

before we can talk about neural inference,

we have to believe machine-learning is doing what it claims!

Multilayer Feedforward Networks are Universal Approximators

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(Received 16 September 1988; revised and accepted 9 March 1989)

Abstract—This paper rigorously establishes that standard multilayer feedforward networks with as few as one hidden layer using arbitrary squashing functions are capable of approximating any Borel measurable function from one finite dimensional space to another to any desired degree of accuracy, provided sufficiently many hidden units are available. In this sense, multilayer feedforward networks are a class of universal approximators.

NNs can approximate any continuous function $f: \mathbb{R}^m \to \mathbb{R}^n$

to arbitrary precision,

given a finite number of free parameters (+ enough training data and time).

neural inference:

extracting parameters of interest from observations using NN approximations of a likelihood(-ratio) or posterior.

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how can we achieve this?

mostly it's a matter of constructing (and minimizing) the correct loss function.

with a simulator s that, given a parameter ϕ , produces observables x

$$s: \phi \to p(x) \equiv p(x | \phi),$$

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if we have many simulated samples (ϕ, x) , then we know how to learn the log-likelihood ratio

$$D_{\phi'}^{\phi}(x) \equiv \log \frac{p(x \mid \phi)}{p(x \mid \phi')}$$

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imagine the case that ϕ only has two possible values: A, B.

$$D_B^A(x) \equiv \log \frac{p(x|A)}{p(x|B)}$$

is simply what a discriminator between A and B learns to emit.

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given a simulator $p(x | \phi)$, we also know, how to learn

$$p(\phi | x) \propto p(x | \phi) p(\phi)$$

for some family of posteriors p_{η} parameterized by η :

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 $p(\phi)$ is the distribution over ϕ used in training,

and we learn the function $f: x \to \eta$ by minimizing the loss

$$L(\phi, x) = -\log p_{\eta = f(x)}(\phi).$$

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too good to be true?

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at least three* open "issues" with neural simulation-based inference (nSBI):







convergence of function approximations



adobe stock

convergence of function approximations

the likelihood-ratio over an entire dataset may require very, very good approximations of the per-event likelihood-ratio to behave properly.

$$L(x | \phi) = \prod_{i} L(x_i | \phi)$$

(for independent observations)

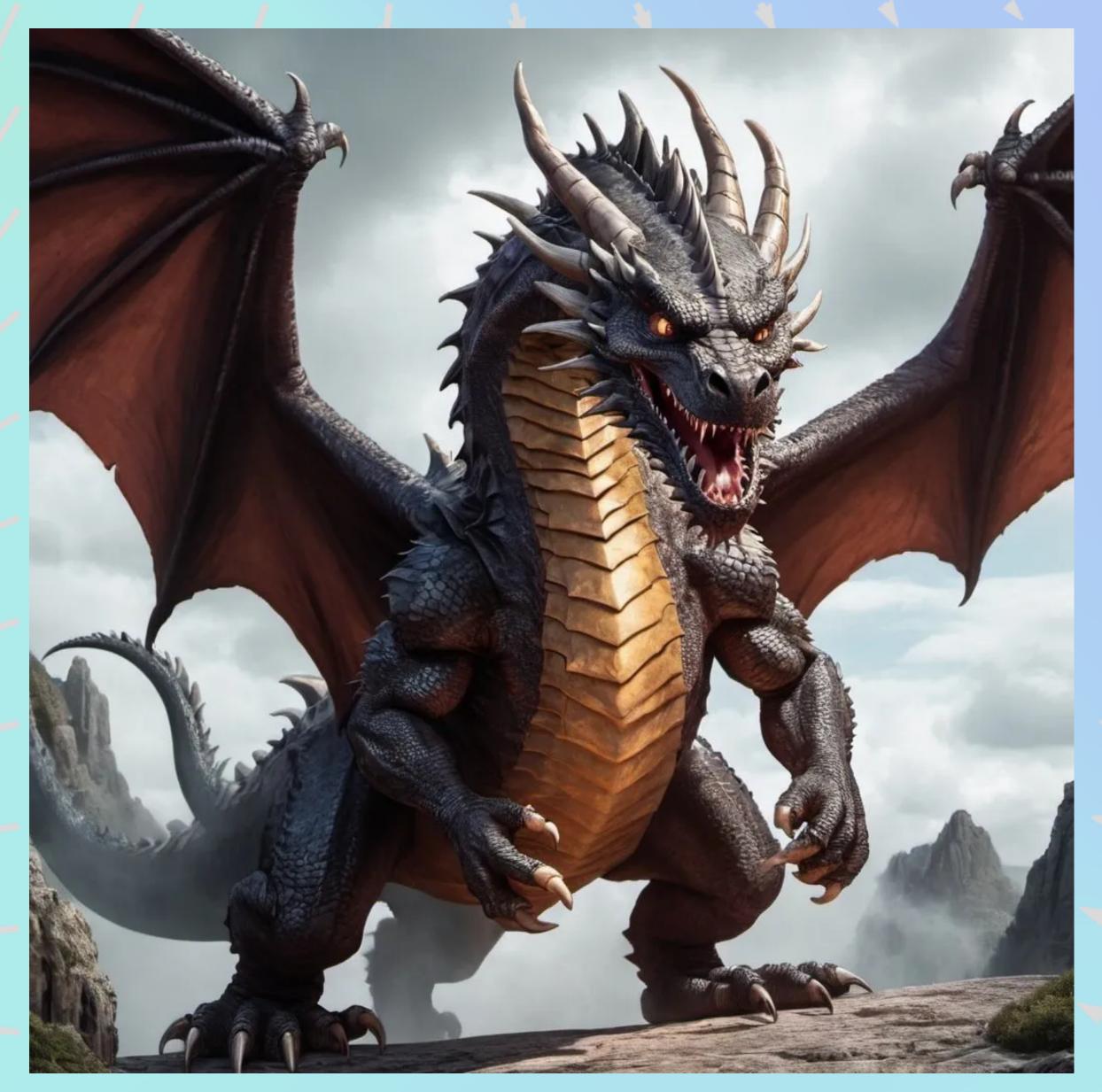
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this may be *difficult*, but at least it can be *verified* on toy datasets.



https://openart.ai/community/NOz2NbLjiXRdF0pRDPuL



in the presence of global nuisance parameters, per-event likelihoods do not easily compose.

we need to be able to profile against or marginalize over global nuisance parameters.



two main approaches:

- 1) learn the likelihood/posterior parameterized in the nuisances and profile/marginalize after the fact, or
 - 2) perform dataset-wide (ensemble) learning.



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works but can be painful

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EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)







CERN-EP-2024-298 December 3, 2024

Measurement of off-shell Higgs boson production in the $H^* \to ZZ \to 4\ell$ decay channel using a neural simulation-based inference technique in 13 TeV pp collisions with the ATLAS detector

The ATLAS Collaboration

A measurement of off-shell Higgs boson production in the $H^* \to ZZ \to 4\ell$ decay channel is presented. The measurement uses 140 fb⁻¹ of proton–proton collisions at $\sqrt{s} = 13$ TeV collected by the ATLAS detector at the Large Hadron Collider and supersedes the previous result in this decay channel using the same dataset. The data analysis is performed using a neural simulation-based inference method, which builds per-event likelihood ratios using neural networks. The observed (expected) off-shell Higgs boson production signal strength in the $ZZ \to 4\ell$ decay channel at 68% CL is $0.87^{+0.75}_{-0.54}$ ($1.00^{+1.04}_{-0.95}$). The evidence for off-shell Higgs boson production using the $ZZ \to 4\ell$ decay channel has an observed (expected) significance of 2.5σ (1.3σ). The expected result represents a significant improvement relative to that of the previous analysis of the same dataset, which obtained an expected significance of 0.5σ . When combined with the most recent ATLAS measurement in the $ZZ \to 2\ell 2\nu$ decay channel, the evidence for off-shell Higgs boson production has an observed (expected) significance of 3.7σ (2.4σ). The off-shell measurements are combined with the measurement of on-shell Higgs boson production to obtain constraints on the Higgs boson total width. The observed (expected) value of the Higgs boson width at 68% CL is $4.3^{+2.7}_{-1.9}$ ($4.1^{+3.5}_{-3.4}$) MeV.

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arXiv:2412.01548

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)



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arXiv:2412.01600v1



An implementation of neural simulation-based inference for parameter estimation in ATLAS

The ATLAS Collaboration

Neural simulation-based inference is a powerful class of machine-learning-based methods for statistical inference that naturally handles high-dimensional parameter estimation without the need to bin data into low-dimensional summary histograms. Such methods are promising for a range of measurements, including at the Large Hadron Collider, where no single observable may be optimal to scan over the entire theoretical phase space under consideration, or where binning data into histograms could result in a loss of sensitivity. This work develops a neural simulation-based inference framework for statistical inference, using neural networks to estimate probability density ratios, which enables the application to a full-scale analysis. It incorporates a large number of systematic uncertainties, quantifies the uncertainty due to the finite number of events in training samples, develops a method to construct confidence intervals, and demonstrates a series of intermediate diagnostic checks that can be performed to validate the robustness of the method. As an example, the power and feasibility of the method are assessed on simulated data for a simplified version of an off-shell Higgs boson couplings measurement in the four-lepton final states. This approach represents an extension to the standard statistical methodology used by the experiments at the Large Hadron Collider, and can benefit many physics analyses.

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arXiv:2412.01600

ATLAS recently published a measurement of off-shell Higgs production using this approach:

- → it does work!
- → very high CPU costs
- → currently limited to relatively small datasets.

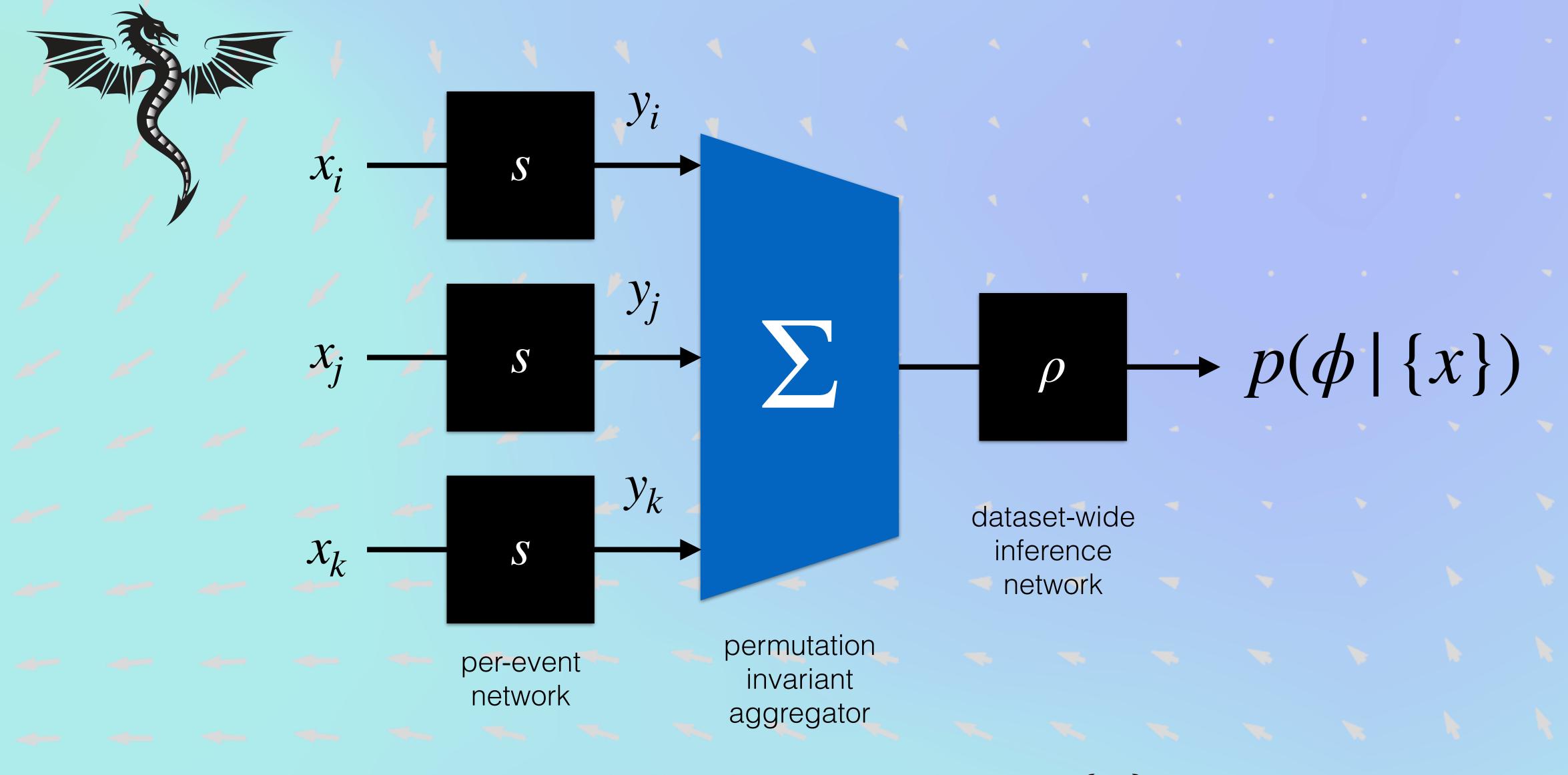


two main approaches:

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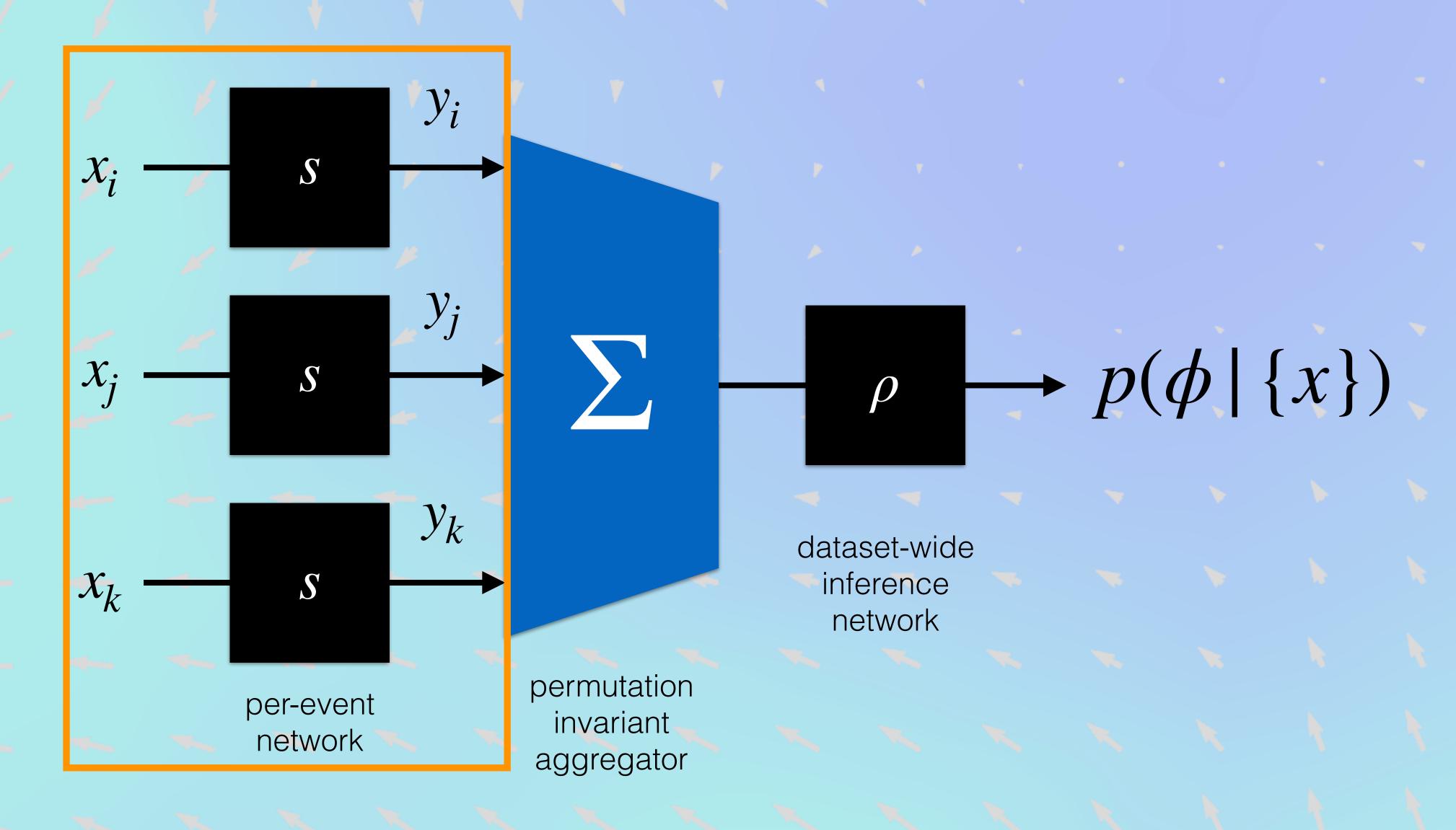
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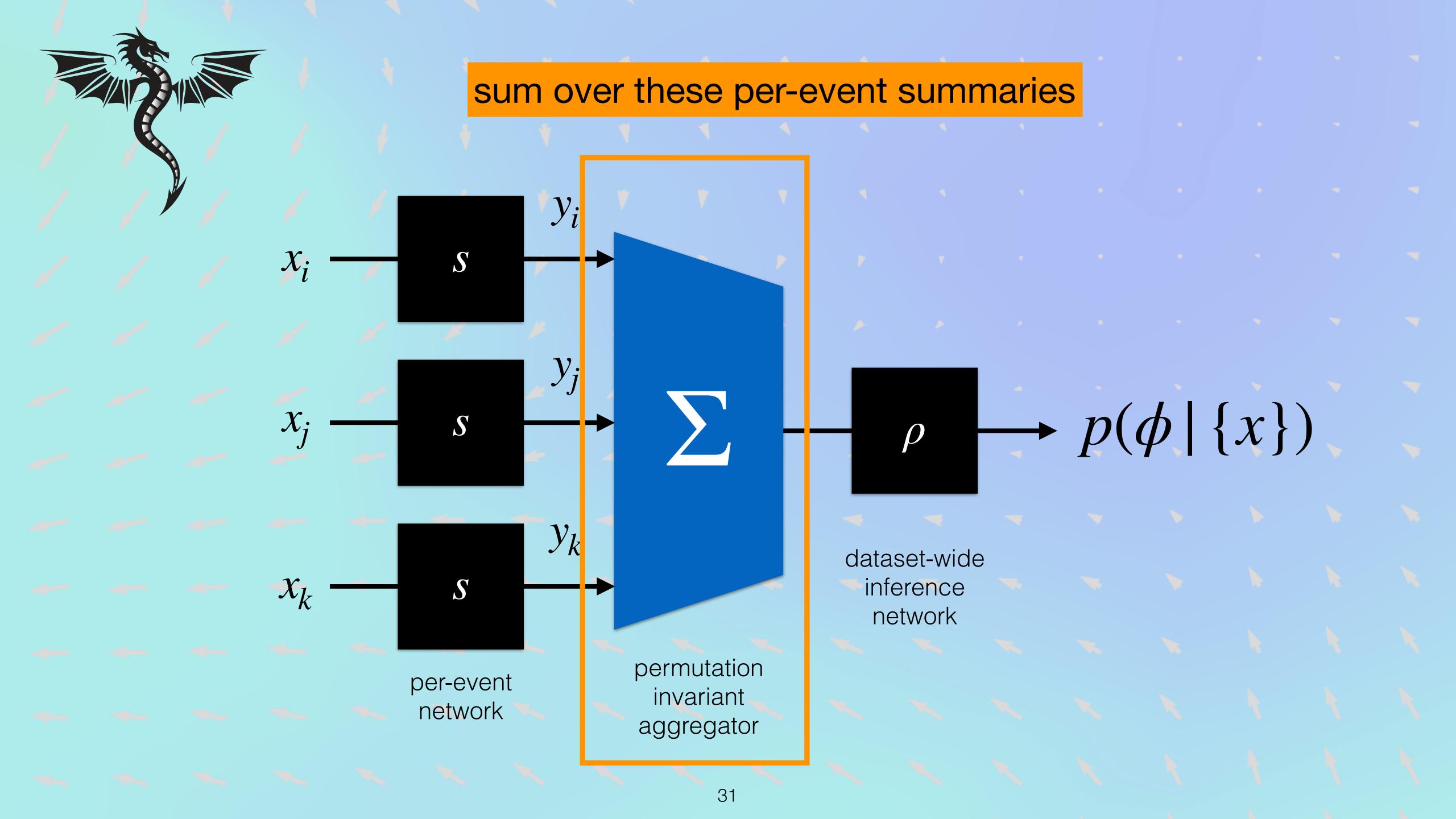
has some very nice properites



