitions
$(C_{9,10} = C_{9,10}^{\mathrm{SM}} + C_{9,10}^{\mathrm{NP}})$

We perform three fits, one for each set of the following set of experimental measurements:
(BRs, angular observables, binned and not binned)

- $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- $B \to K^*\mu^+\mu^-$
- $B_s \to \phi\mu^+\mu^-$

Combined fit would be very challenging $\longrightarrow$ 130 nuisance parameter

Global fit to $b \to s\mu^+\mu^-$ (results)

we **obtain good fits**, agreement between the three fits

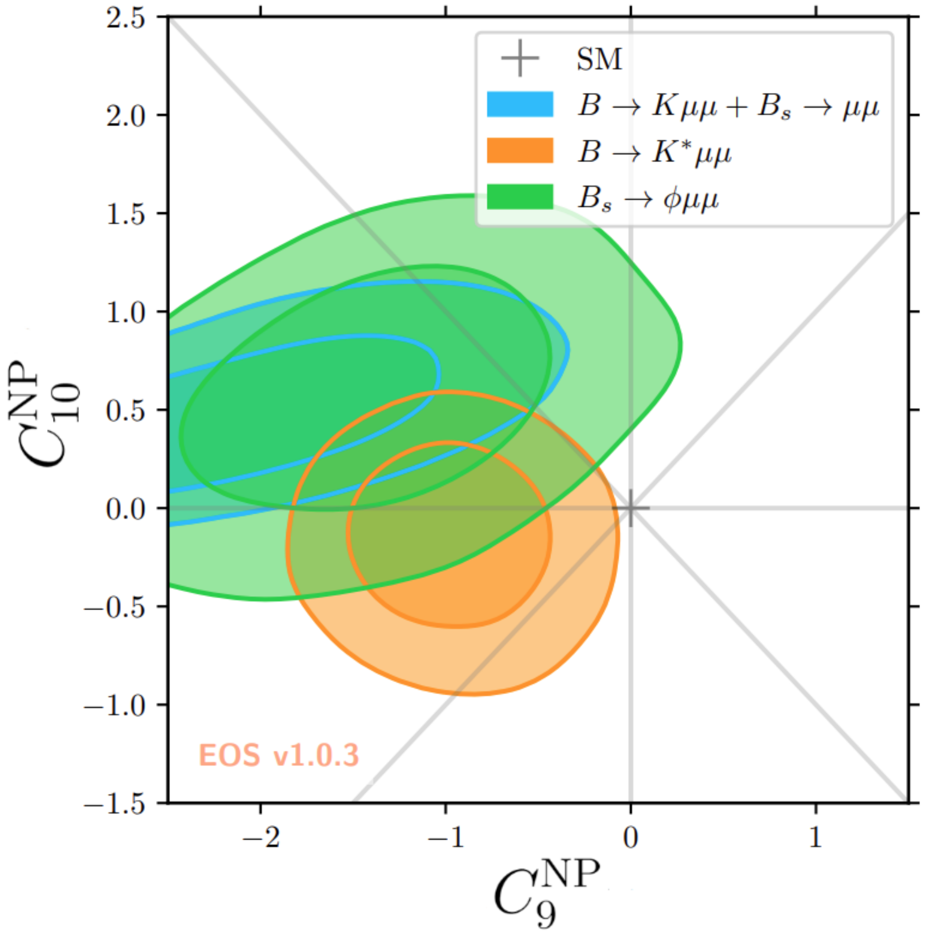
Substantial tension w.r.t. SM (in agreement with the literature)

Pulls (p value of the SM hypothesis):

- 5.7σ for $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- 2.7σ for $B \to K^*\mu^+\mu^-$
- 2.6σ for $B_s \to \phi\mu^+\mu^-$

Local FFs $\mathcal{F}_\lambda$ main uncertainties

Non-local FFs $\mathcal{H}_\lambda$ cannot explain this tension

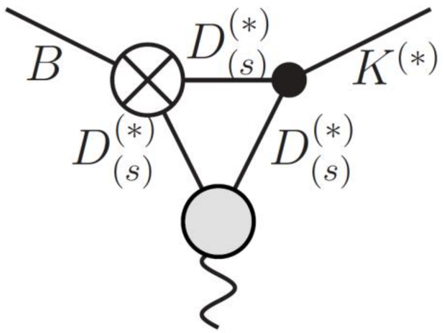


Open issues

Rescattering effects

Missing contributions?

Ciuchini et al. 2022 (also way before) claim that $B \to \bar{D} D_s \to K^{(*)} \ell^+ \ell^-$ rescattering might have a sizable contribution $\Longrightarrow \mathcal{O}(20\%)$ at amplitude level



LCOPE contains (implicitly) rescattering effects

partonic calculation does not yield large contribution (LP OPE and NLO α_s)

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

$\mathcal{C}_\lambda$ is complex valued for any q^2 value due to branch cut in $p^2 = M_B^2$ as expected

[Asatrian/Greub/Virto 2019]

Large quark-hadron duality violation?

Slow convergence of the LCOPE?

