Velocity distribution of dark matter

Malcolm Fairbairn

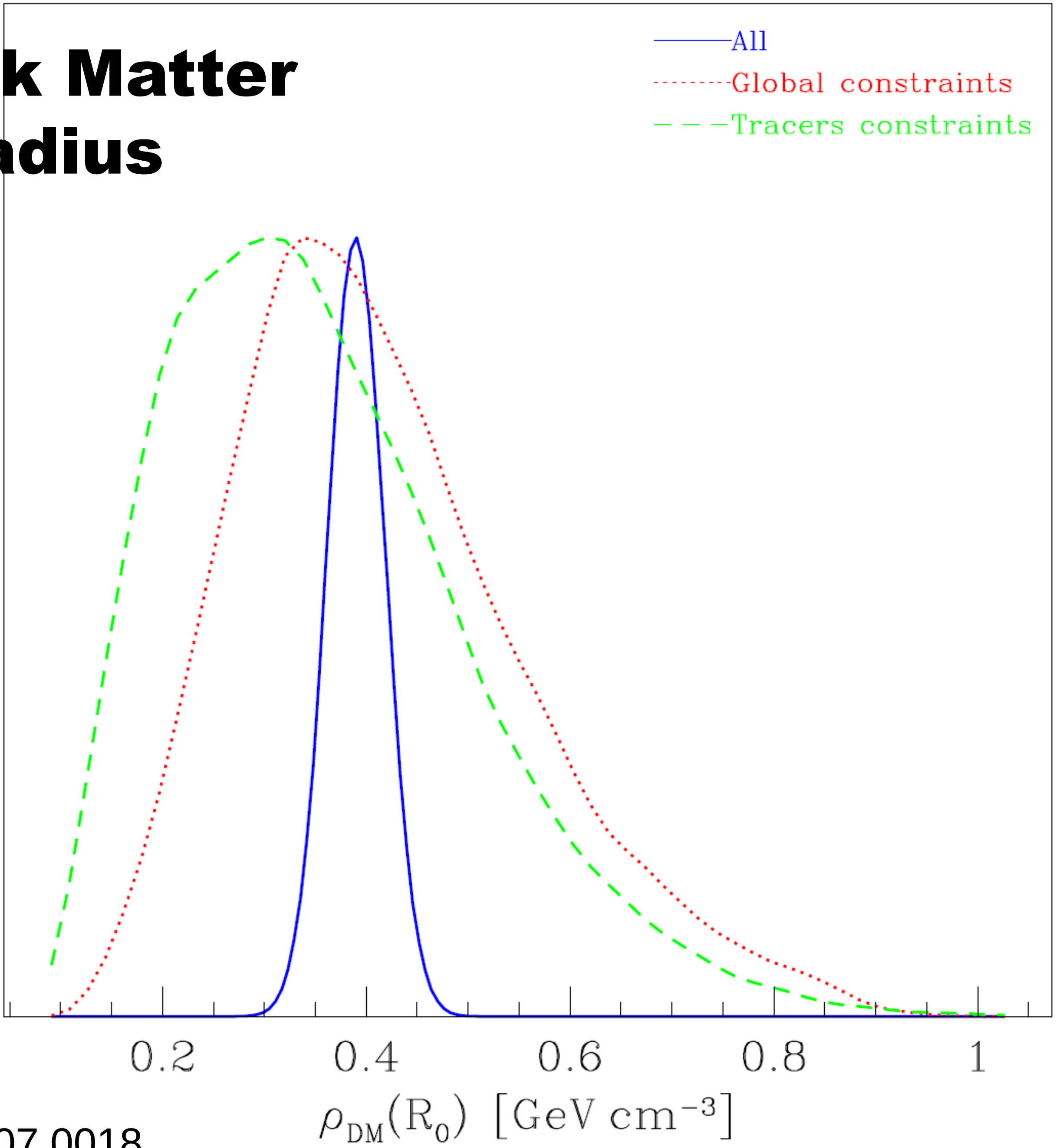
King's College London
University of London

Outline

1. Dark matter density uncertainties
2. Dark matter anisotropy uncertainties
3. Is it smooth?
4. Is it Maxwellian?

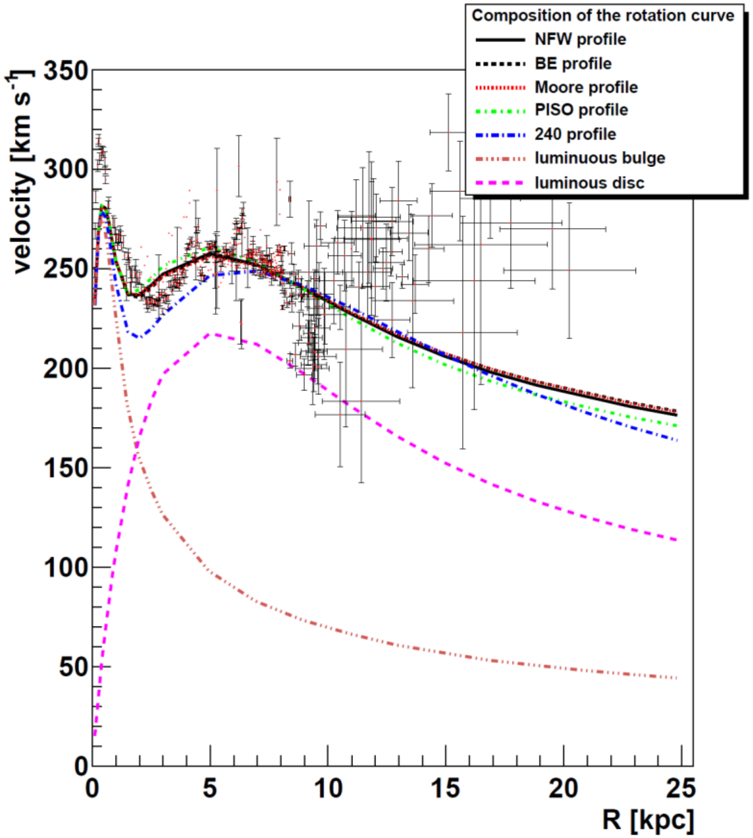
Density of Dark Matter at Solar Radius

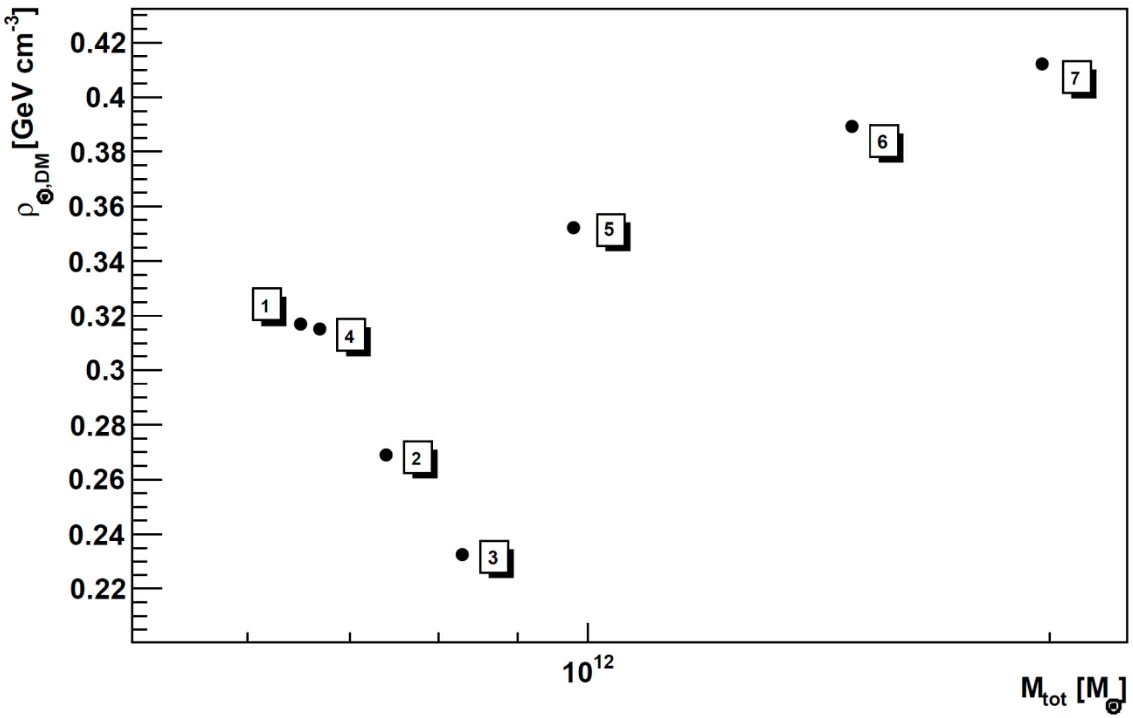
Relatively under control?



Catena and Ullio arXiv:0907.0018

...but is it really understood?