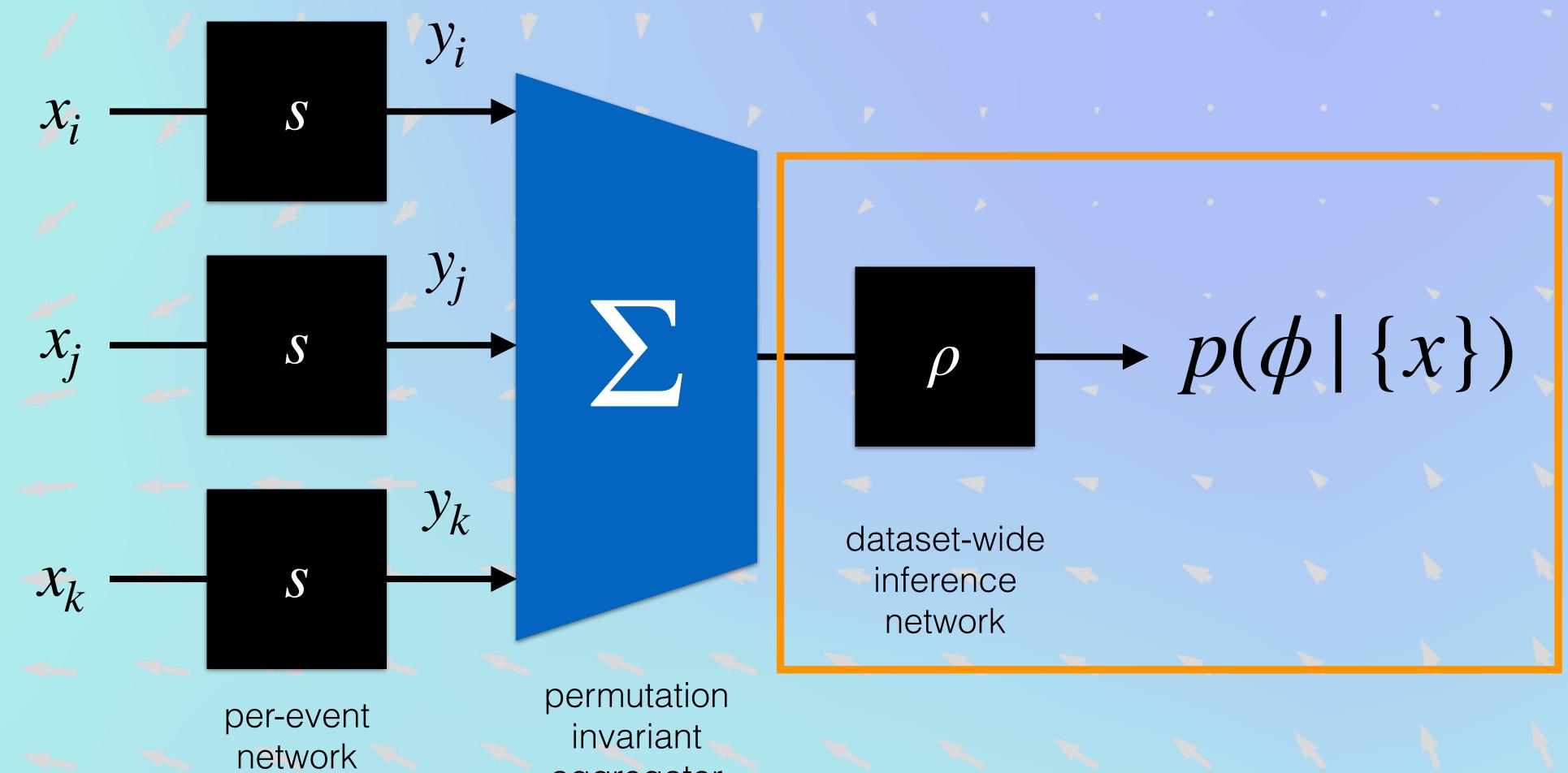
inference over an ensemble, $\{x\}$, via the deep set architecture







perform variational inference on this dataset-wide summary



aggregator



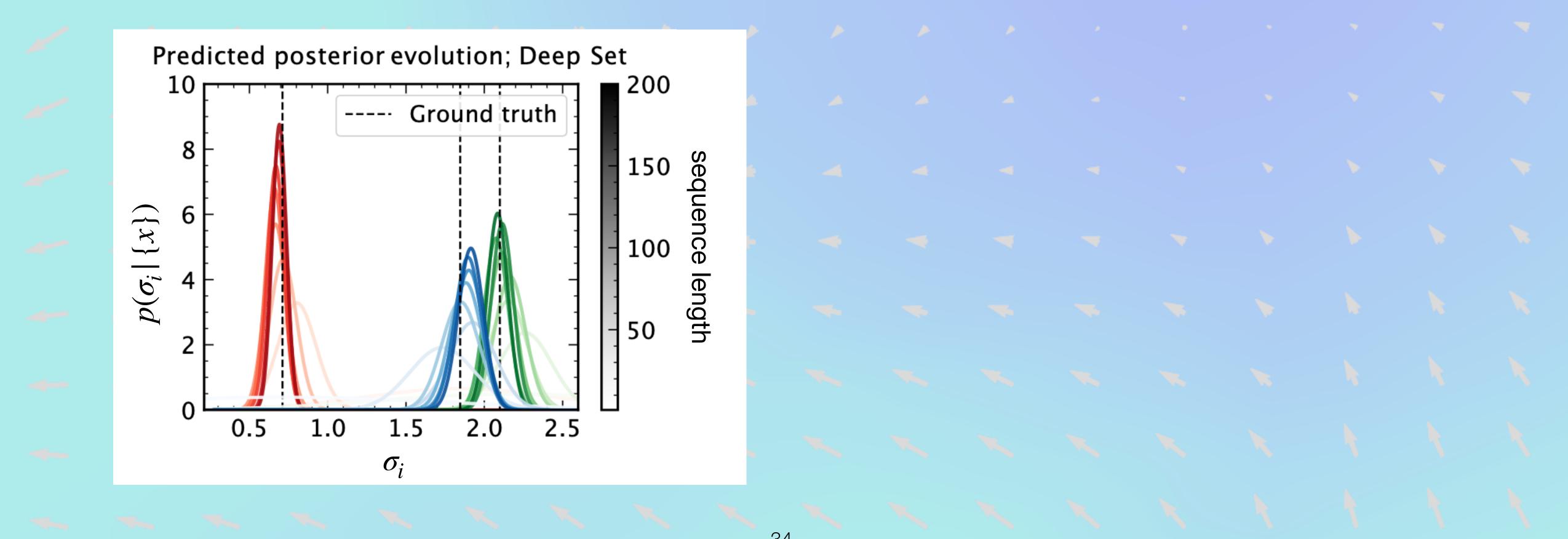
very simple case: inferring diagonal components of $\vec{\sigma}$ from observations

$$x \sim \mathcal{N}(\overrightarrow{\mu}, \operatorname{diag}(\overrightarrow{\sigma}^2))$$



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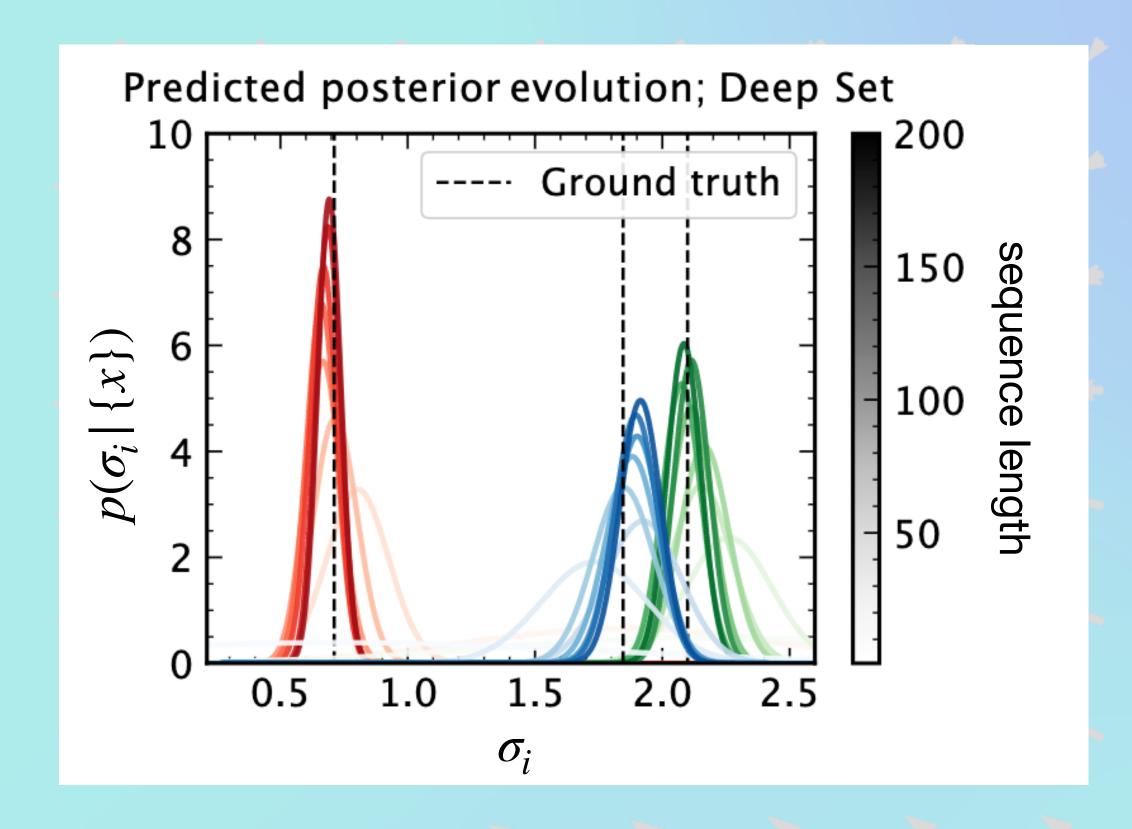
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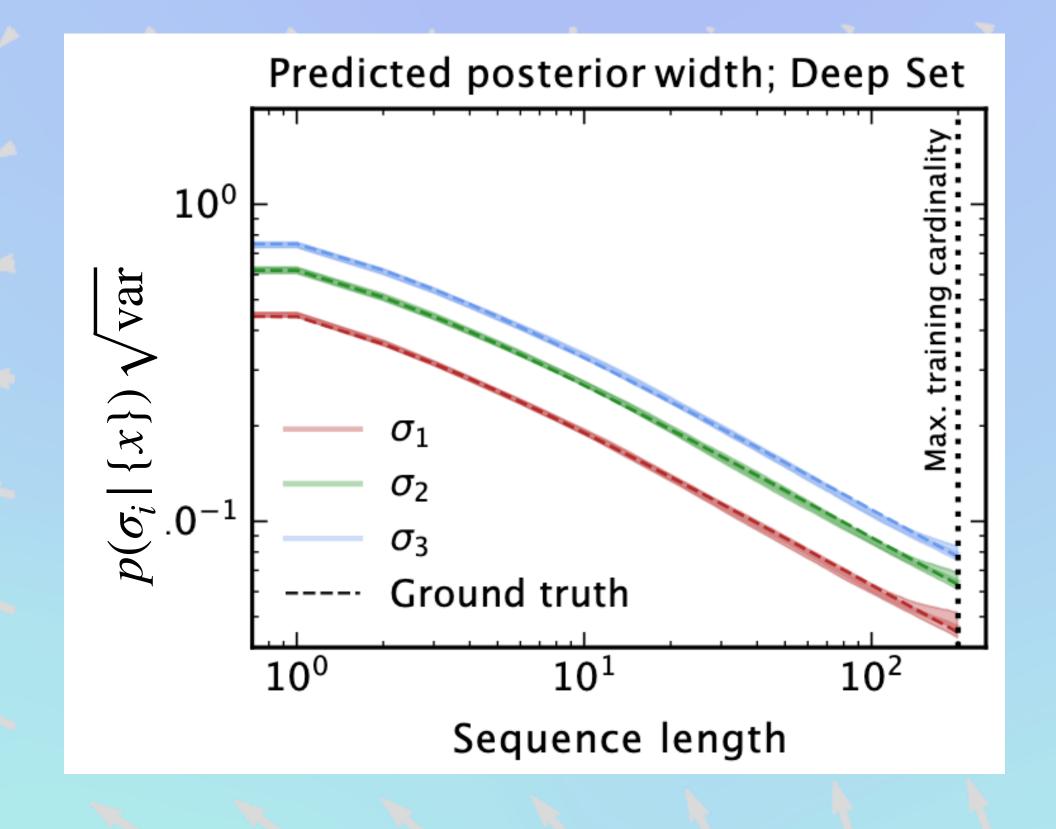




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particle-physics "inspired" example:

goal: extract signal fraction θ ,

- without knowing a priori the signal location, θ_{ν} .
 - (θ_{ν}) is a global NP.)

plus a wide background contribution.