Alternative approach $\Longrightarrow$ directly calculate rescattering effects using hadronic methods

Anomalous branch cuts

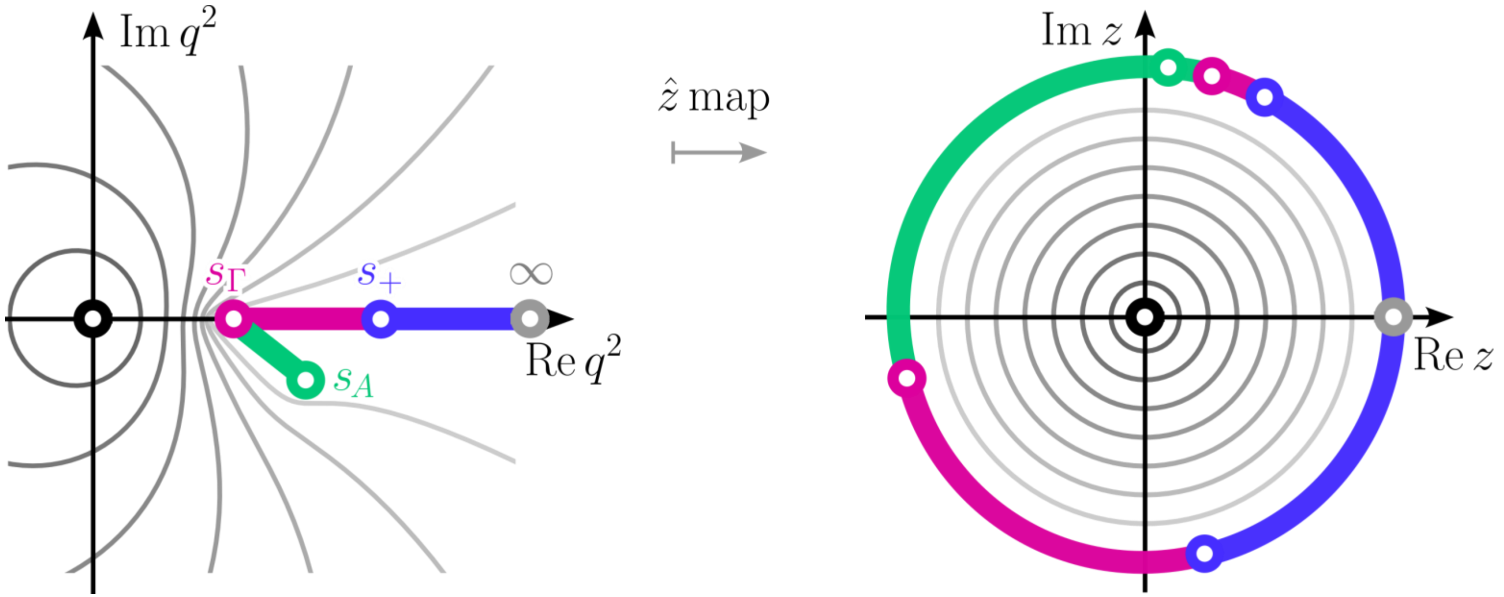
Non-local FFs may present have anomalous branch cuts that extend into the complex plane
Example $B \to DD_s^* \to K\ell^+\ell^-$ rescattering

$$s_+ = (m_B + m_K)^2 \qquad s_\Gamma = 4m_D^2 \qquad s_A = 24.1 - 3.5i$$

[Mutke et al. 2024]

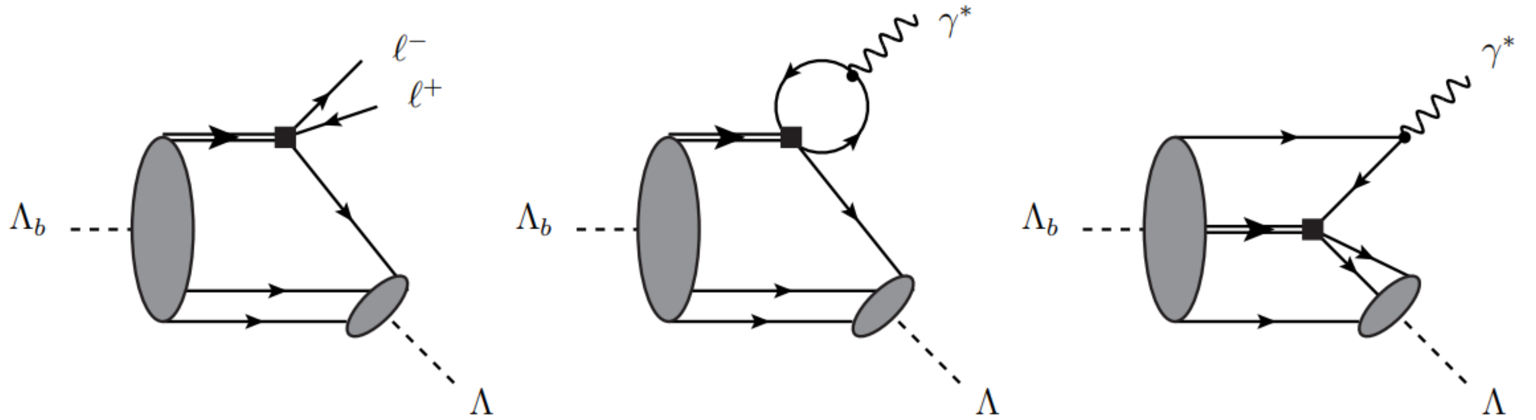
Apply the same procedure as for the subthreshold branch cuts, but:

- $\hat{z}$ map is very hard to obtain (existence guaranteed by the Riemann Mapping Theorem)
- derivation of unitarity bounds extremely challenging



[Gopal/NG 2024]

$\Lambda_b \to \Lambda \ell^+ \ell^-$ decays



[Feldmann/NG 2023]

If $b \to s\mu^+\mu^-$ anomalies are due to New Physics $\Longrightarrow$ same shift expected in $\Lambda_b \to \Lambda\mu^+\mu^-$ but rescattering effects are different

Already measured by LHCb $\Longrightarrow$ new and more precise measurements on the way

Progress needed in theory calculations (no estimate of charm-loop beyond naïve factorization)

First calculation of “annihilation” contributions in [Feldmann/NG 2024]

Possible issues on local FFs

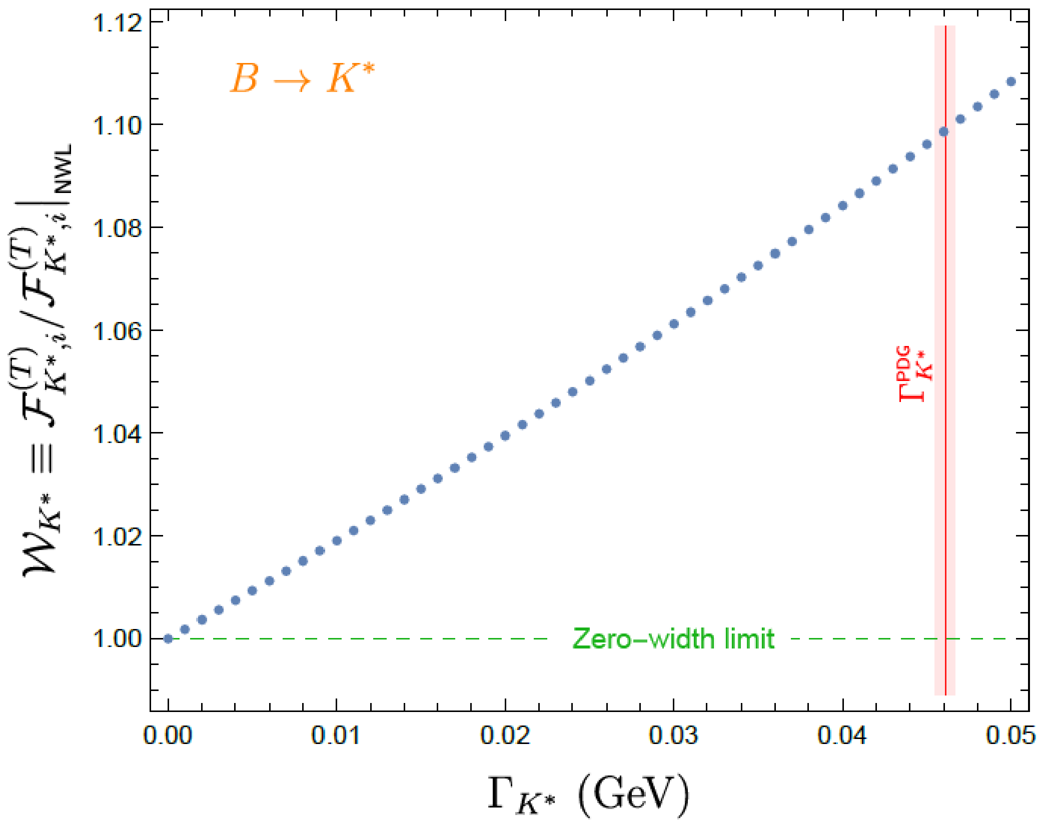
Precise LQCD calculations for local $\mathcal{F}_\lambda$ FFs at low q^2 are **essential** to have better theoretical predictions

Already available for $B \to K\ell^+\ell^-$ [HPQCD 2022]

w.i.p. for $B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$

K^* has a sizable width
$\Longrightarrow$ $B \to K\pi\ell^+\ell^-$ local FFs calculation
first steps in [Descotes-Genon et al. 2019] using LCSRs

Clear path to solve these issues



[Descotes-Genon et al. 2019]

Summary and outlook

Summary and outlook

1. Improved parametrization for local FFs $\mathcal{F}_\lambda$ (consider below threshold branch cuts)

   Combine LQCD (and LCSRs) inputs to get results for $\mathcal{F}_\lambda$ in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$

2. Calculate $\mathcal{H}_\lambda$ with LCOPE and use unitarity bounds

   Need to include anomalous branch cuts

3. SM predictions for observables in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

   Coherent deviations between SM and data in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

4. **Progress on the theory side needed** more than ever

Thank you!