Weber and de Boer arXiv:0910.4272

Default halo model for dark matter direct detection

1. Isothermal halo
2. Gaussian velocity distribution
3. Isotropic velocity distribution

PROBABLY NONE OF THESE ARE TRUE !

So what?

Direct detection of dark matter

Look for recoil of DM-nucleus scattering:

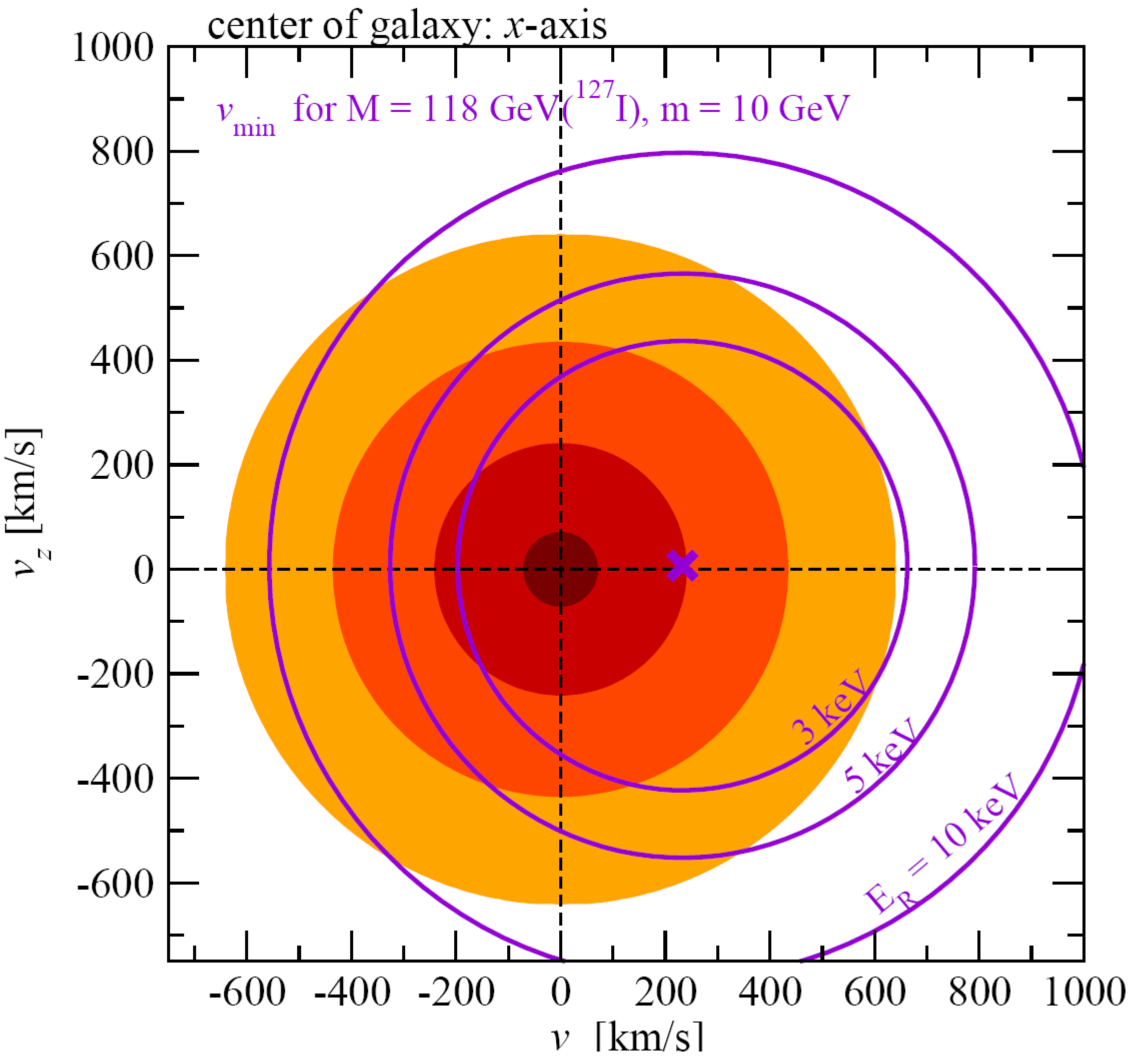
$$\chi + N \rightarrow \chi + N$$

cnts / kg-detector mass / keV recoil energy E_R:

$$\frac{dN}{dE_R}(t) = \frac{\rho_\chi}{m_\chi} \frac{\sigma(q)}{2\mu_\chi^2} \int_{v>v_{\min}} d^3v \, \frac{f_\oplus(\vec{v}, t)}{v}$$

ρ_χ	DM energy density, default: 0.3 GeV cm^{-3}
$q = \sqrt{2ME_R}$	momentum transfer
$\mu_\chi = m_\chi M/(m_\chi + M)$	reduced DM/nucleus mass

Solar Orbit signal



$$\int_{v>v_{\min}} d^3v\, \frac{f_{\oplus}(\vec{v},t)}{v}$$

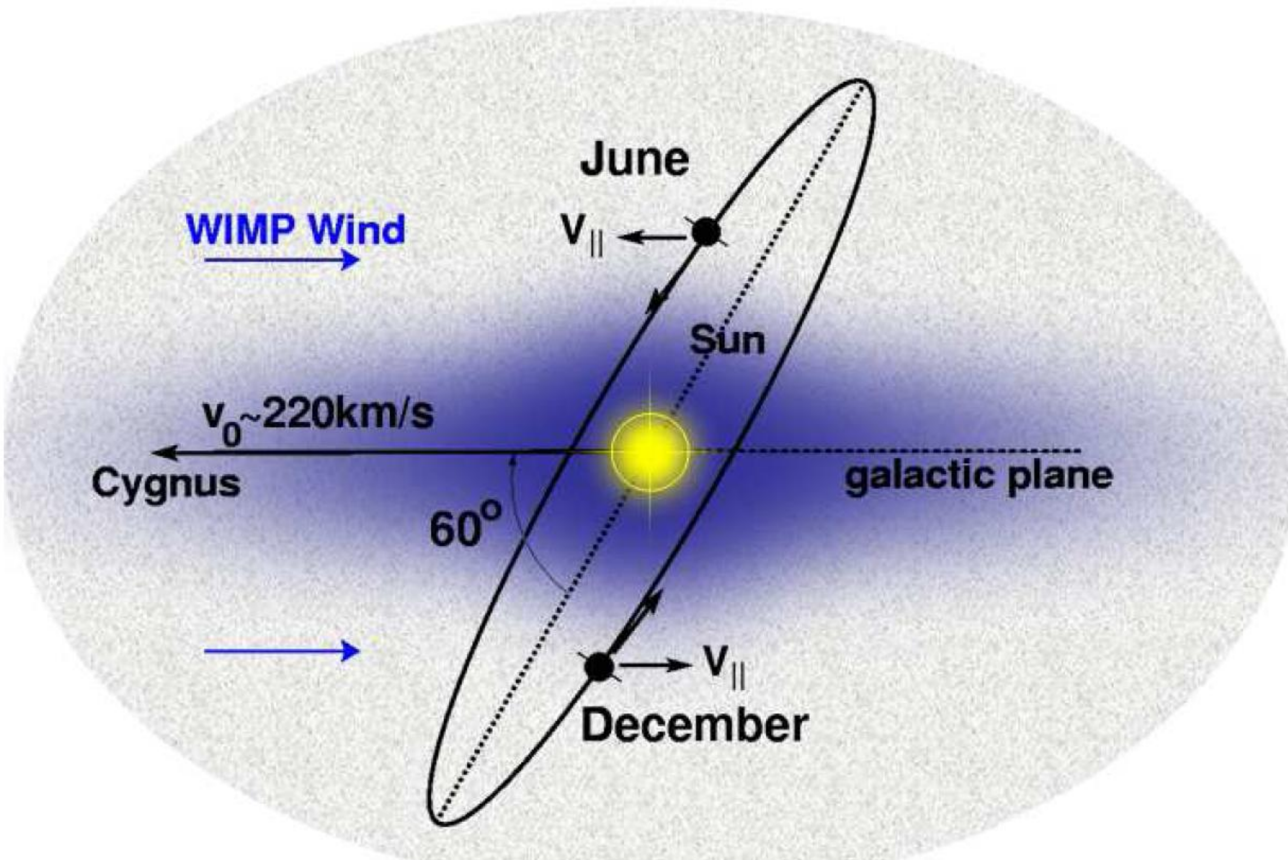
$$v_{\min} = \sqrt{\frac{M E_R}{2\mu_\chi^2}}$$