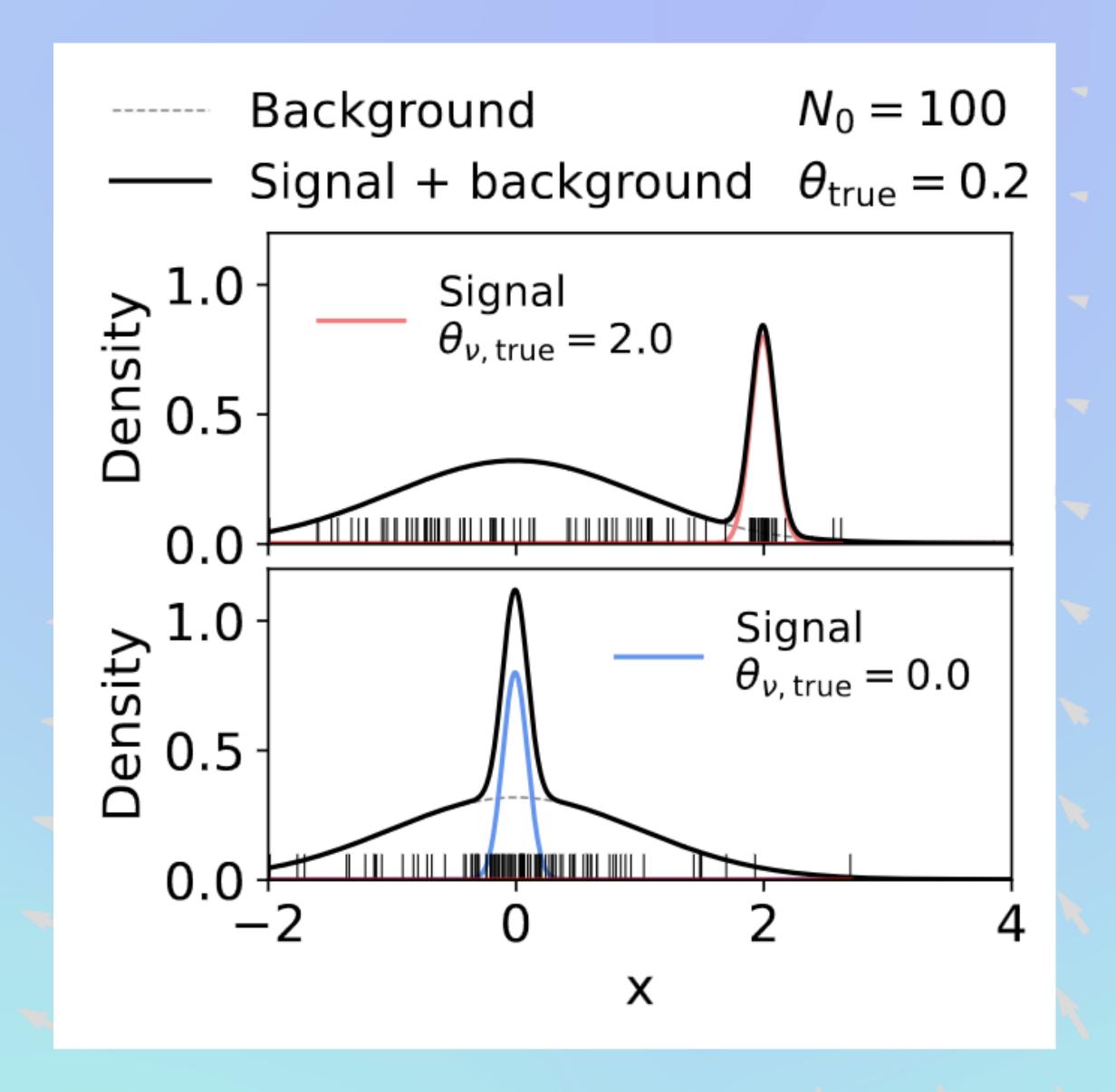
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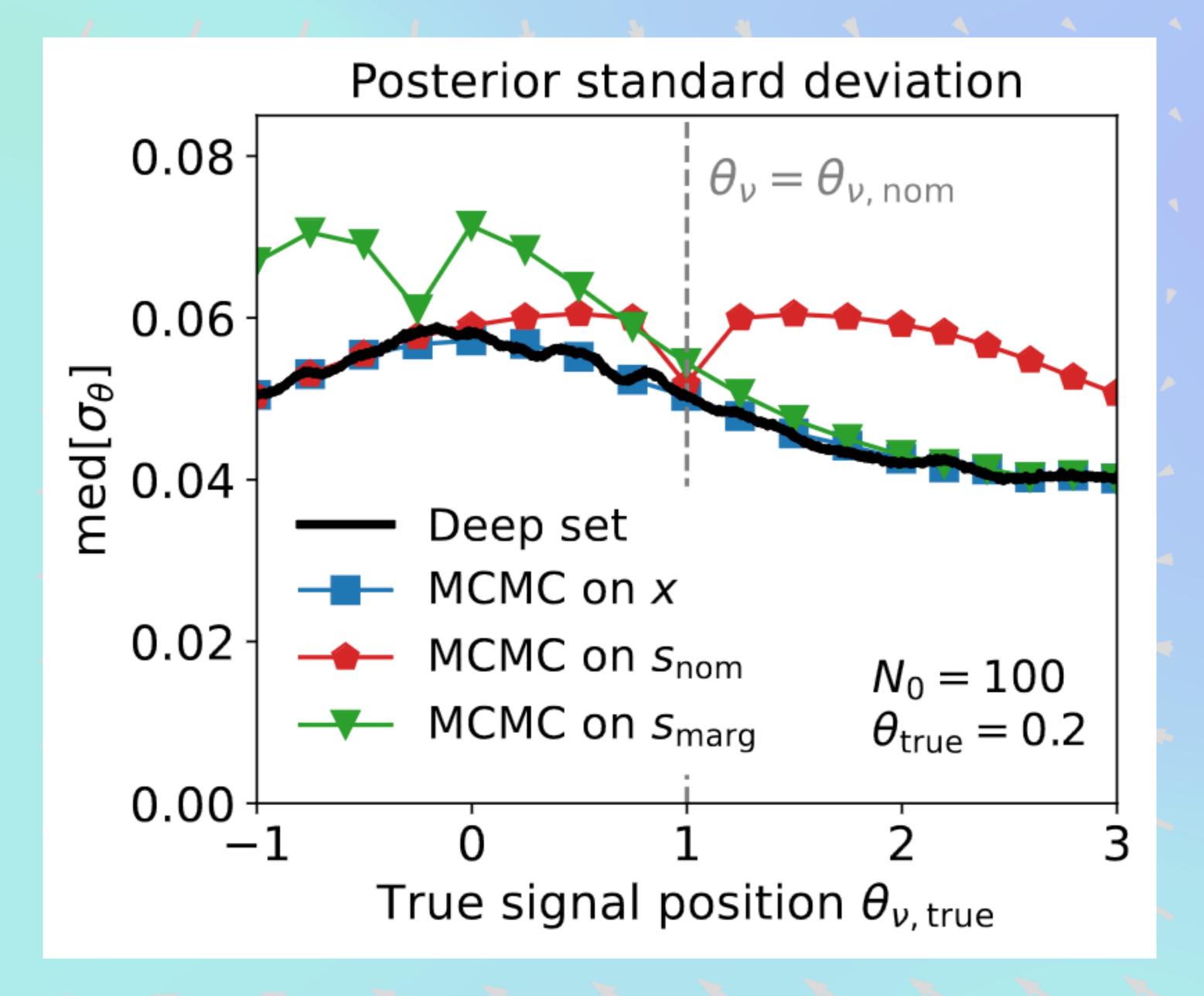
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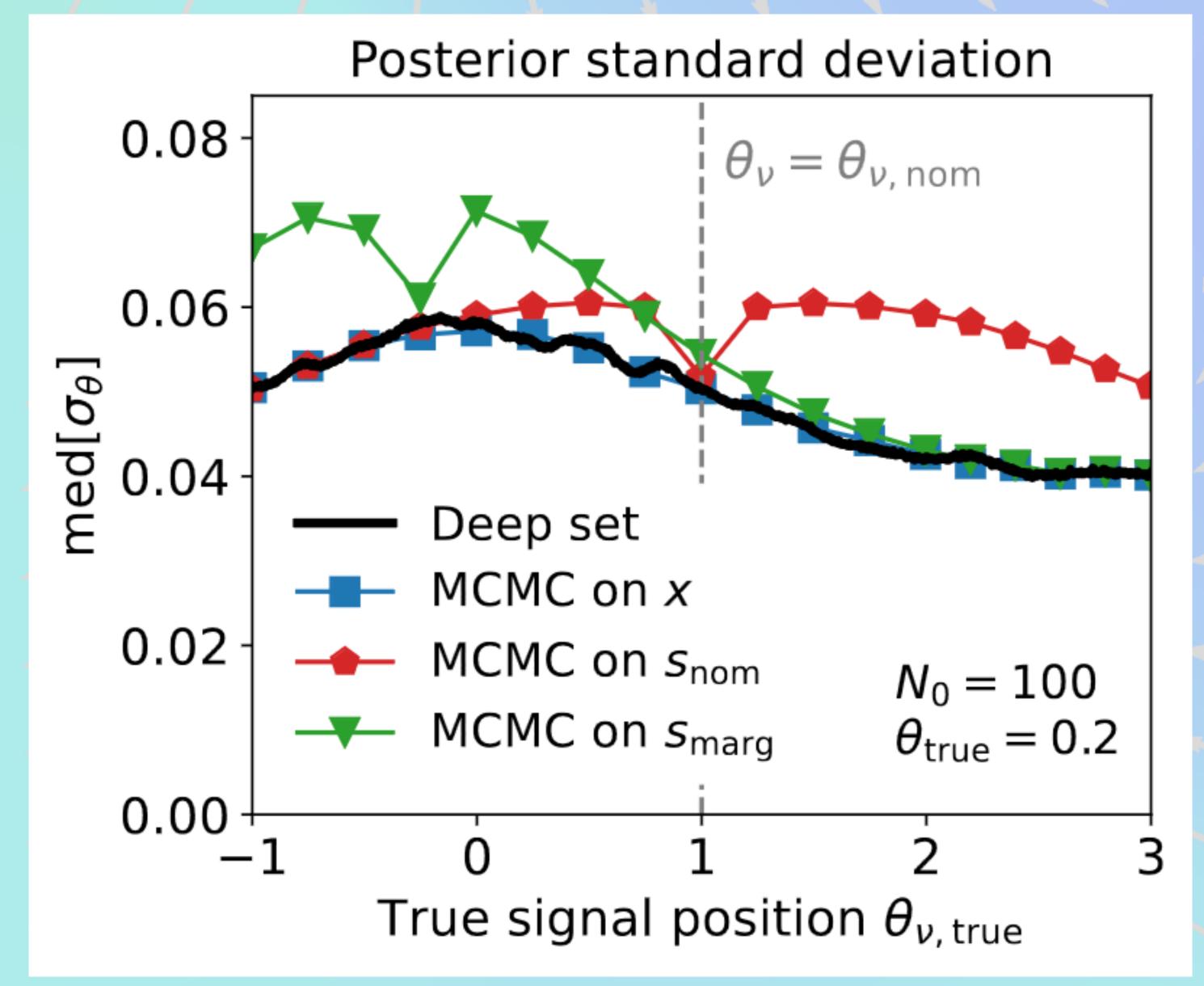
as we sweep $\theta_{
u}$,

the deep-set posterior

agrees with

brute-force (MCMC)

for posterior estimation on $\{x\}$.

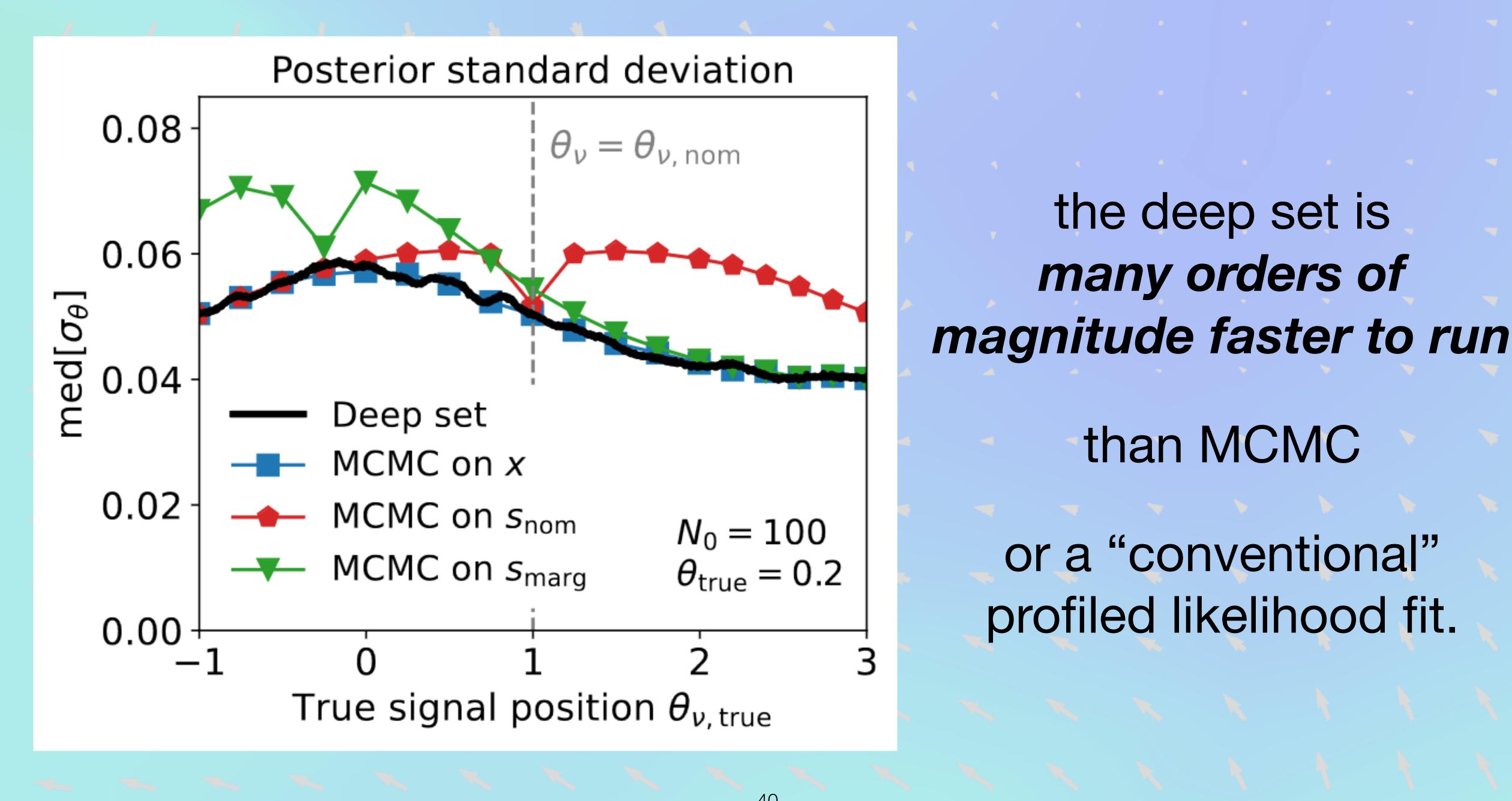


discriminators learned per-event on x

at the nominal signal position

and marginalized over $heta_
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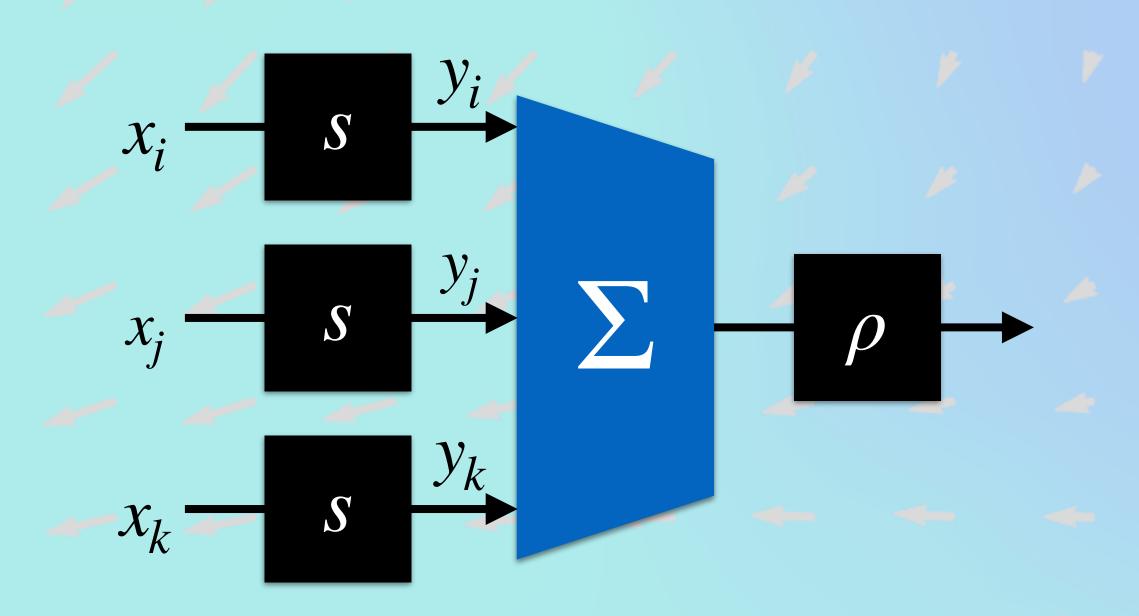
do not preserve enough information to build the correct posterior $\forall \theta_{\nu}$.