Earth goes round Sun, Sun goes round Galaxy

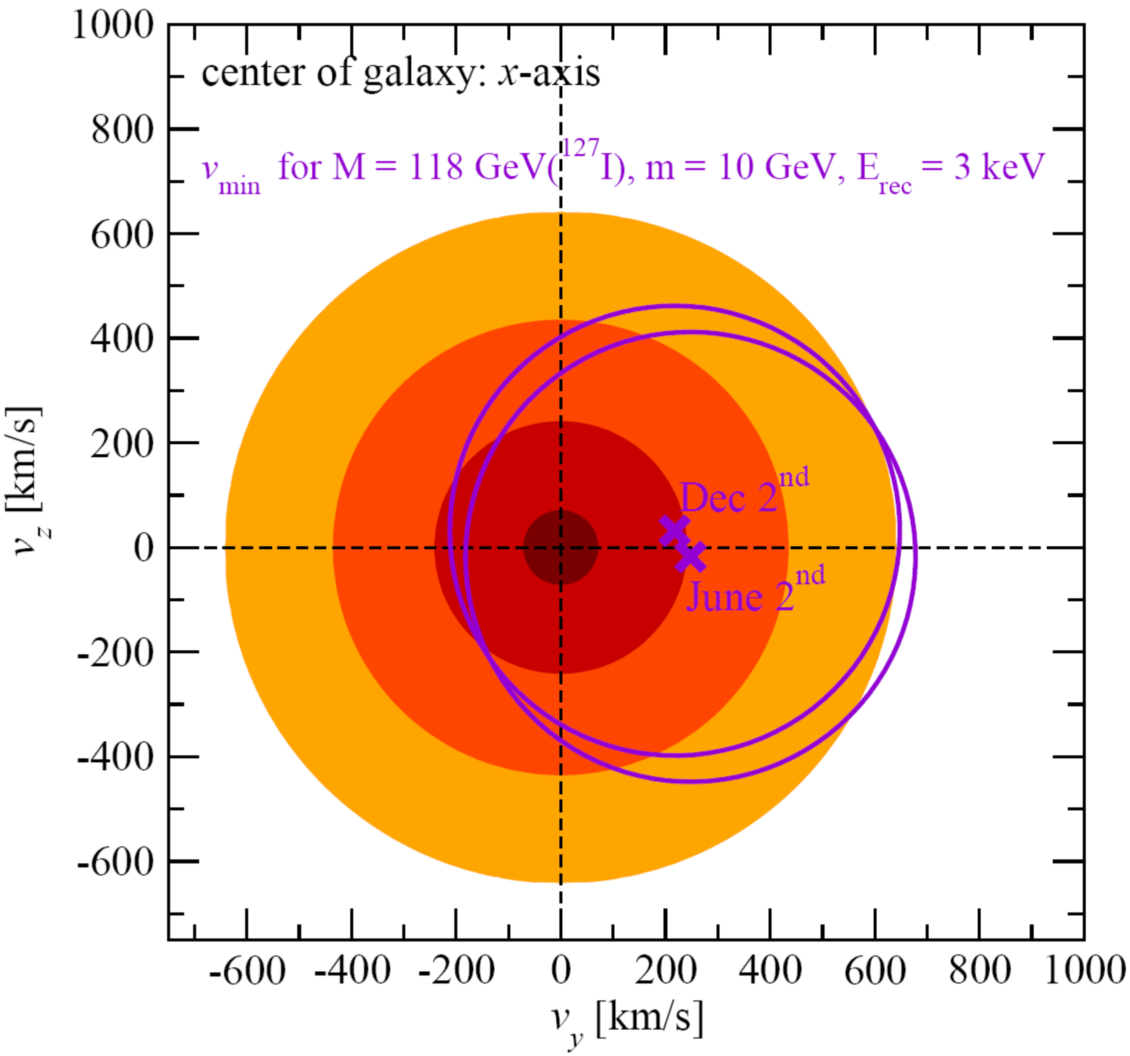
$$f_\oplus(\vec{v}, t) = f_{\rm gal}(\vec{v} + \vec{v}_\odot + \vec{v}_\oplus(t))$$