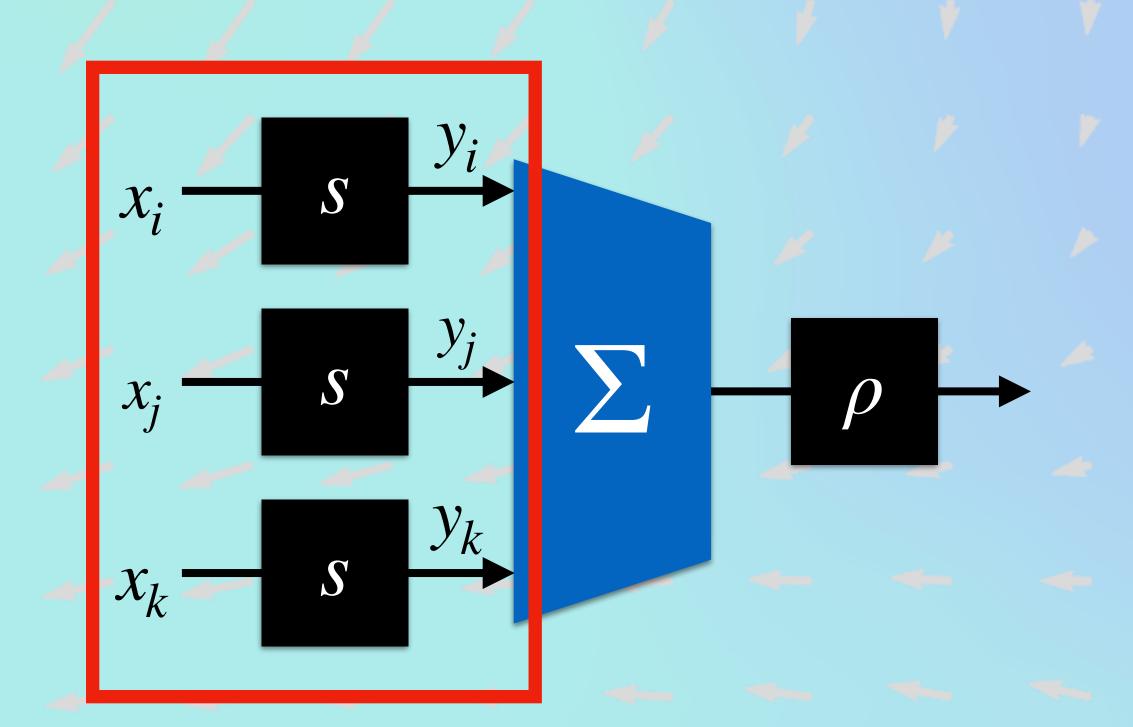
the deep set is many orders of magnitude faster to run

than MCMC

or a "conventional" profiled likelihood fit.



coming back to our network architecture

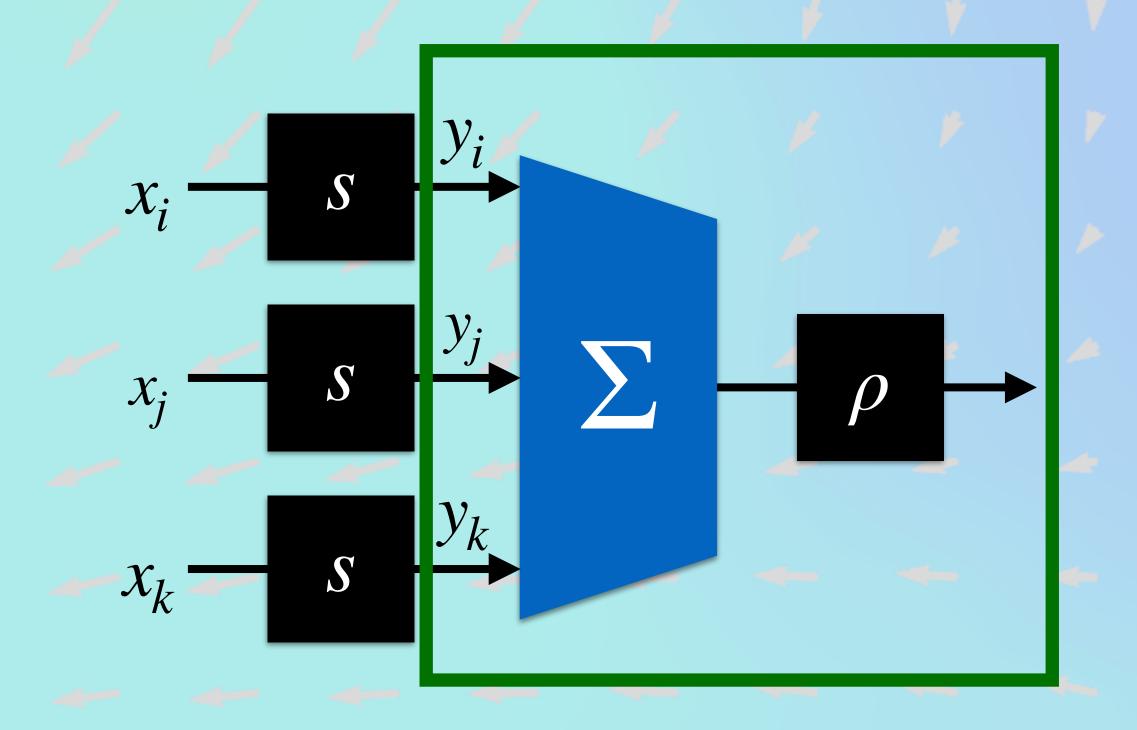


the per-event embedding

$$y \equiv s(x)$$

is information-preserving w.r.t. the posterior or likelihood over θ ,

even in the presence of global NPs.

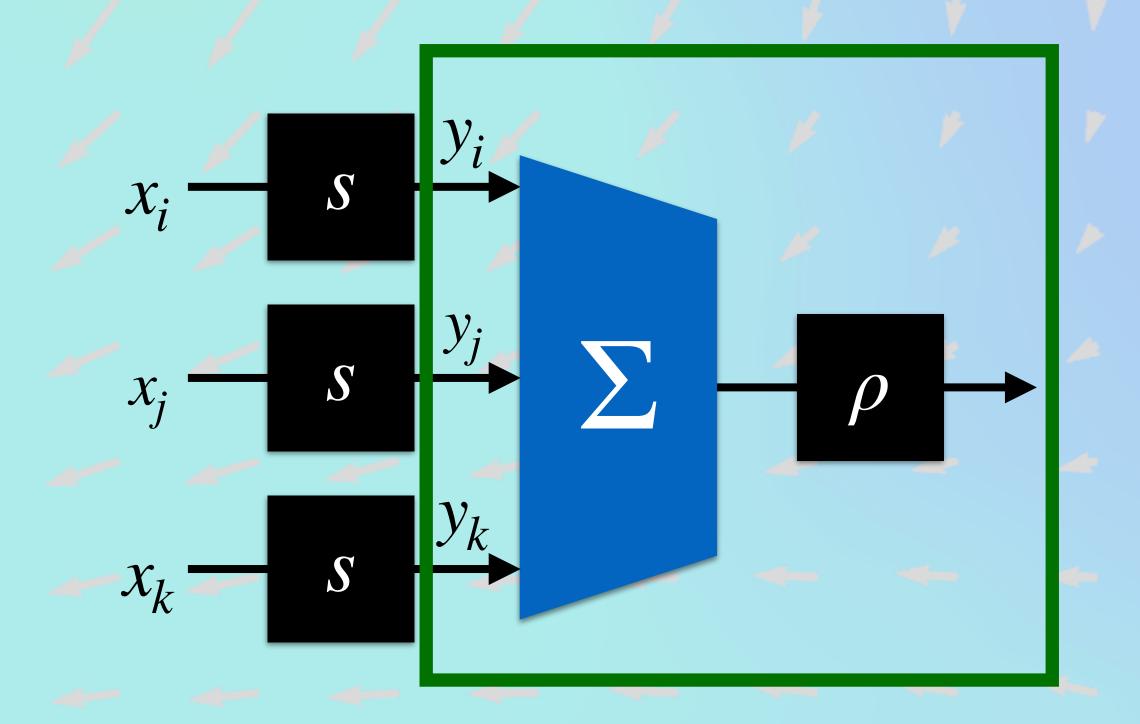


updating the posterior with new data is simple:

$$y_{\{x\}+x_0} = y_{\{x\}} + y_{x_0}$$

then feed this to ρ .

this is extremely fast: can run full inference ~in real time.

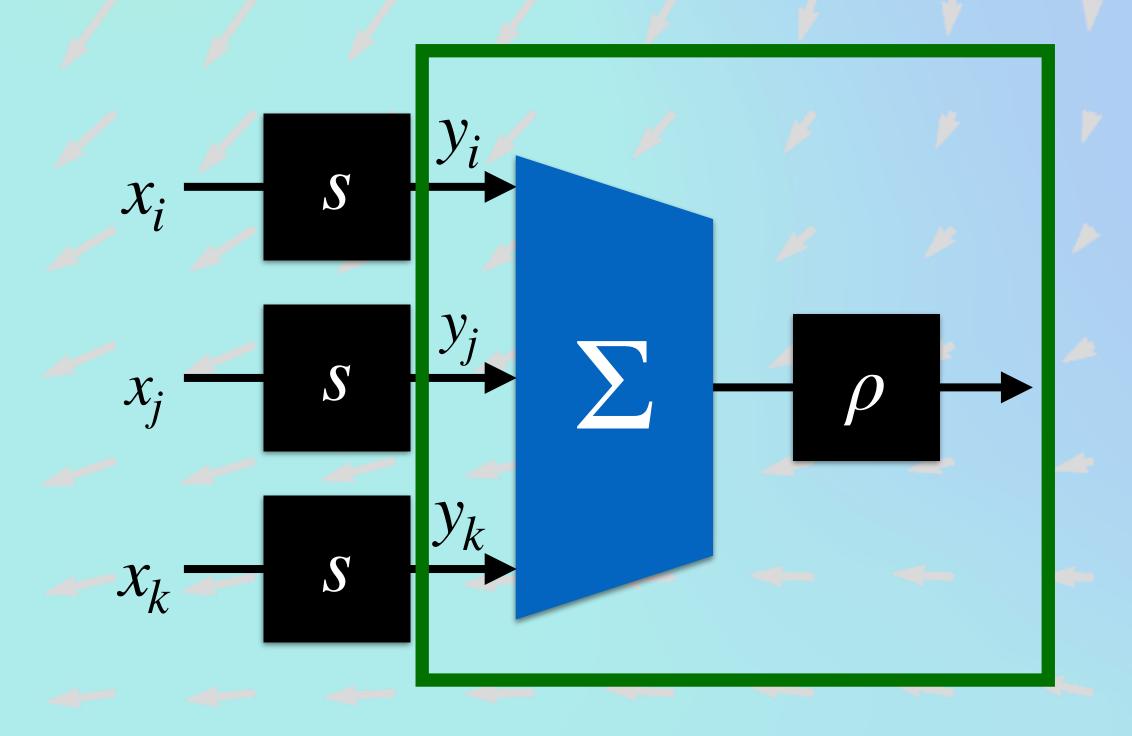


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ongoing work: scaling up to datasets with millions of events +



https://animegenius.live3d.io/features/dragon-ai-art-generator



summary:

if we believe NNs work the way they should...

then given a simulator $p(x | \phi)$

arbitrarily complex!

we can learn an arbitrarily precise approximation of

the entire analysis is basically "done."

$$\log \frac{p(x|\phi)}{p(x|\hat{\phi})} \text{ or } p(\phi|x).$$

leads to *best* possible constraints

basically "done."

 $p(x \mid \phi)$

constraints

we claimed for

"arbitrarily complex x and ϕ "

given a simulator $p(x | \phi)$

we can perform "correct" inference.



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of course the answer is "no"!

how do we validate/calibrate very complex observables x that enable strong, correct constraints?



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leads to best possible

in general a very difficult problem, but we are working on it...

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we have a recipe for reweighting a density p(x) (sim) to q(x) (data):

- ullet train a discriminator $D_p^q(x)$ and weight each sample x by $\exp D_p^q(x)$.
- allows weight-based calibration for complex observables.

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in many instances it is preferable to move events to perform a calibration:

$$T: \vec{x} \to \vec{x} \implies T_{\#}p \approx q$$

- \odot for any p and q, at least one T exists, and it is unitary.
- we usually want to change the simulation as little as possible:
 - lacktriangle we need to find the unique \hat{T} that minimally (or "optimally") morphs p into q.

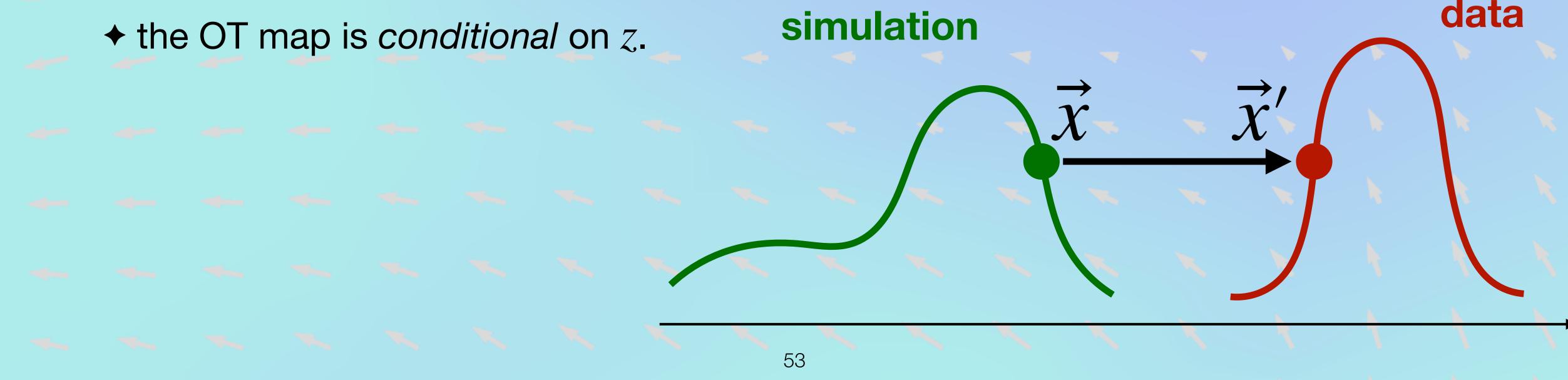
we have been implicitly using optimal transport (OT) for decades in HEP:

- \odot correct simulated scale of a gaussian density \leftrightarrow move the mean of simulated distribution. neural OT generalizes this process.
- for euclidean spaces, the OT map is the gradient of some convex potential:

$$\hat{T}\vec{x} \equiv \overrightarrow{\nabla}\phi(\vec{x}).$$

ullet we have recently managed to learn $\phi_{_{7}}$ and therefore $\hat{T}_{_{7}}$ in concrete use-cases:

 \bullet the OT map is conditional on z.



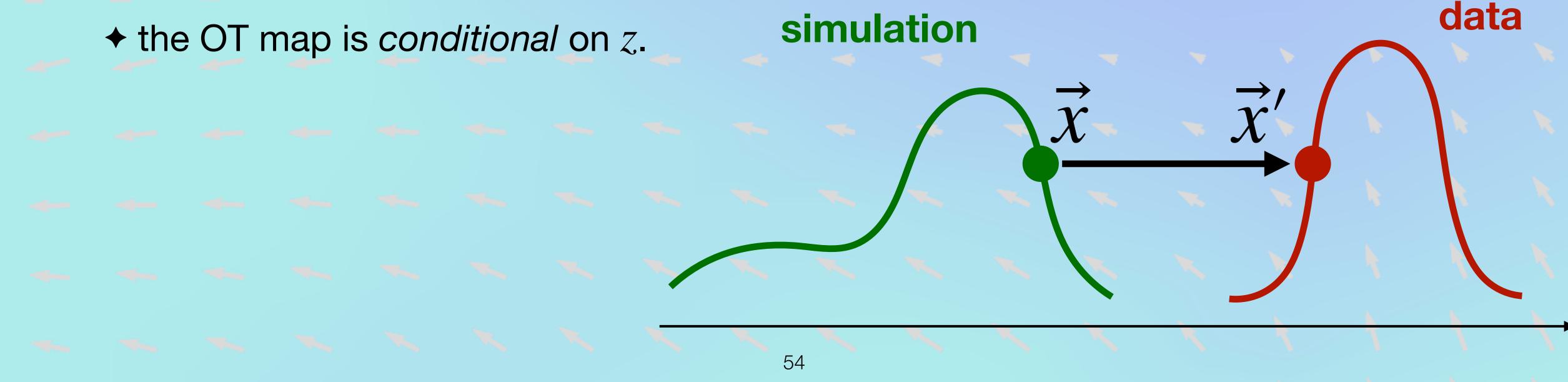
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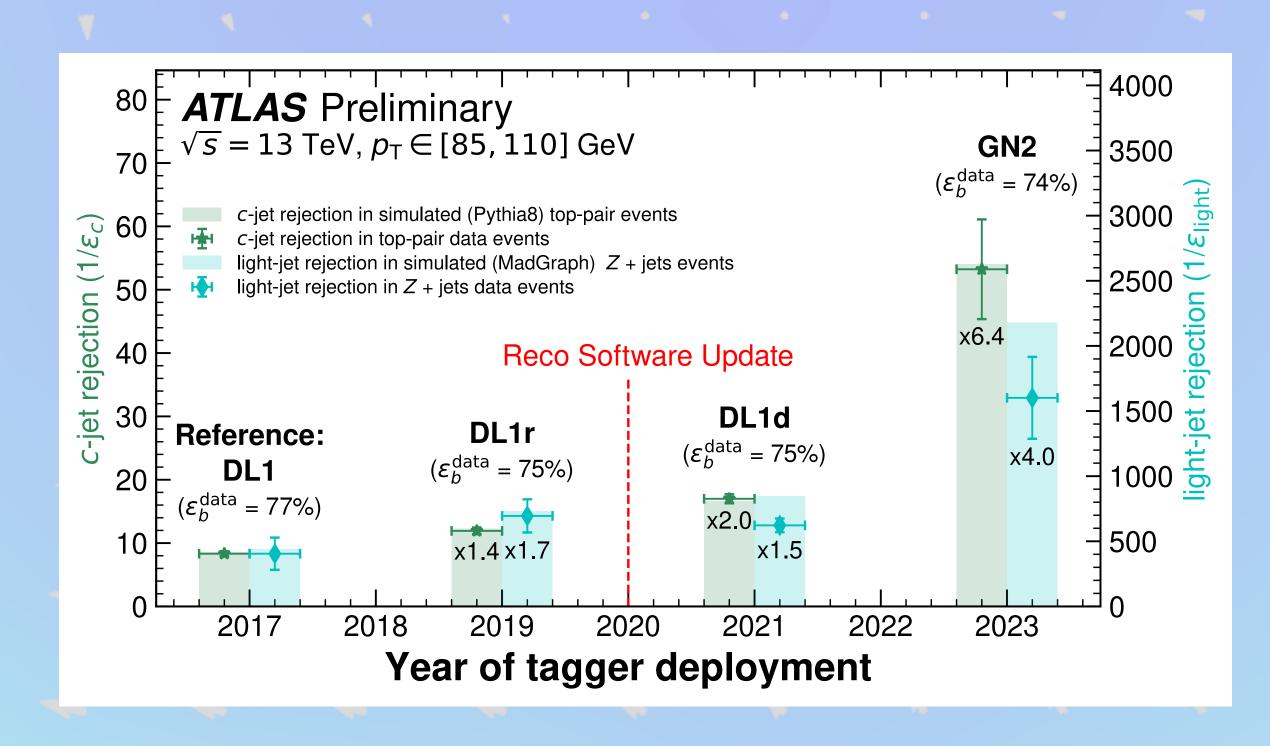
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a concrete example: jet flavor-tagging in ATLAS

jet flavor-tagging is a classification problem:

- ATLAS's classifiers emit the probability of a jet to contain a b-hadron, c-hadron, or neither (p_b, p_c, p_u) .
- modern algorithms are very complex:
 - charged-particle tracks as inputs.
 - incredible separating power, but clear mismodeling
- until now, we had no direct calibration for these probabilities.
 - ♦ to do so, we set $q_i \equiv \log it p_i$, treat \vec{q} as euclidean, and calibrate via OT.