sun velocity: $\vec{v}_\odot = (0, 220, 0) + (10, 13, 7)$ km/s

earth velocity: $\vec{v}_\oplus(t)$ with $v_\oplus \approx 30$ km/s

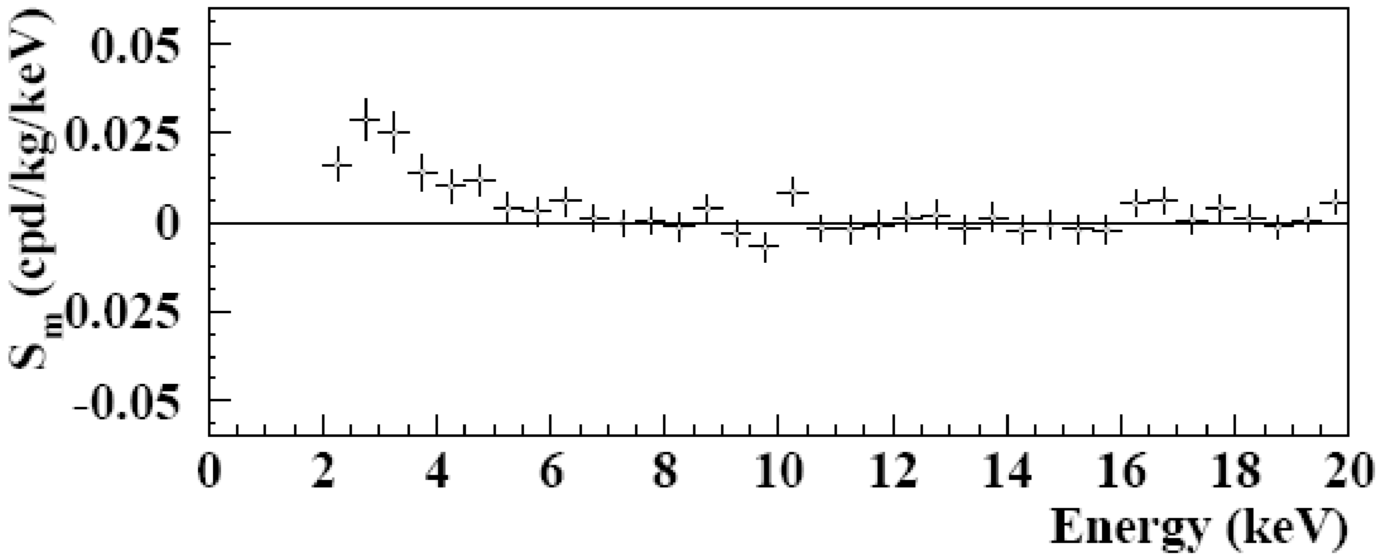


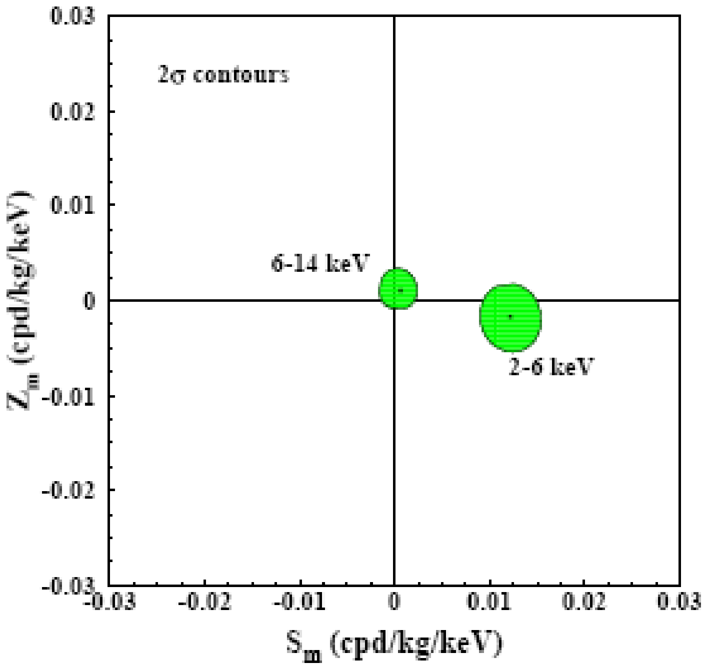
Annual modulation signal



$$\int_{v>v_{\min}} d^3v \, \frac{f_{\oplus}(\vec{v}, t)}{v}$$

DAMA/LIBRA results





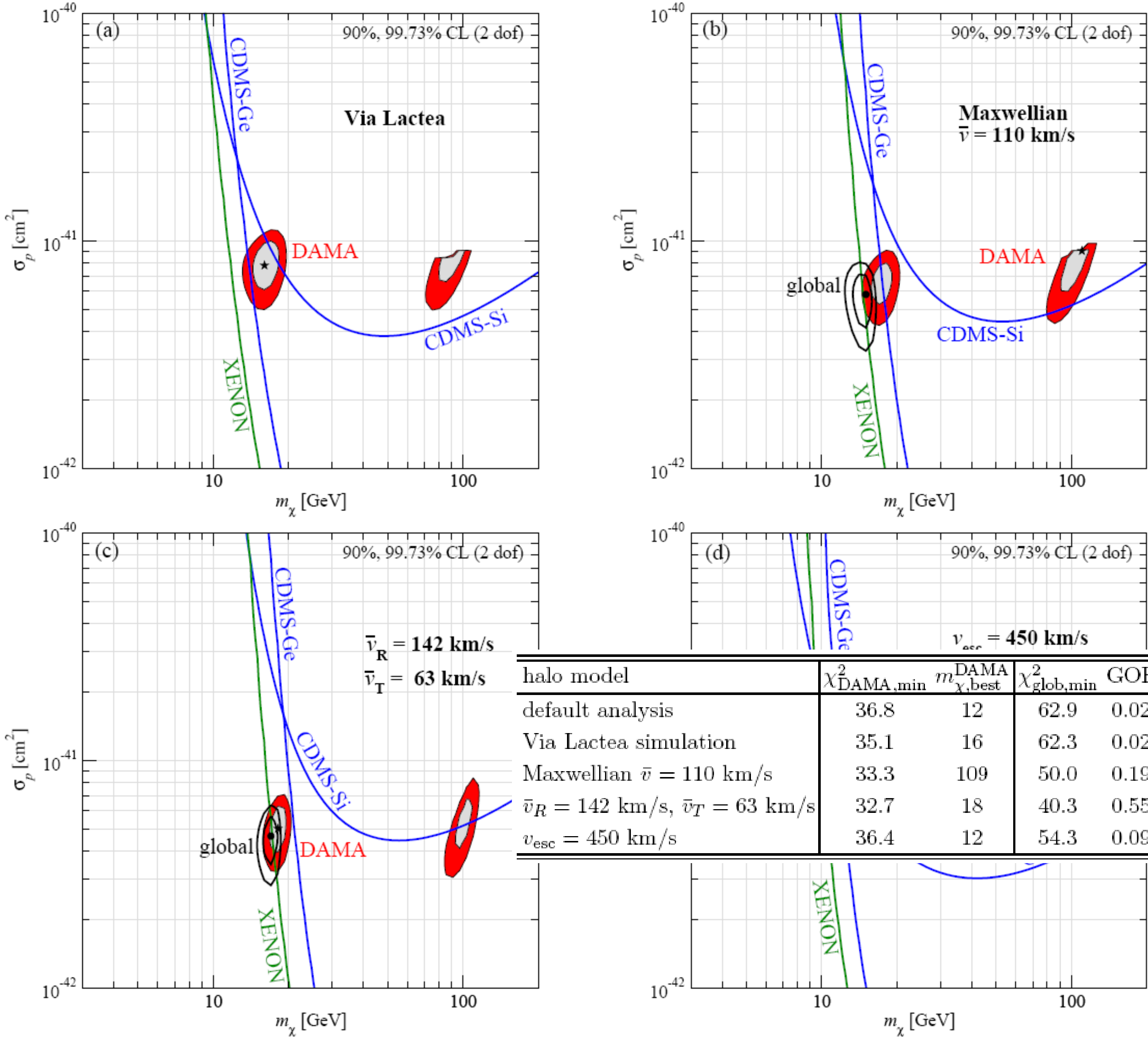
modulation signal at $2-6$ keV

above 6 keV no modulation

fitting:

$$S_0 + S_m \cos\omega(t - t_0) + Z_m \sin\omega(t - t_0)$$

with $t_0 = 152$, $T = 1$ yr

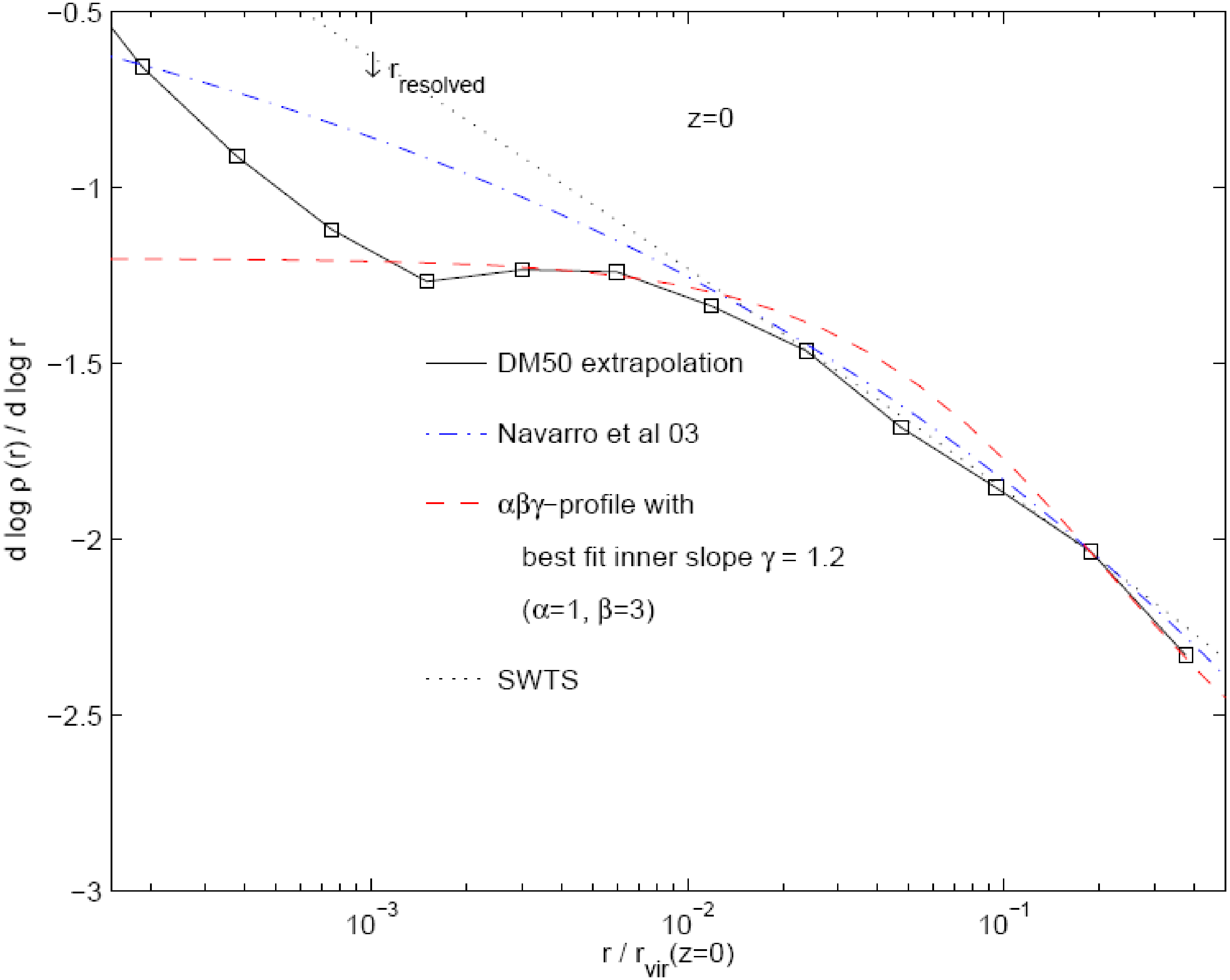


halo model	$\chi^2_{\rm DAMA,min}$	$m^{\rm DAMA}_{\chi,\rm best}$	$\chi^2_{\rm glob,min}$	GOF	$\chi^2_{\rm PG}$	PG	$m^{\rm glob}_{\chi,\rm best}$
default analysis	36.8	12	62.9	0.02	26.1	2×10^{-6}	8.6
Via Lactea simulation	35.1	16	62.3	0.02	27.2	1×10^{-6}	12
Maxwellian $\bar{v} = 110$ km/s	33.3	109	50.0	0.19	16.7	2×10^{-4}	15
$\bar{v}_R = 142$ km/s, $\bar{v}_T = 63$ km/s	32.7	18	40.3	0.55	7.6	0.02	17
$v_{\rm esc} = 450$ km/s	36.4	12	54.3	0.09	17.9	2×10^{-4}	9.7

Simulations show halos not isothermal

$$\rho(r) = \frac{\rho_0}{(r/r_s)^\gamma \left[1 + (r/r_s)^\alpha\right]^{\frac{\beta-\gamma}{\alpha}}}$$

Typical values obtained from simulations are $1 < \gamma < 1.5$, $\beta = 3$



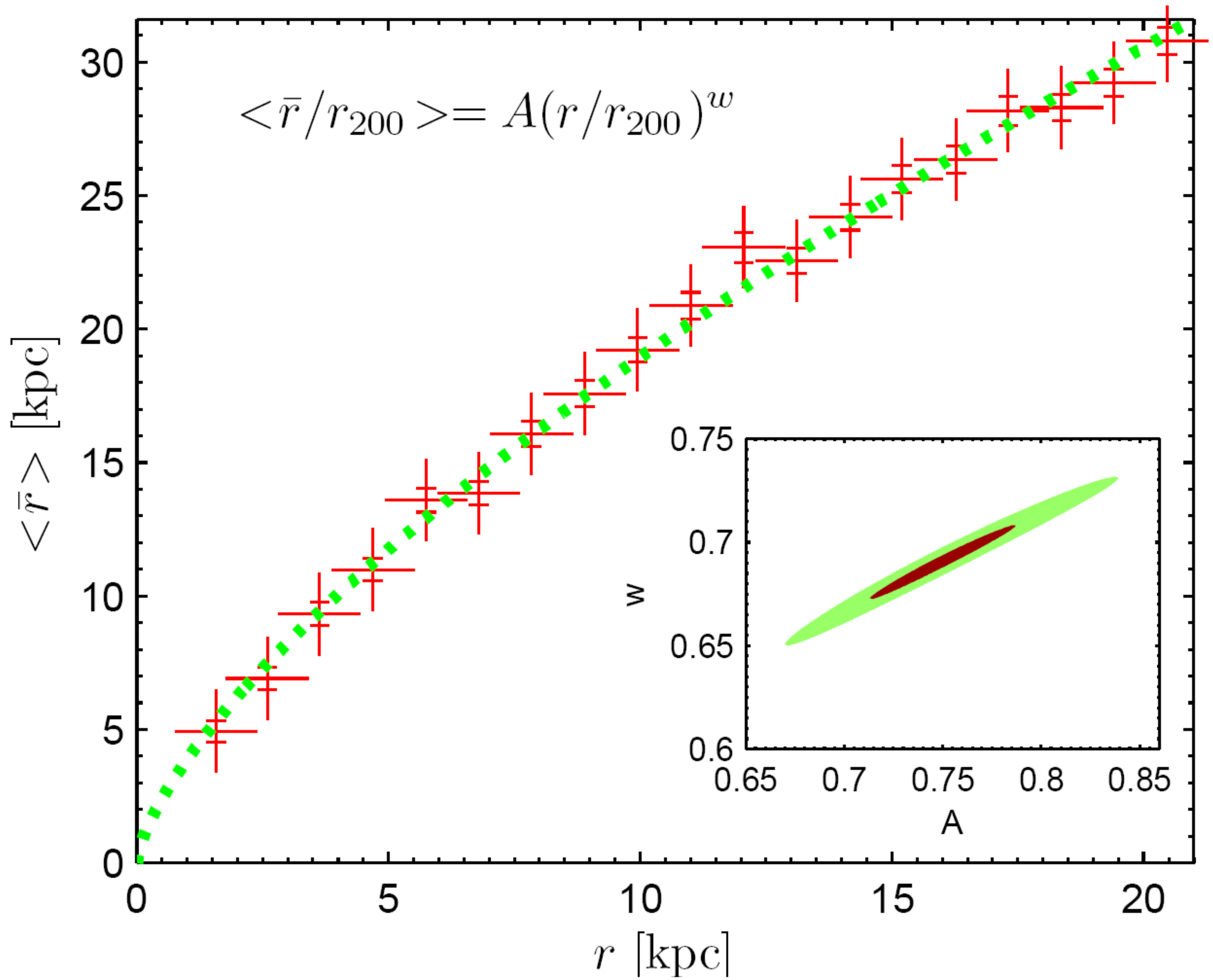
Diemand et al. (2005)