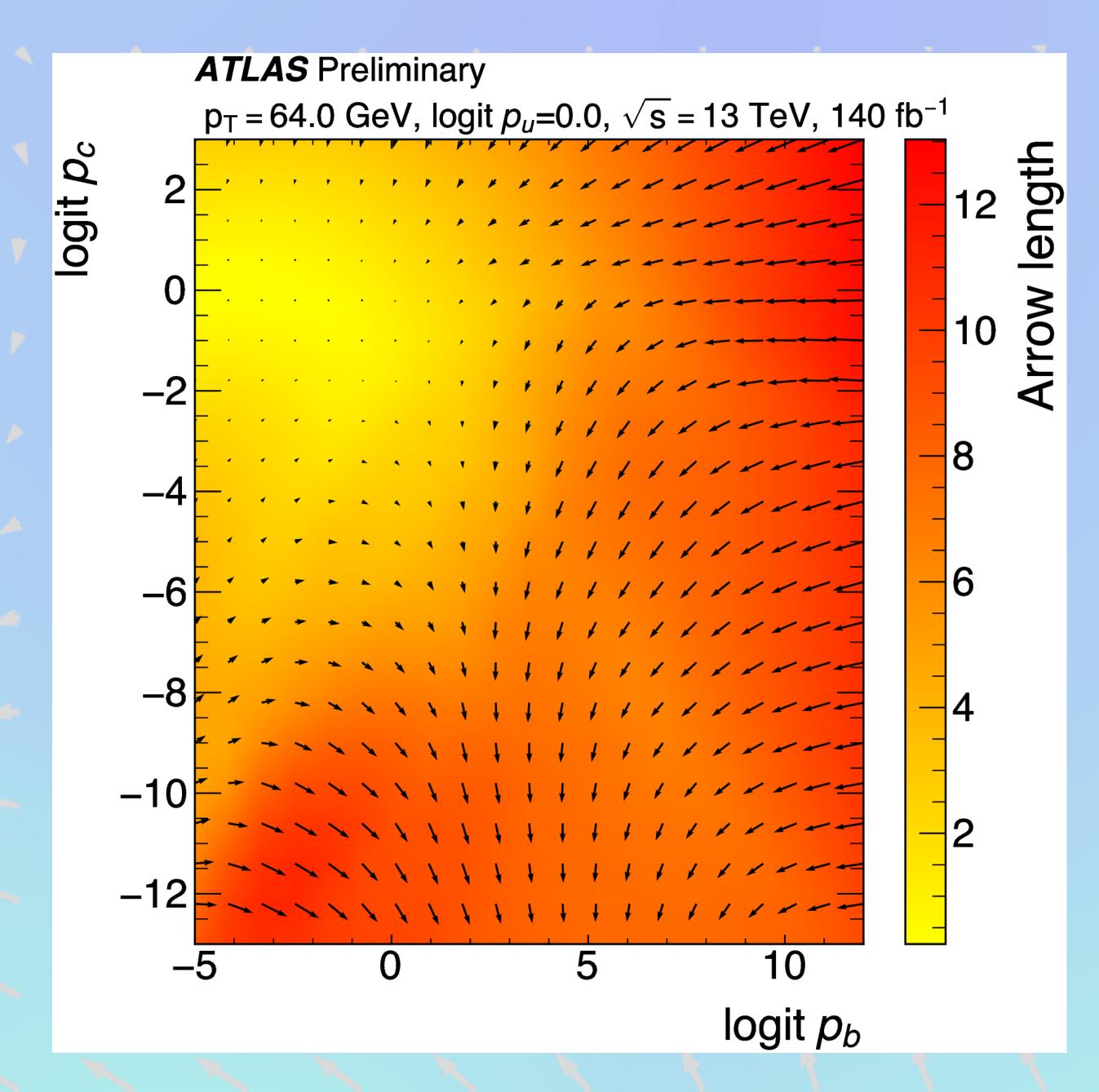


notation:

- $q_i \equiv \log it p_i$: flav. class. scores
- $p_{\text{sim}}(\vec{q} \mid p_T) \equiv p_{\text{sim}}(\vec{q} \mid p_T) p_{\text{data}}(p_T)$
- $\hat{T}_{\#} \equiv p_T$ -dependent OT map

result:

- we obtain the full 3D OT maps in \vec{q} space s.t. $\hat{T}_{\#}p_{\rm sim} \approx p_{\rm data}$,
 - lacktriangle derived as a function of jet p_T .
- here we show a 2D slice for $q_b \times q_c$ at fixed $q_u \times p_T$.



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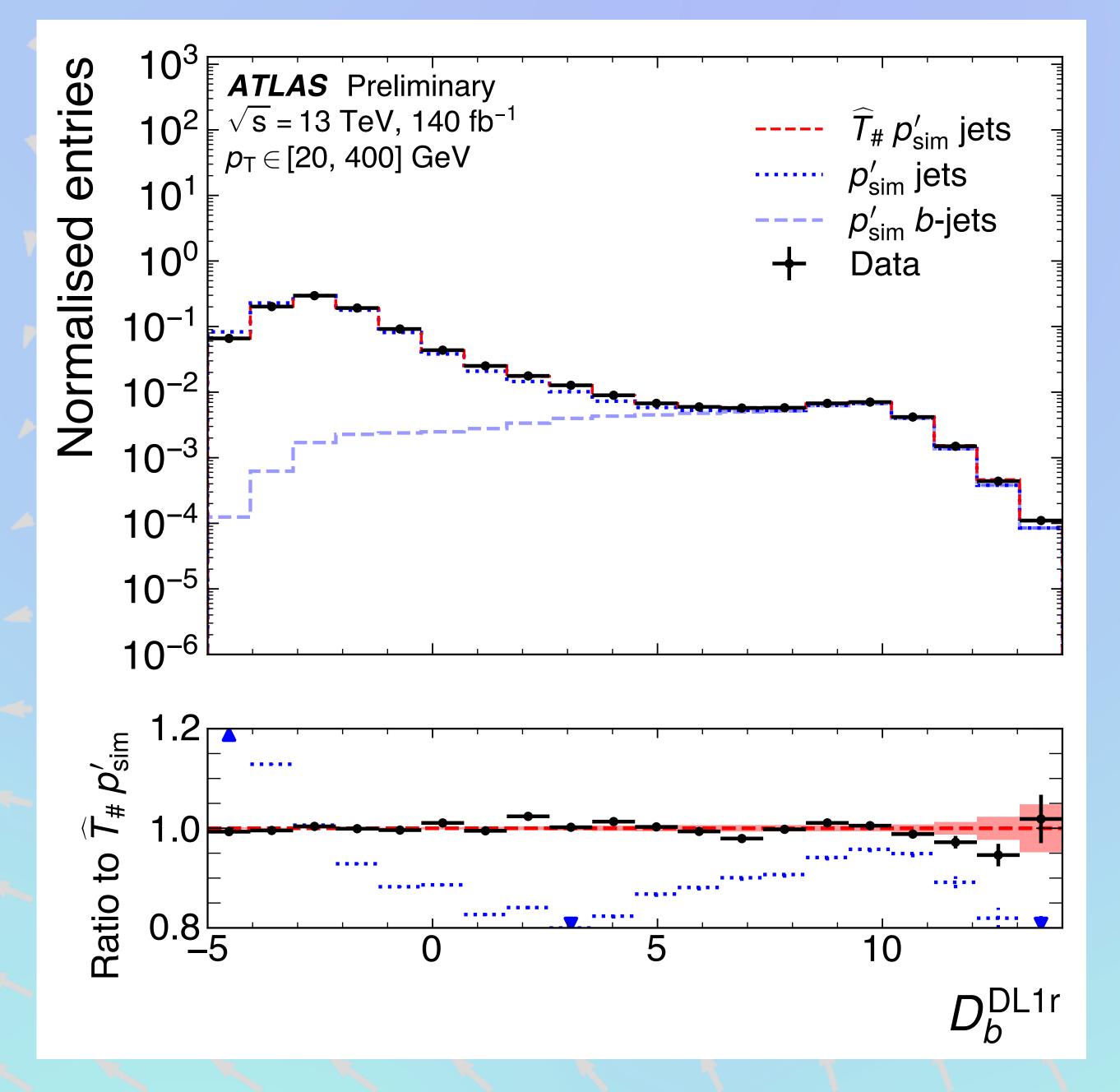
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the technology works!

- before calibration, poor modeling of many quantities...
 - → including the b-tagging discriminator

$$D_b \equiv \log \frac{p_b}{f_c p_c + (1 - f_c) p_u}$$

- after calibration, very good agreement even for this non-trivial quantity:
 - ullet higher-order correlations between p_i are properly corrected.



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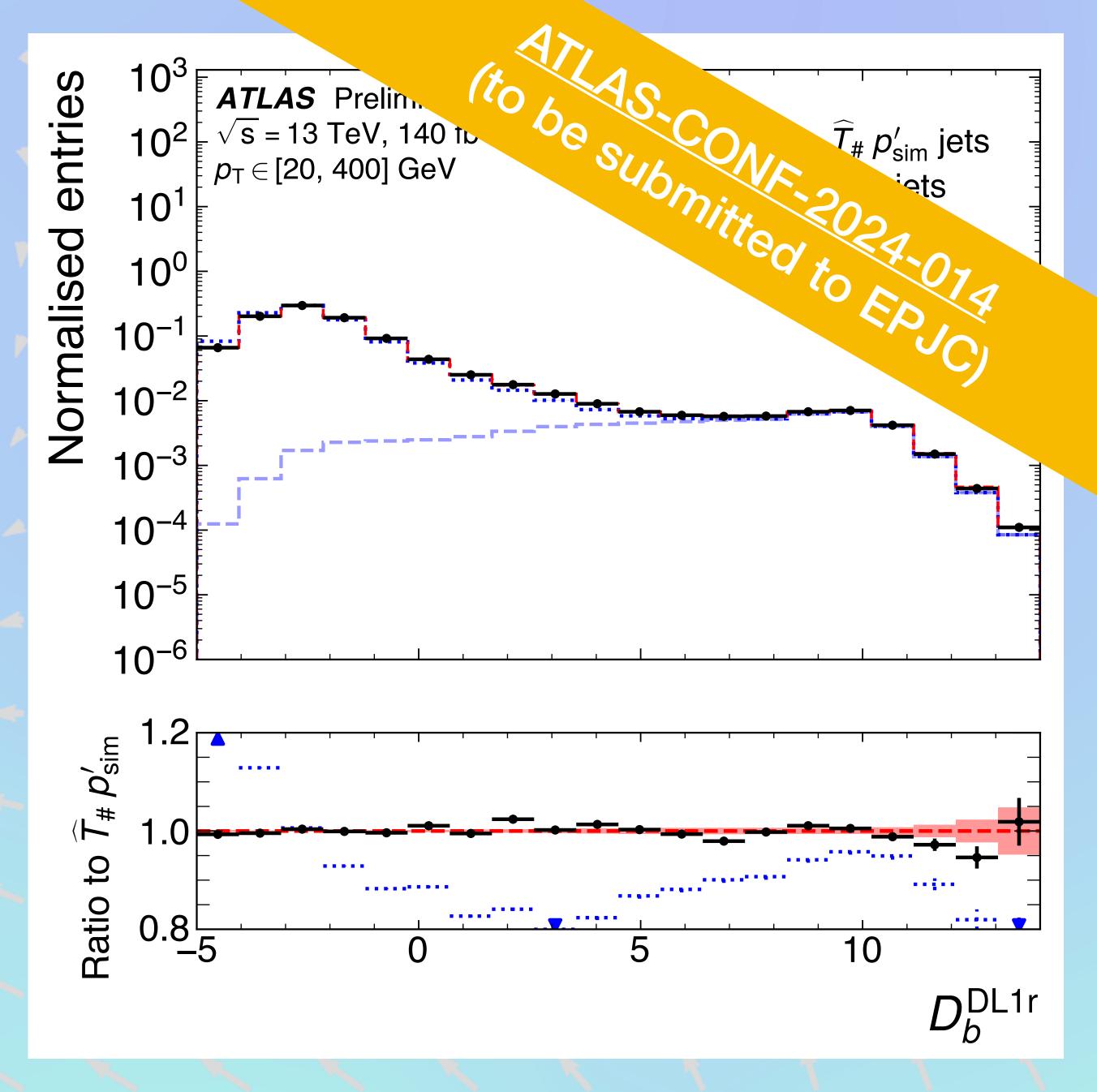
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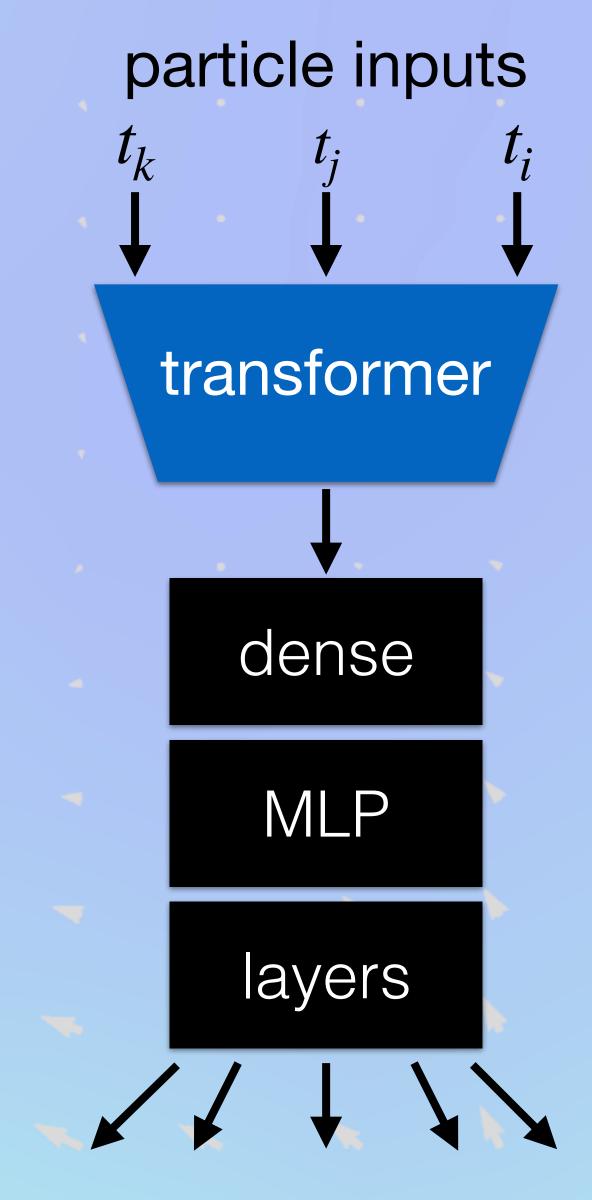
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the future (?): scaling this up

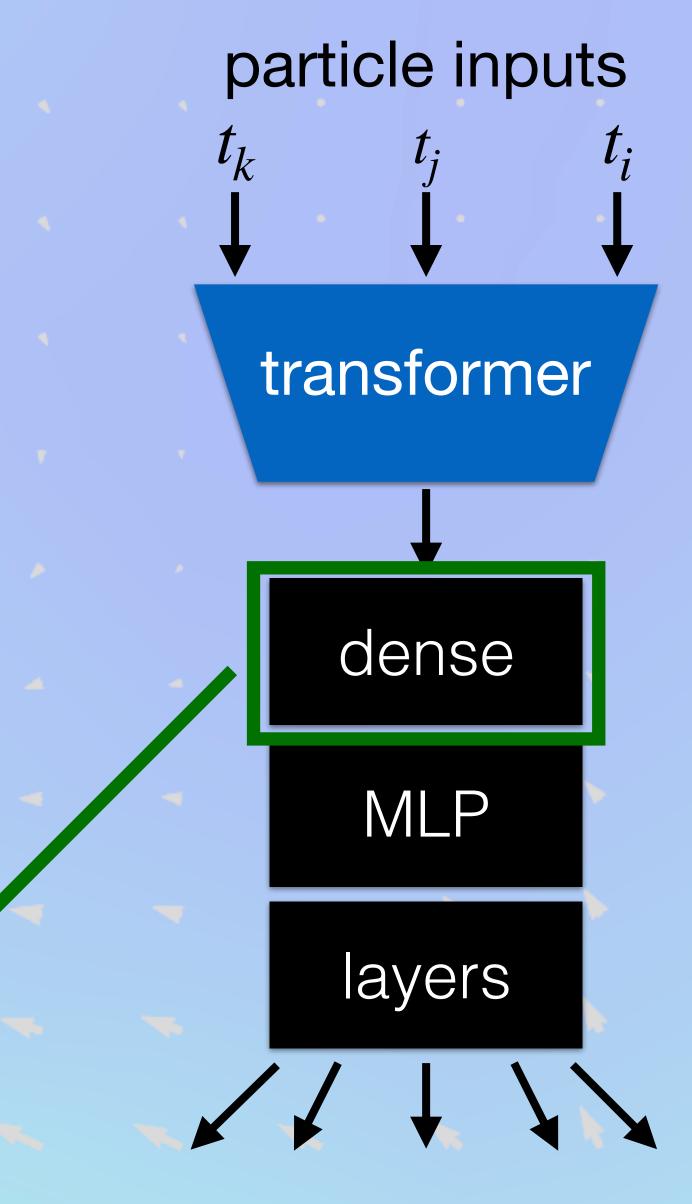
- preliminary results show that this scales nicely to very high dimensions:
- start with a JetClass-like classifier that discriminates between 10 distinct types of large-radius jets:
 - → H → bb, H → cc, t → bqq', inclusive ("QCD") jets, etc



multi-class outputs:

$$p_{t \to bqq'}, p_{H \to b\bar{b}}, p_{QCD}, \dots$$

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- calibrate the internal 128-dim representation of the jet information

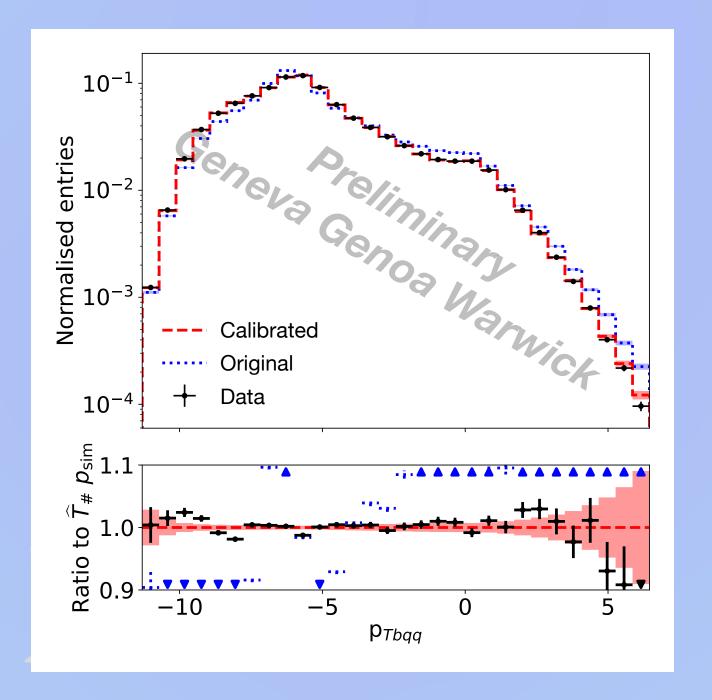


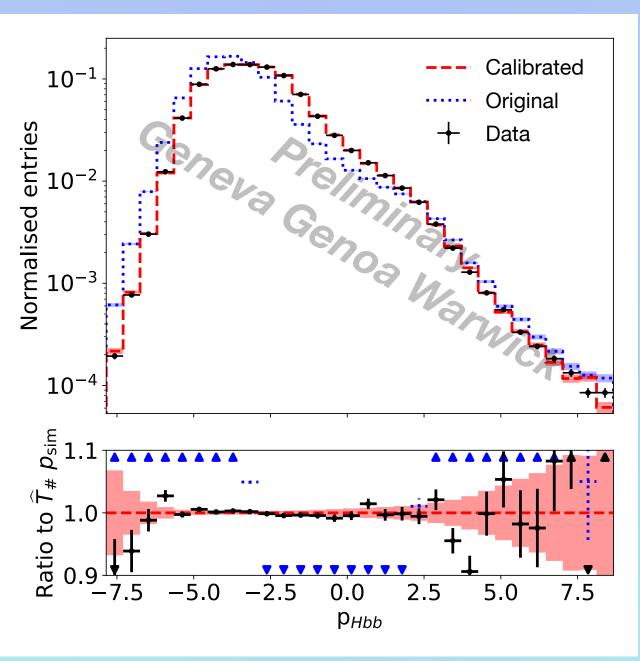
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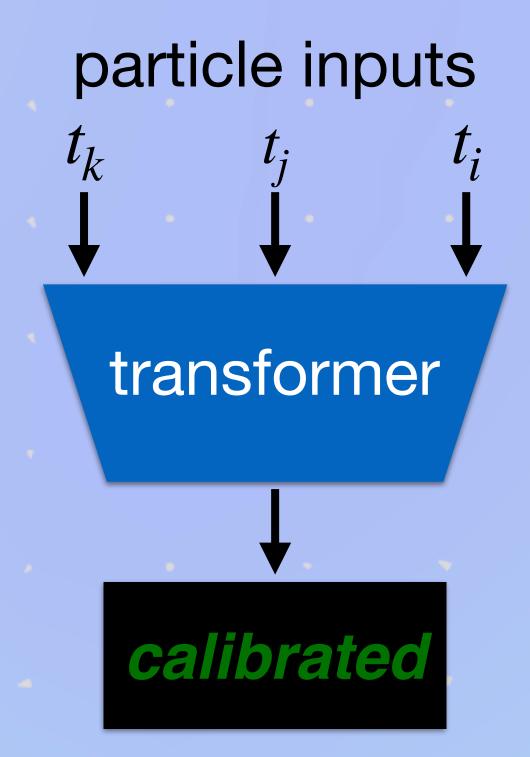
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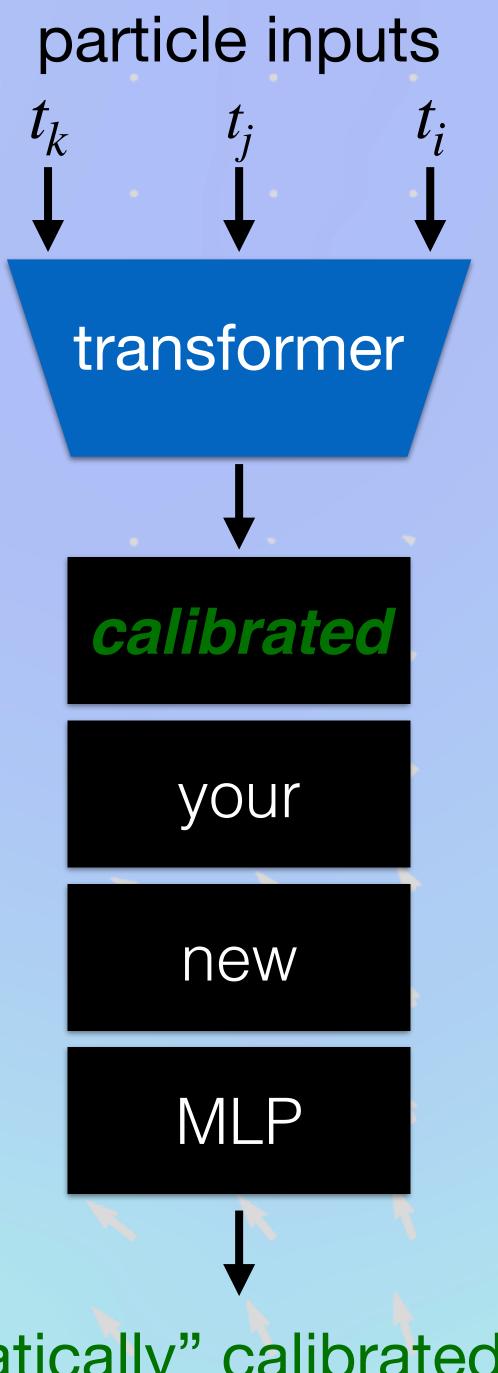




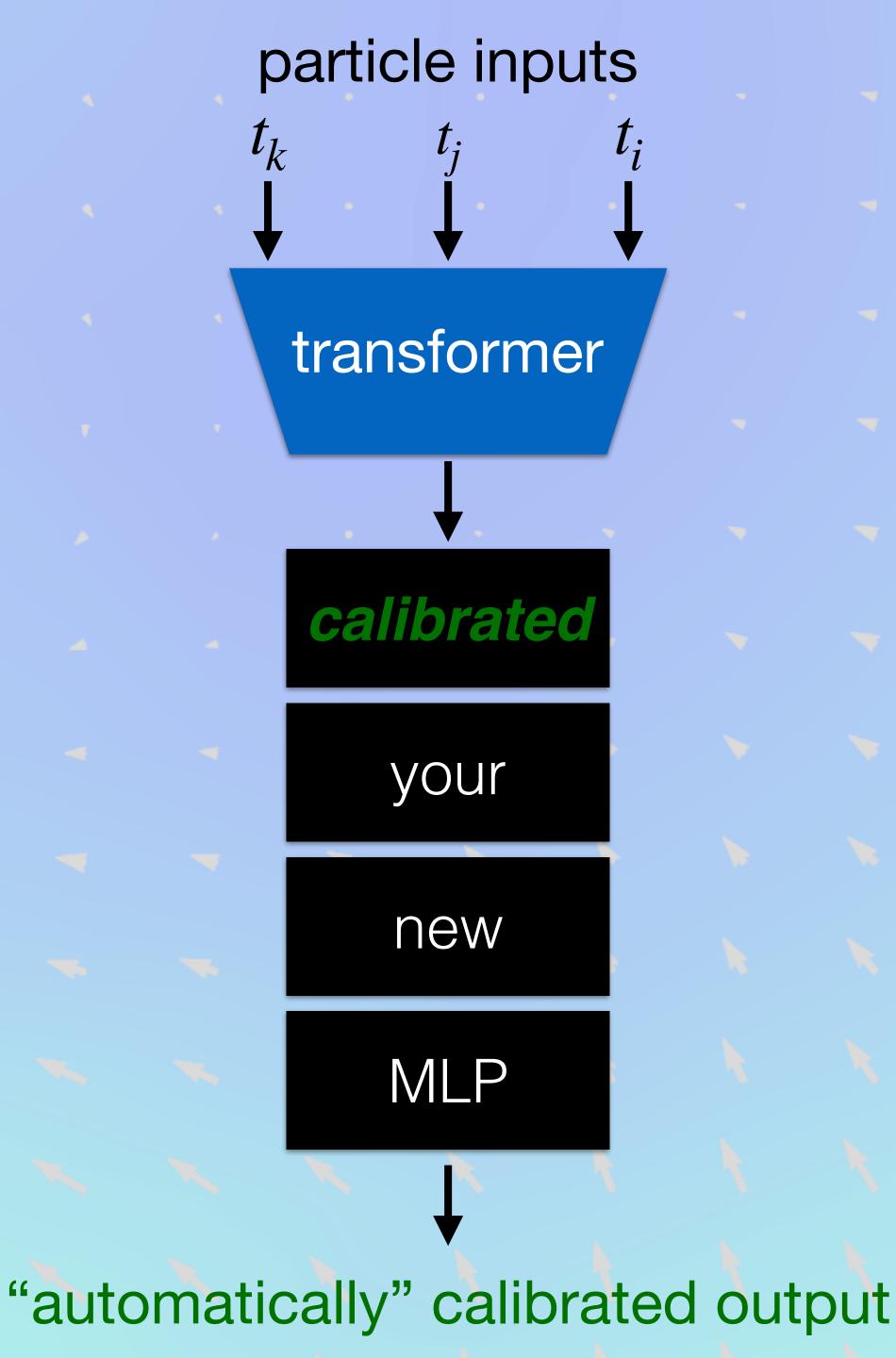
- with the 128-dim "latent representation" of the jet calibrated,
 - we observe that the original 10 classification scores close very well.
- but this enables "arbitrary" use of the information contained in that representation:



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- with the 128-dim "latent representation" of the jet calibrated,
 - we observe that the original 10 classification scores close very well.
- but this enables "arbitrary" use of the information contained in that representation:
 - allows calibration of "foundation" models for broad use.
 - useful well beyond particle physics!



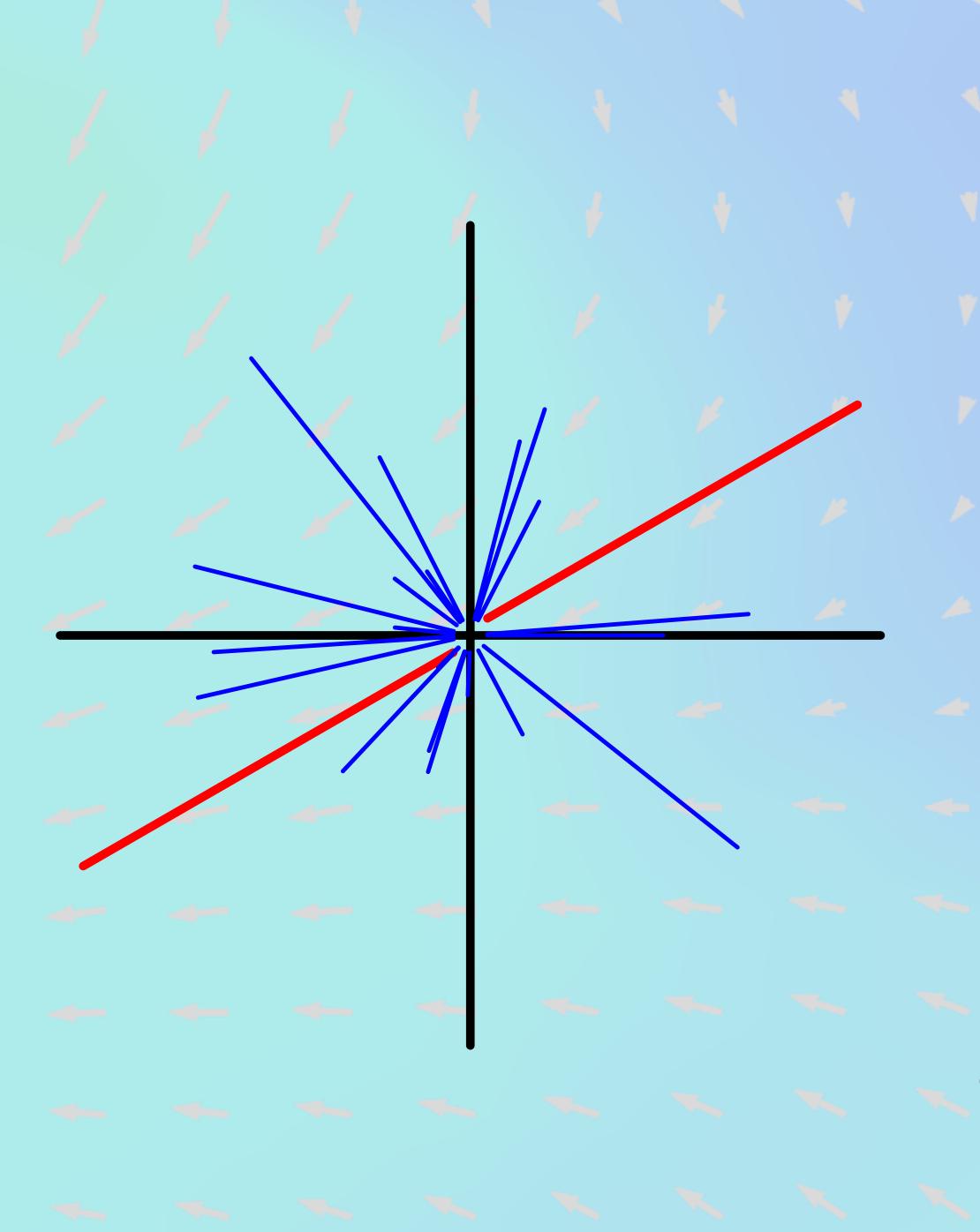
nSBI promises to yield the best possible constraints on a parameter ϕ given an observable x.

- that's a big claim but also worth pursuing...
- \odot all the information of x relevant to ϕ in the simulator $p(x | \phi)$ can be used for inference:
 - lacktriangle allows more complex x and ϕ than standard approaches.
 - potential huge gains in cases where NPs are very correlated with Pols.
 - + requires very good (correct!) simulation.

nSBI promises to yield the best possible constraints on a parameter ϕ given an observable x.