Tangential Velocity Dispersion

$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

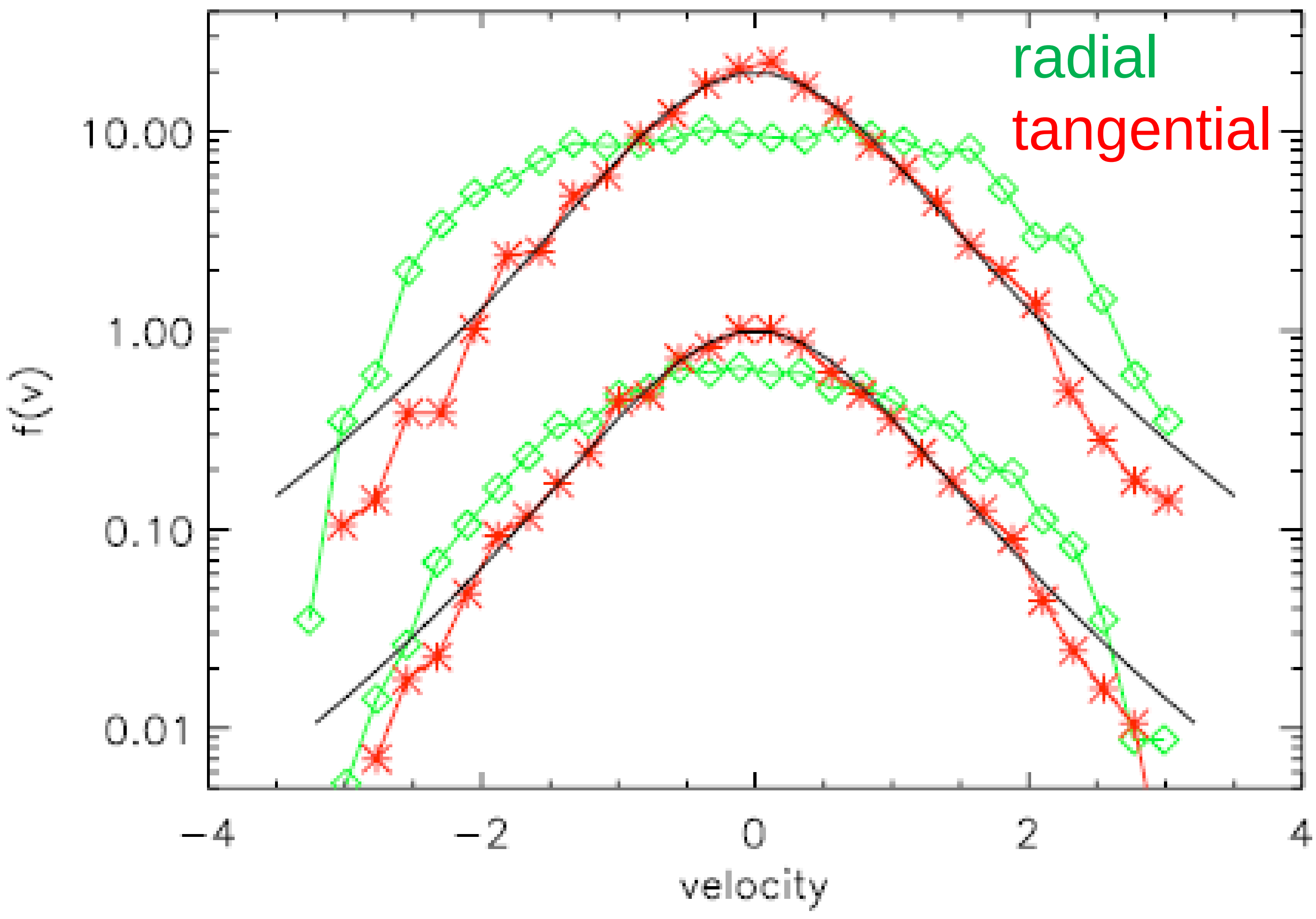
Radial Velocity Dispersion

Dark matter particle orbits far from circular



Gustafsson, MF and Sommer-Larsen arXiv:0608634

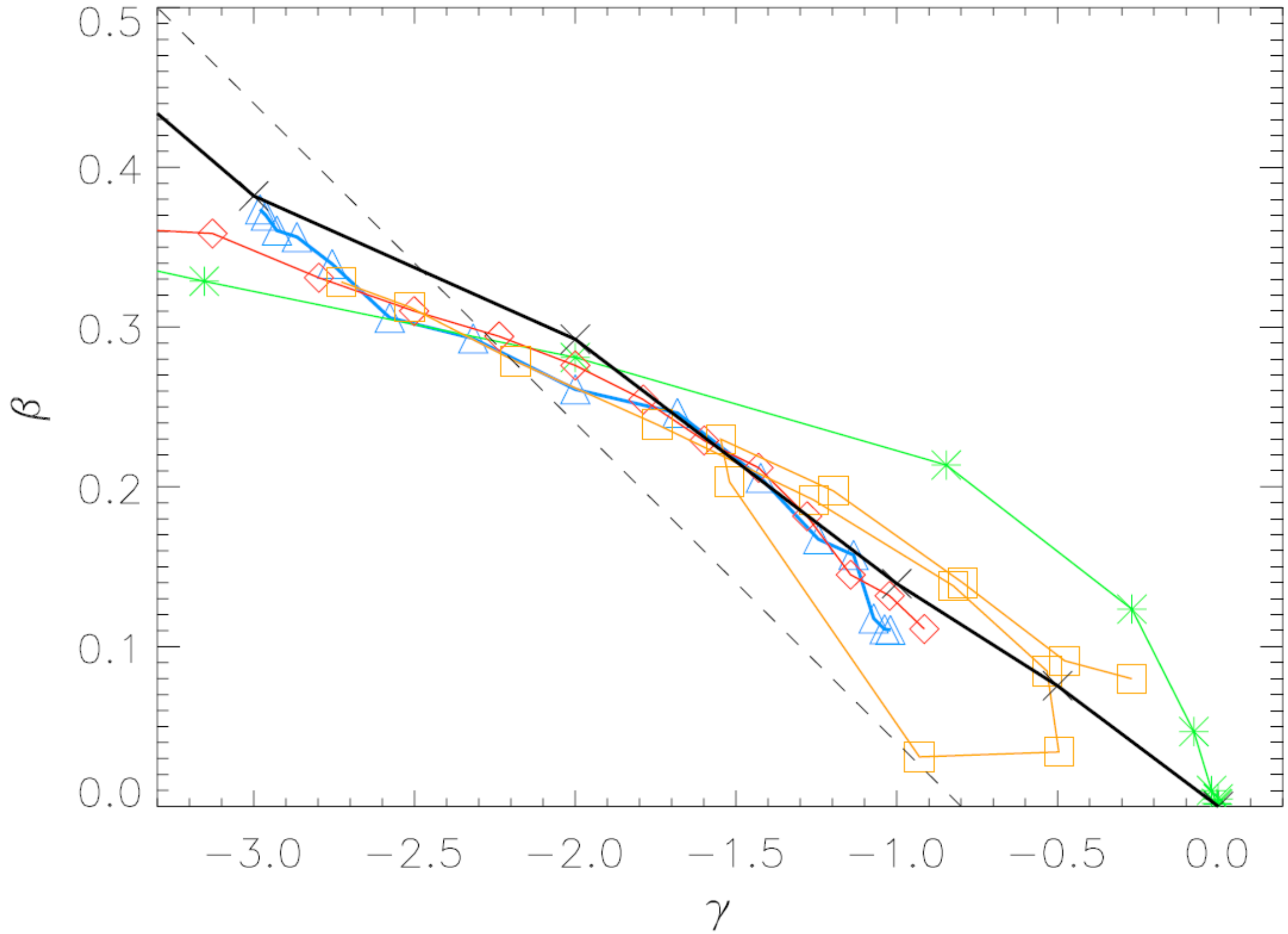
Dark Matter halos anisotropic



0812.1048, Sommer-Larsen astro-ph/0602595, astro-ph/0204366

Anisotropy vs density gradient

$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$



Hansen 0812.1048

$$\gamma \equiv \frac{d\ln\langle\rho\rangle}{d\ln\langle r\rangle}$$

Velocity Anisotropy in Galaxy Clusters

$$\sigma_r^2 \left(\frac{d \ln \rho_{\rm DM}}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2\beta \right) = -\frac{GM(r)}{r}$$

Jeans equation

$$\frac{k_B T_{\rm gas}}{\mu m_H} \left(\frac{d \ln n_e}{d \ln r} + \frac{d \ln T_{\rm gas}}{d \ln r} \right) = -\frac{GM(r)}{r}$$

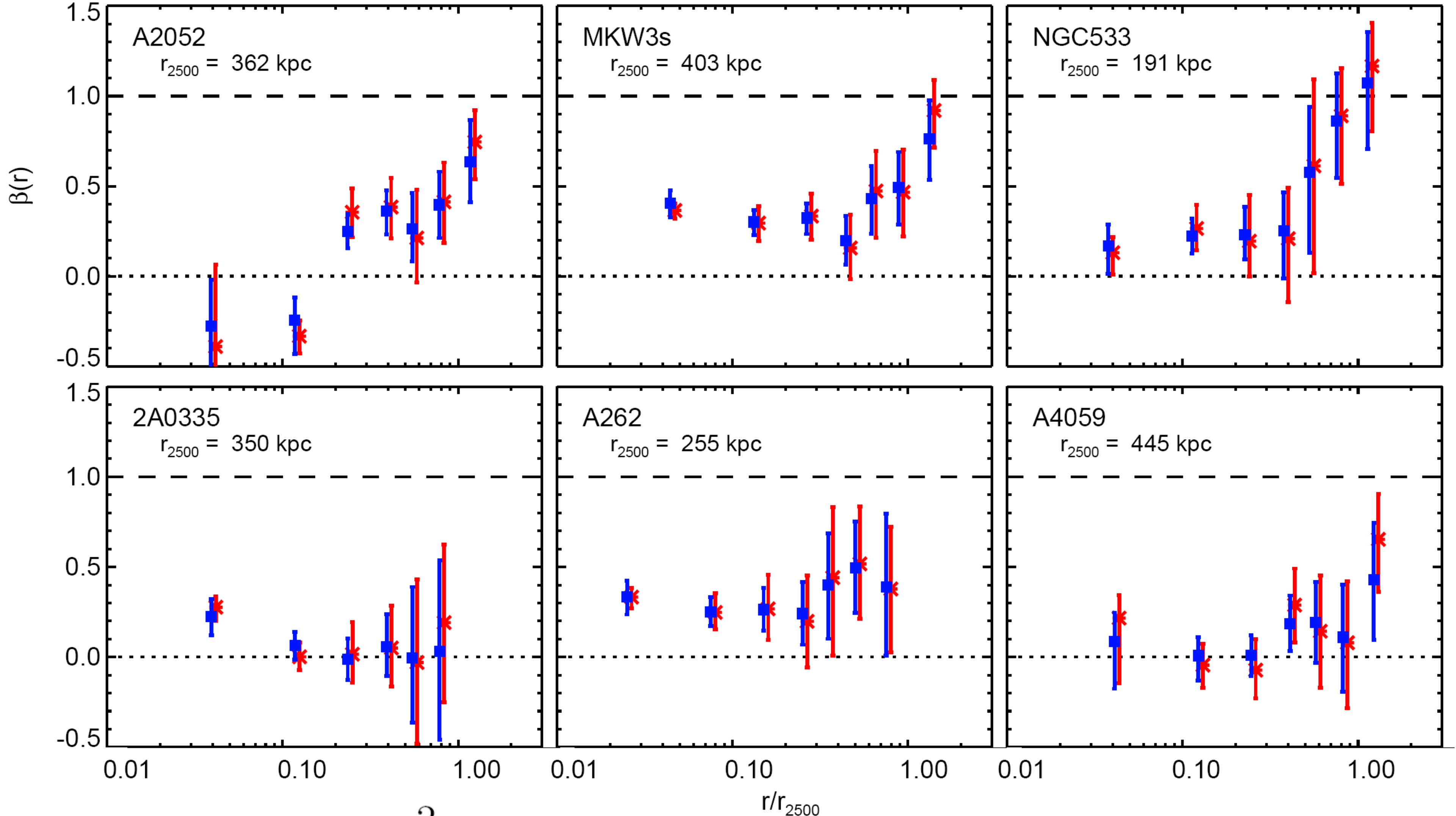
hydrostatic equilibrium

Can use plus assumptions to derive *$\beta(r)$.*

Authors have tested method using simulated clusters.

Host et al. arXiv:0808.2049

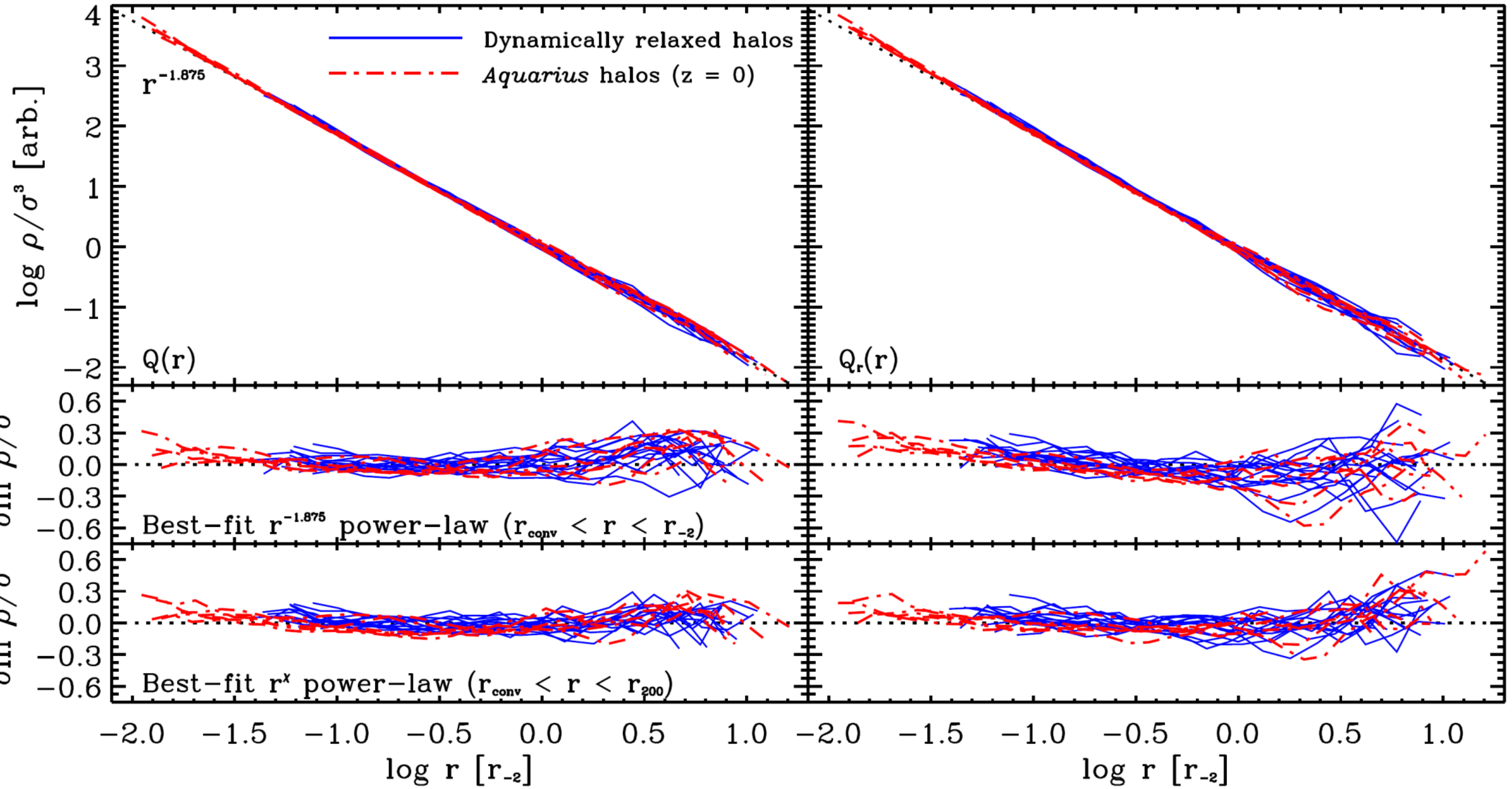
Velocity Anisotropy in Galaxy Clusters



$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

Host et al. arXiv:0808.2049

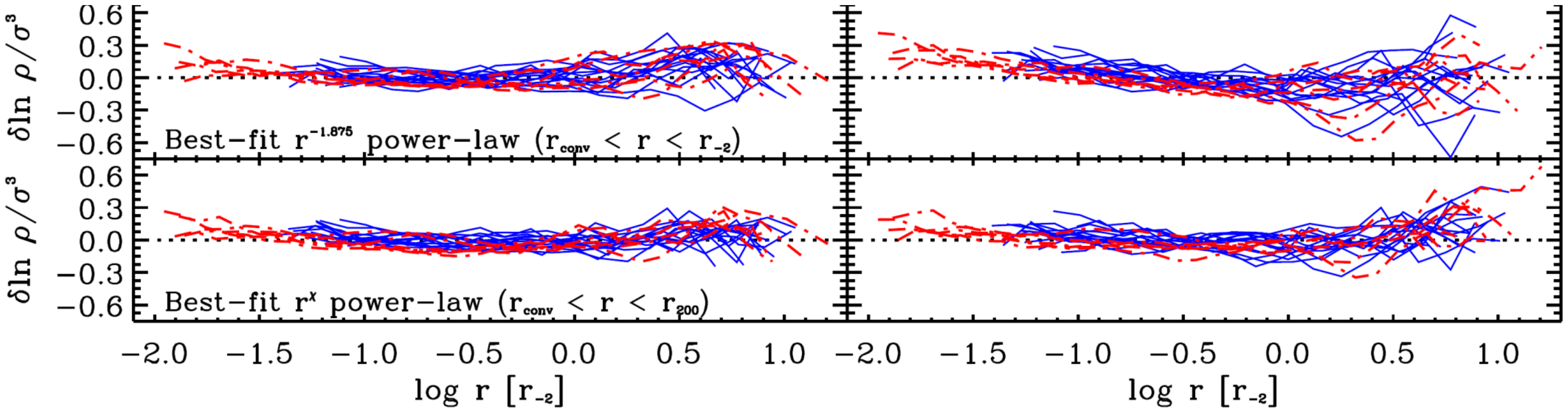
Pseudo Phase-space Density



Ludlow et al.arXiv:1001.2310

$$\frac{\rho}{\sigma^3} \propto r^{-1.875}$$

Trying to model this



$$\frac{\rho}{\sigma^3}\frac{\sigma_c^3}{\rho_c} = \exp\left[-\frac{\chi}{\omega}\left(\chi^{\omega}-1\right)\right]$$

$$\frac{\rho}{\sigma^3}\frac{\sigma_c^3}{\rho_c} = \exp\left[-\frac{\chi_R}{\omega_R}\left(\chi_R^{\omega_R}-1\right)\right]$$

Try to reconstruct Einasto profile using Jean's equation and our model for Pseudo Phase Space Density

$$\frac{1}{\rho}\frac{\partial}{\partial r}\left(\rho\sigma_R^2\right)+2\frac{\beta\sigma_R^2}{r}=-\frac{GM}{r^2}$$

$$\gamma=\frac{d\ln\rho}{d\ln r}$$

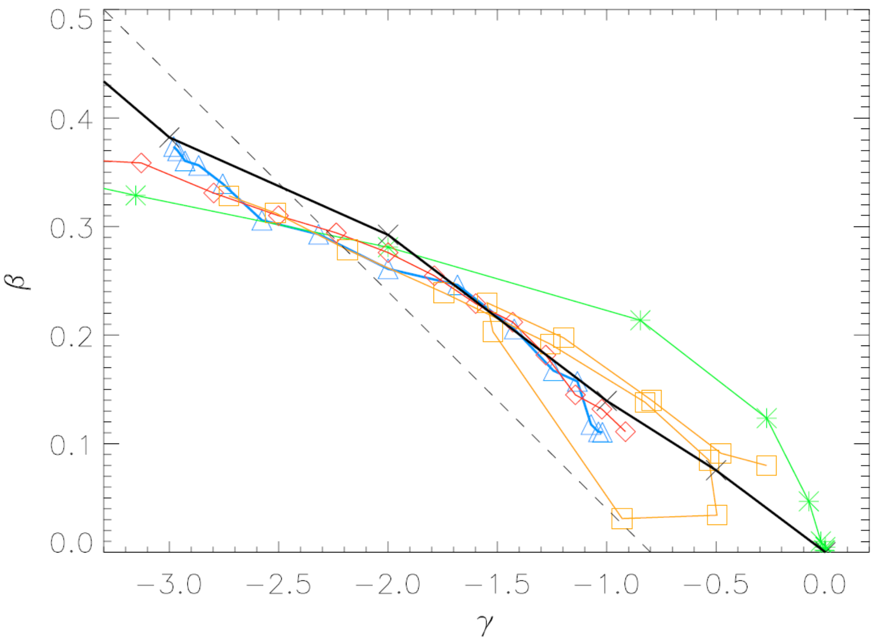
We integrate this

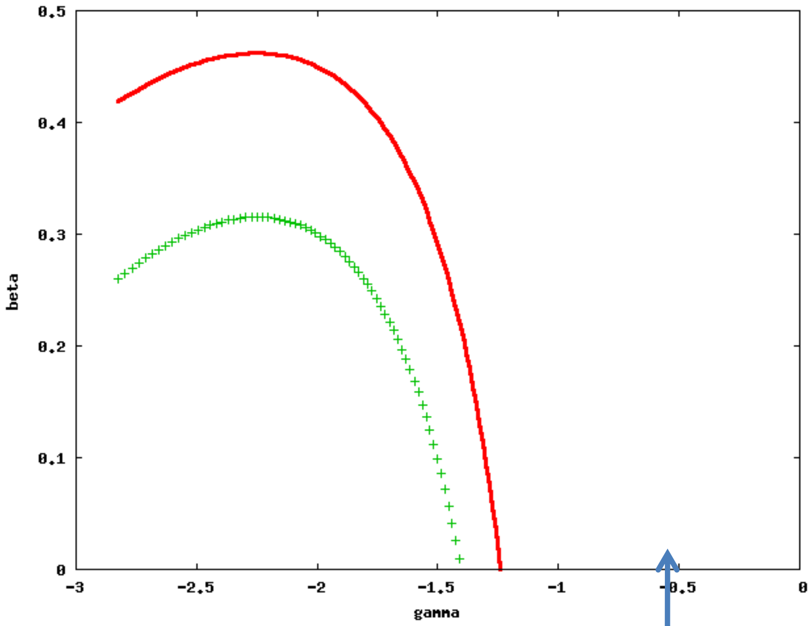
$$\gamma=-\frac{2}{5}\chi_R x^{\omega_R}-\frac{6}{5}\beta\ -\frac{K}{\bar{\rho}^{2/3}x}\exp\left\{-\frac{2}{3}\frac{\chi_R}{\omega_R}\left(x_R^{\omega}-1\right)\right\}\int_0^x\bar{\rho}x^2dx$$

And (try to) fit to get Einasto profile!!

See also Dehnen and McLaughlin for related approach.