- lots of ongoing work to make this a reality:
 - to contend with "technical" problems like convergence and global NPs,
 - ◆ to calibrate complicated observables x to enable correct constraints.
- large potential speed-up in inference time.
- \circ I'm hopeful that in $\mathcal{O}(5)$ years there will be "over the counter" solutions available but it will take effort to get there.
 - ◆ It's a very interesting space with lots of work to do!





why do we need summaries / embeddings / "physics objects"?

tractability: ~millions of detector channels to read out per LHC bunch crossing.

correctness: difficult to construct a simulator that adequately describes all details of the data!

indeed, this is perhaps the main "point" of constructing jets:

- we cannot correctly predict the details of QCD with arbitrary accuracy;
- we can predict the "large-scale structure" of the fragmentation of partons.



"large-scale structure": calculable features

calibration region

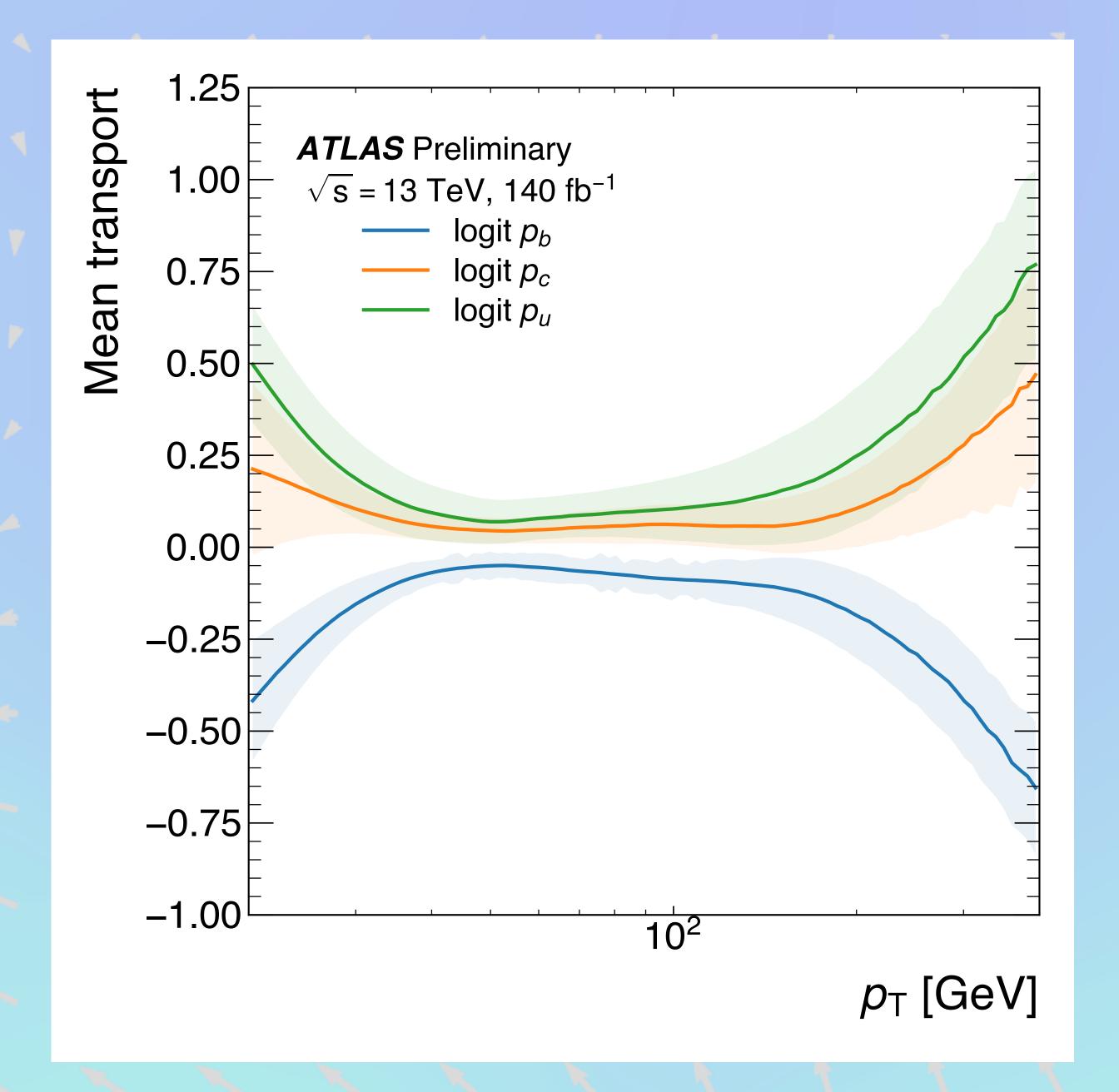
analysis region

even so, having access *more* relevant details can enable better constraints.

- we can often *measure* the density over some features better than we can predict it.
- a calibration region may be used to measure said density and to correct the simulation.

notation:

- $q_i \equiv \log it p_i$: flav. class. scores
- $p_{\text{sim}}(\vec{q} \mid p_T) \equiv p_{\text{sim}}(\vec{q} \mid p_T) p_{\text{data}}(p_T)$
- $\hat{T}_{\#} \equiv p_T$ -dependent OT map
- for b-jets: $\hat{T}p_b < p_b$, while the reverse is true for p_c and p_u .
 - the simulation overstates its classification power.
- we have much more information about what aspects of the simulation are incorrect than before.

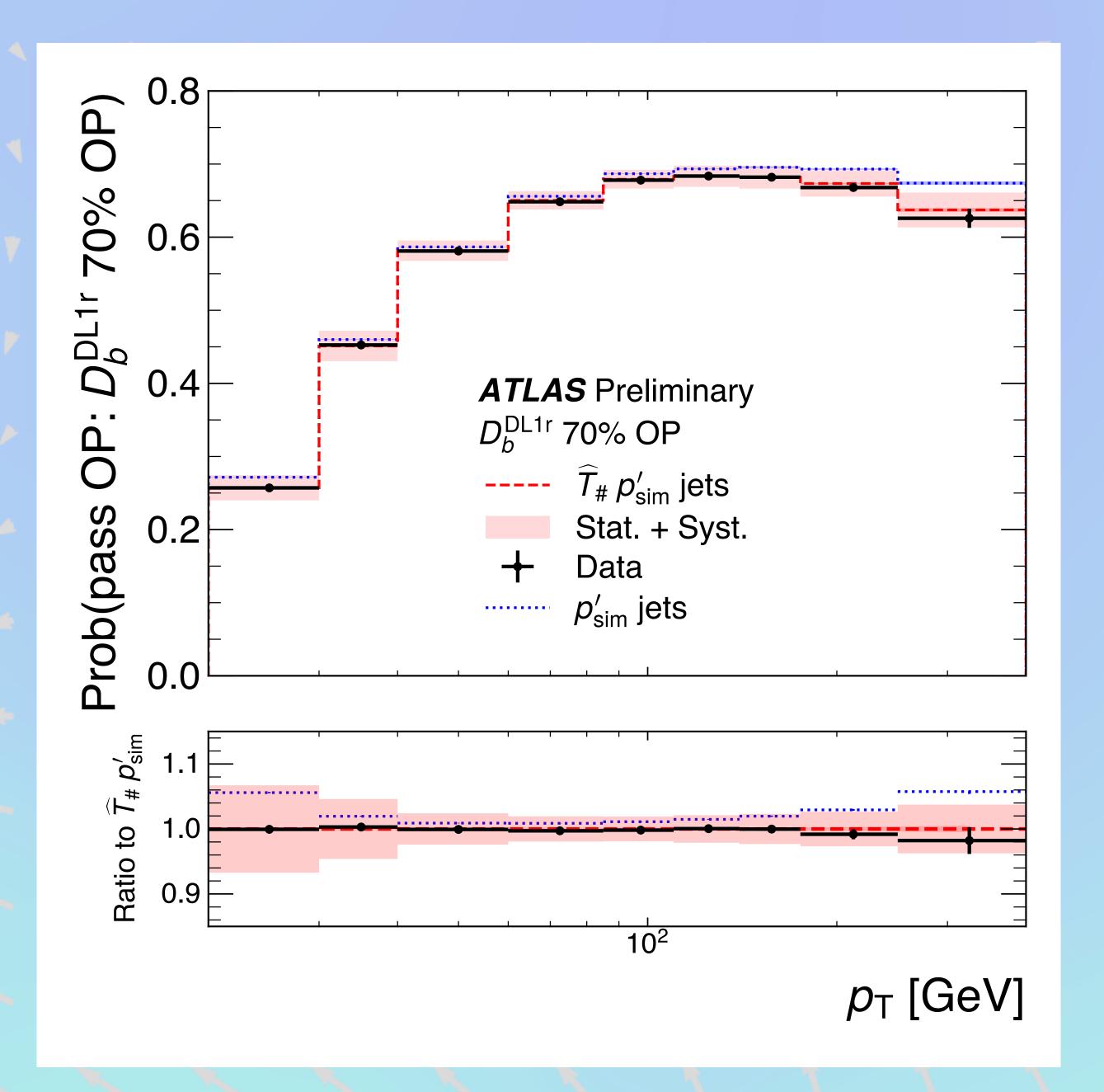


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conventional operating points are "automatically" corrected.

- excellent closure observed for the "standard" b-tagging discriminant points
 - lacktriangle used to observe Higgs couplings to W, b, t.

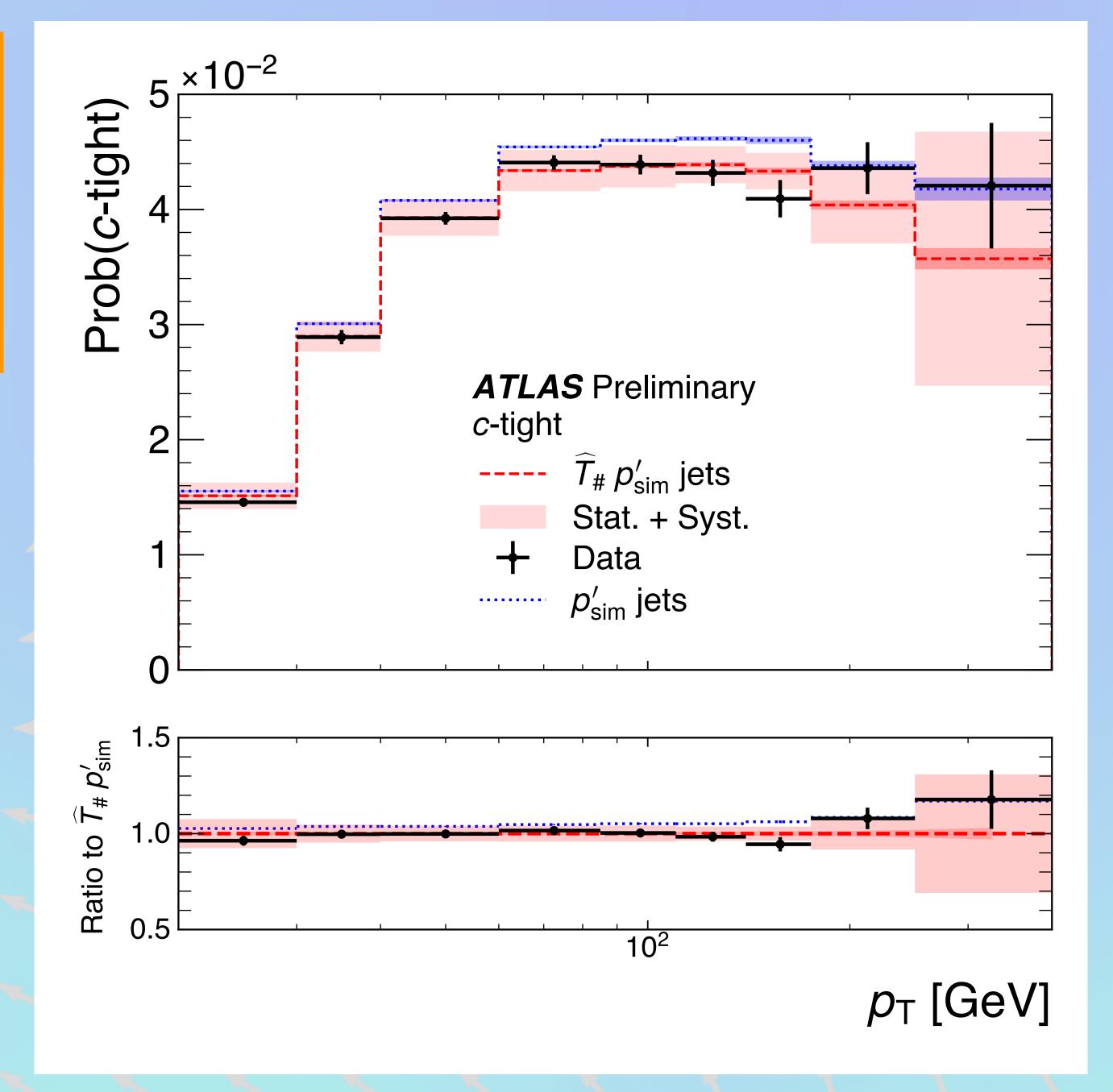


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more general uses of flavortagging become possible.

- charm-tagging discriminator used to constrain the $H \leftrightarrow c$ couplings shows good agreement.
- closure depends on the full 3D density $\hat{T}_{\#} p_{\text{sim}}(\vec{q} \mid p_T)$ agreeing with data.



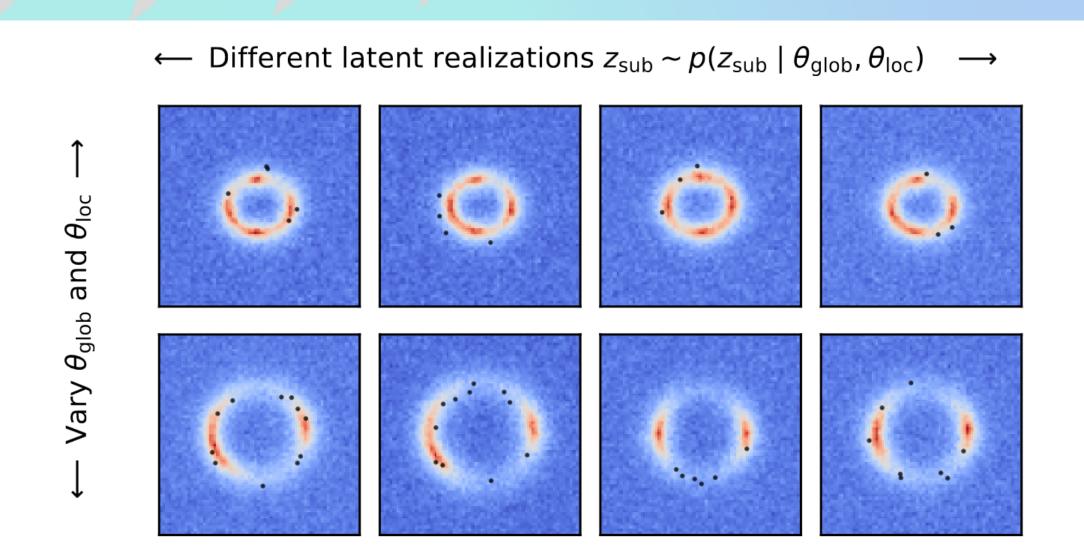
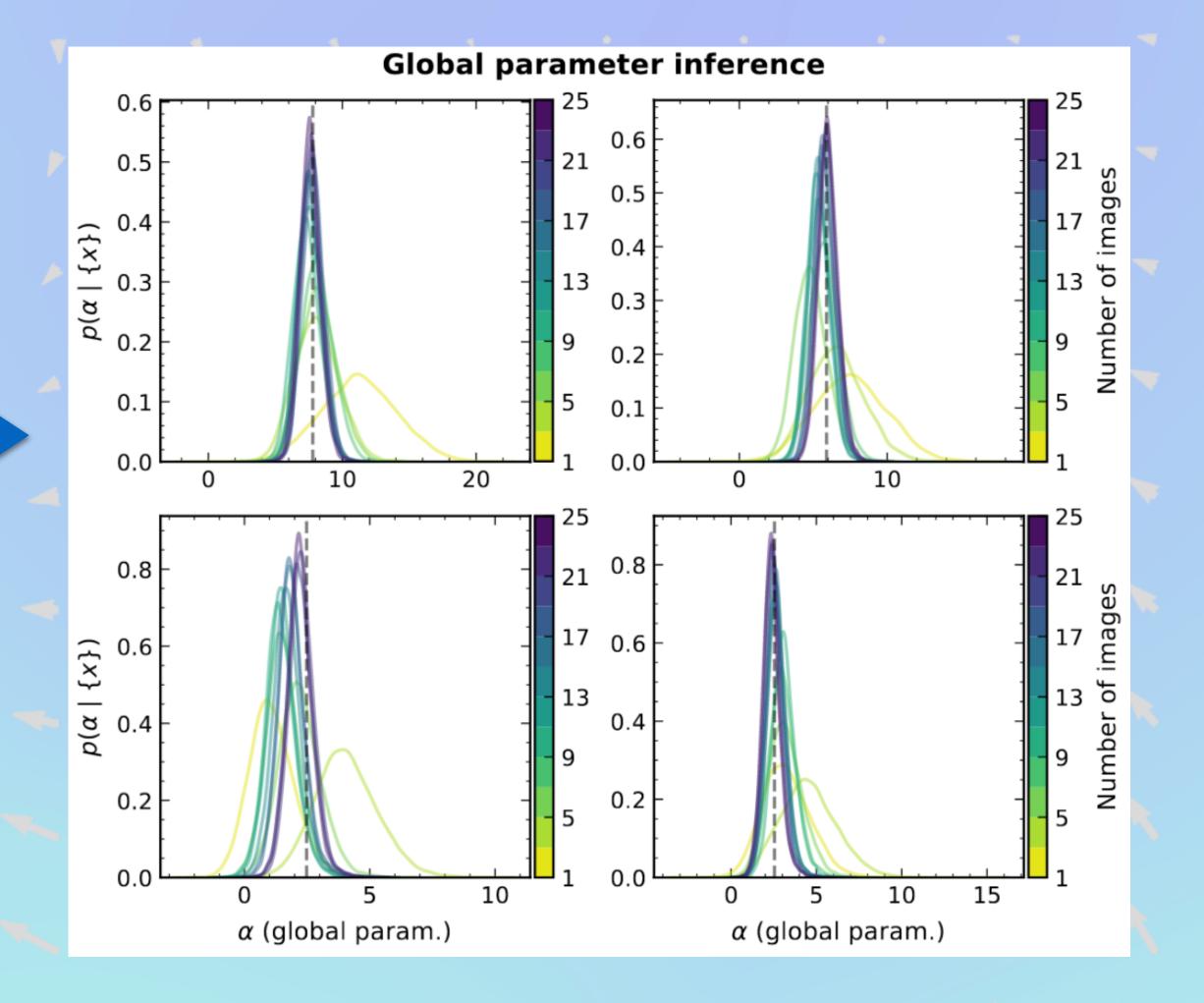


Figure 5: Illustrative samples from the lensing model. The rows show two different choices of local (per-event) parameters, while the different columns show variations on the global (set-wide) parameters for the fixed choice of local parameters. Sample-to-sample variation induced by the global parameters, which control the abundance of a subhalo population in the lens, can be seen. The scatter points shows the location of individual subhalos in each image.



need to write down likelihood

need to do MCMC or something from scratch each time

have to manually derive summary statistics

loss of

information

beseeched by curse of dimensionality

LIKELIHOOD-BASED INFERENCE

no need to write down explicit likelihood. as long as u can simulate it, youre good

no loss of information

once trained, do inference in milliseconds

neural network finds the best summary statistics for you

handily beat curse of dimensionality

SIMULATION BASED INTERENCE

credit: Siddarth Mishra-Sharma

before we can talk about neural inference,

we have to believe machine-learning is doing what it claims!

why does it work? (from the perspective of a physicist...)

$$|\text{continuous}(\mathbb{R}^m \to \mathbb{R}^n)| = |\mathbb{N} \to \mathbb{R}|$$

continuous real functions \leftrightarrow functions from $\mathbb N$ to $\mathbb R$.

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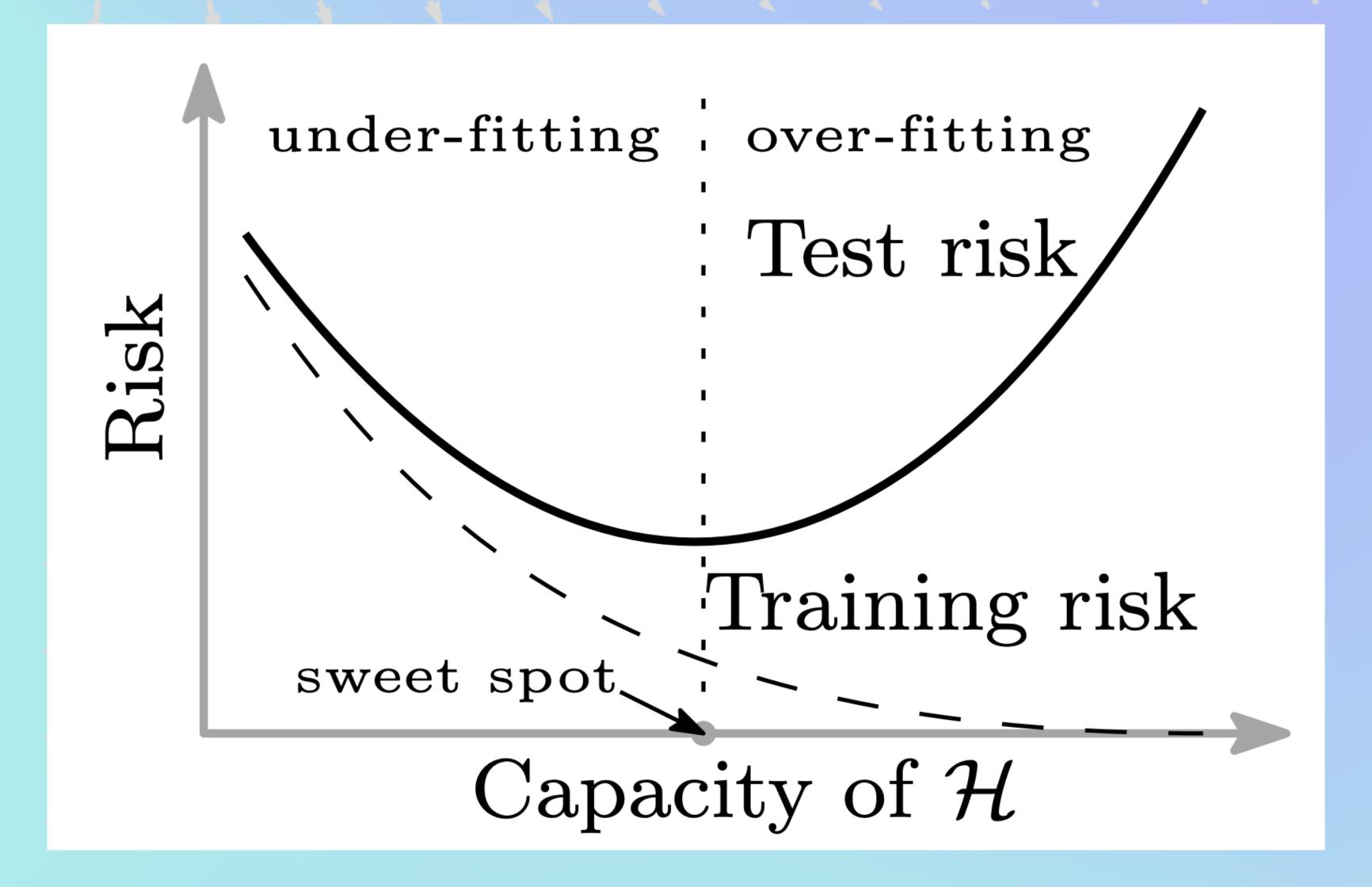
i.e. any continuous function can be *parameterized* by a countable number of real numbers.

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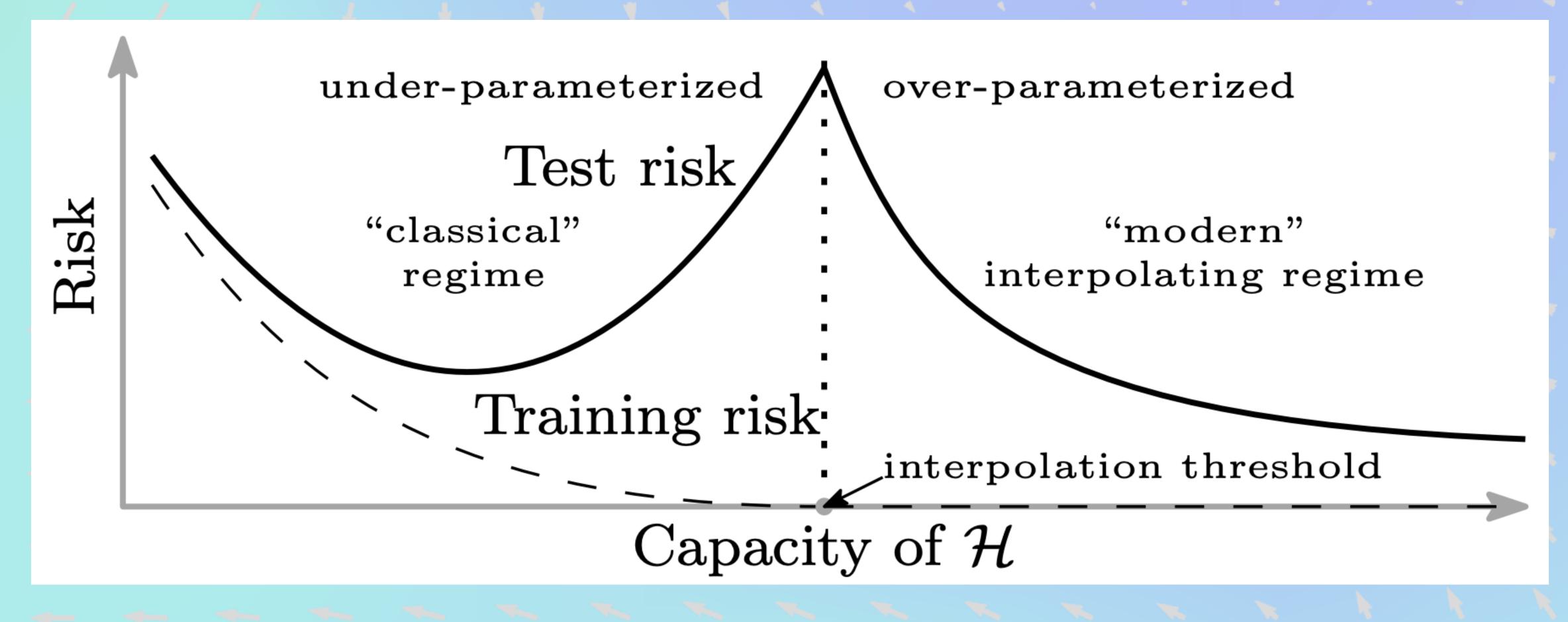
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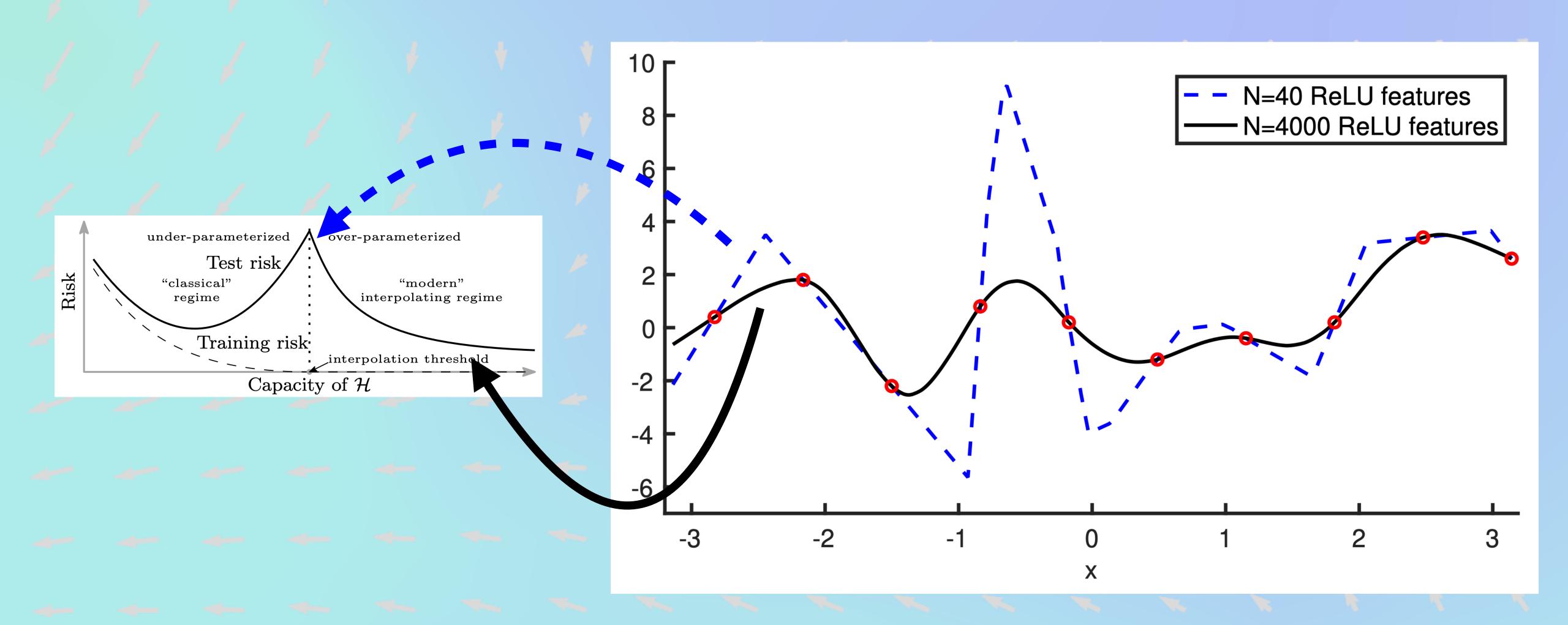
(underpins e.g. Fourier series, eigenvector decomposition, etc)



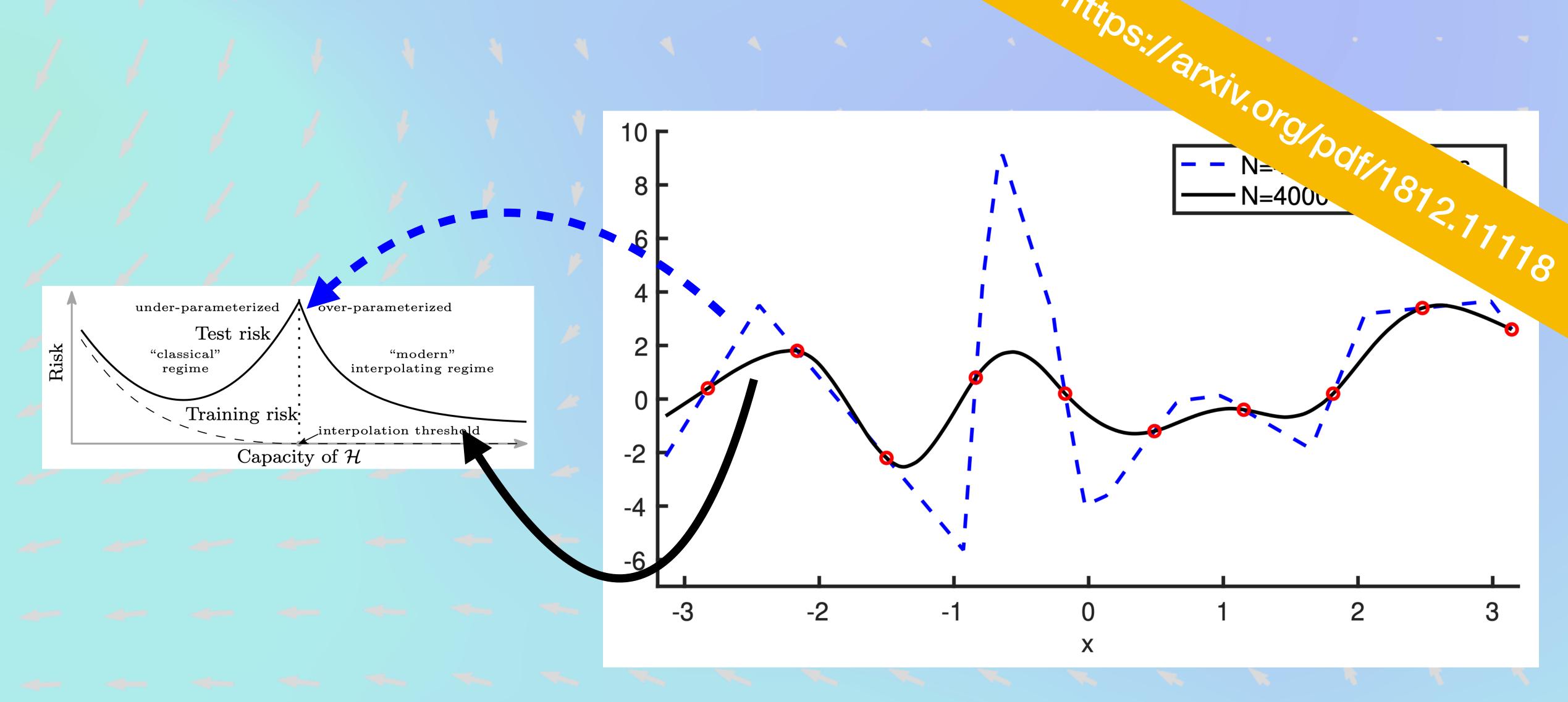
"classical" view of fitting data



growing (empirical) evidence: "classical" under- and over-fitting ideals do not apply.



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to summarize:

- 1) NNs approximate any real function to arbitrary precision with a finite number of parameters (and enough training data).
- 2) over-parameterization yields high-quality (very predictive) fits.
- 3) over-parameterization *increases* the odds the fit converges to a "good" local minimum.

mounting evidence that NNs really are learning what they claim.

to summarize:

- 1) NNs approximate any real function to arbitrary precision with a finite number of parameters (and enough training data).
- 2) over-parameterization yields high-quality (very predictive) fits.
- → growing confidence that NNs are learning what they claim to.