Does it explain observed γ–β relationship?

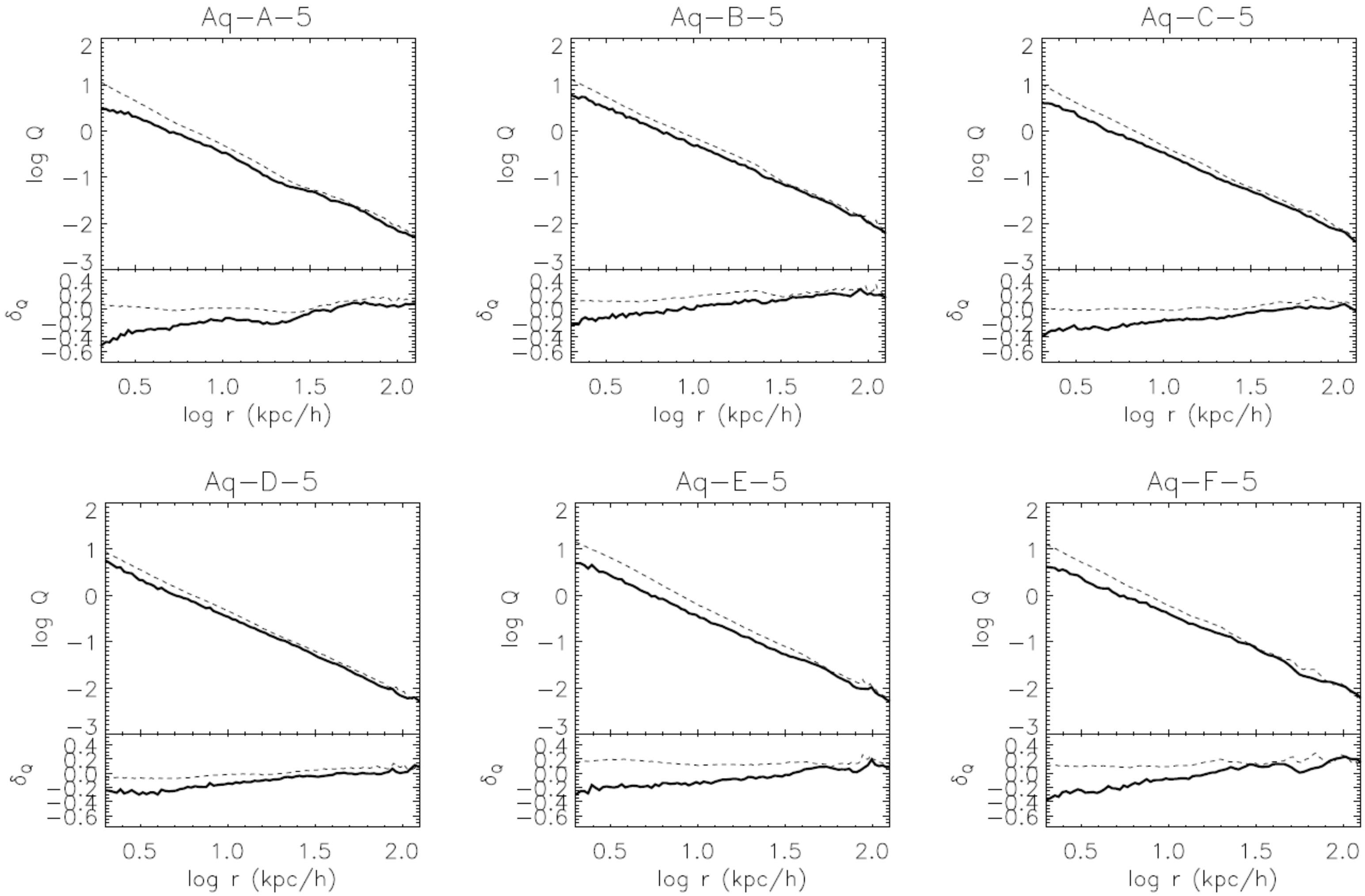




well kind of.......

Expect things to go wrong in the middle anyway

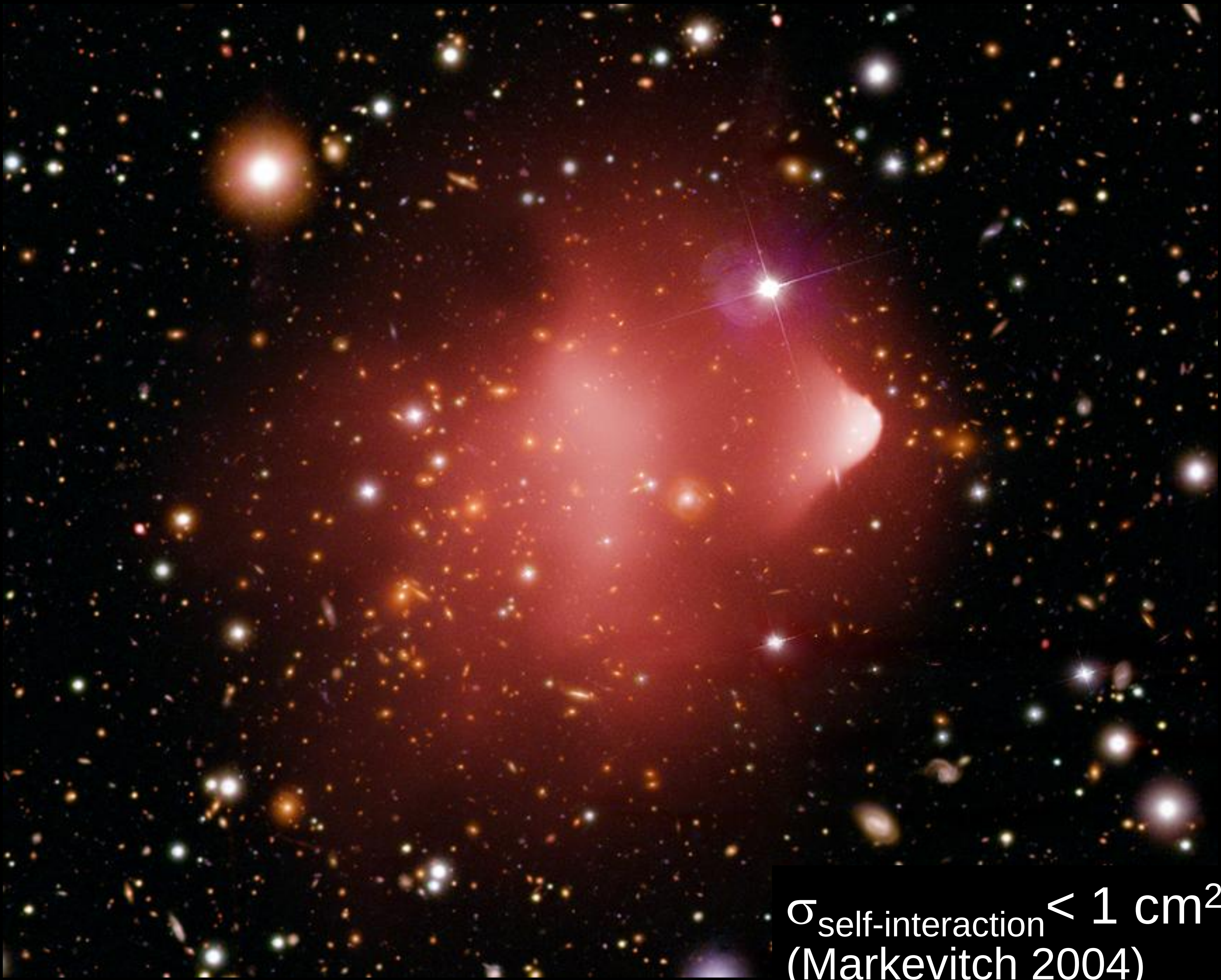
Phase space density with baryons



Tissera et al. arXiv:0911.2316

It gets worse

The distribution is non-Gaussian



$\sigma_{self\text{-}interaction} < 1\ cm^2/g$
(Markevitch 2004)

Modelling train delays with q-exponential functions

Keith Briggs

Complexity Research Group, BT, Adastral Park, IP5 3RE, UK

Christian Beck

School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK

Abstract

We demonstrate that the distribution of train delays on the British railway network is accurately described by q-exponential functions. We explain this by constructing an underlying superstatistical model.

Non Extensive Statistics

At any given time, probability of a particular delay time, t, given by

$$P(t|\beta) = \beta e^{-\beta t}$$

Over course of a year, β varies quite a lot due to seasonal factors.
Therefore, over the long term, one needs to include fluctuations in β

$$p(t) = \int_0^\infty f(\beta) p(t|\beta) d\beta = \int_0^\infty f(\beta) \beta e^{-\beta t}$$

Non Extensive Statistics

if β is sum over Gaussian random variables ...

$$\beta = \sum_{i=1}^{n} X_i^2$$

leads to a χ^2 distribution ...

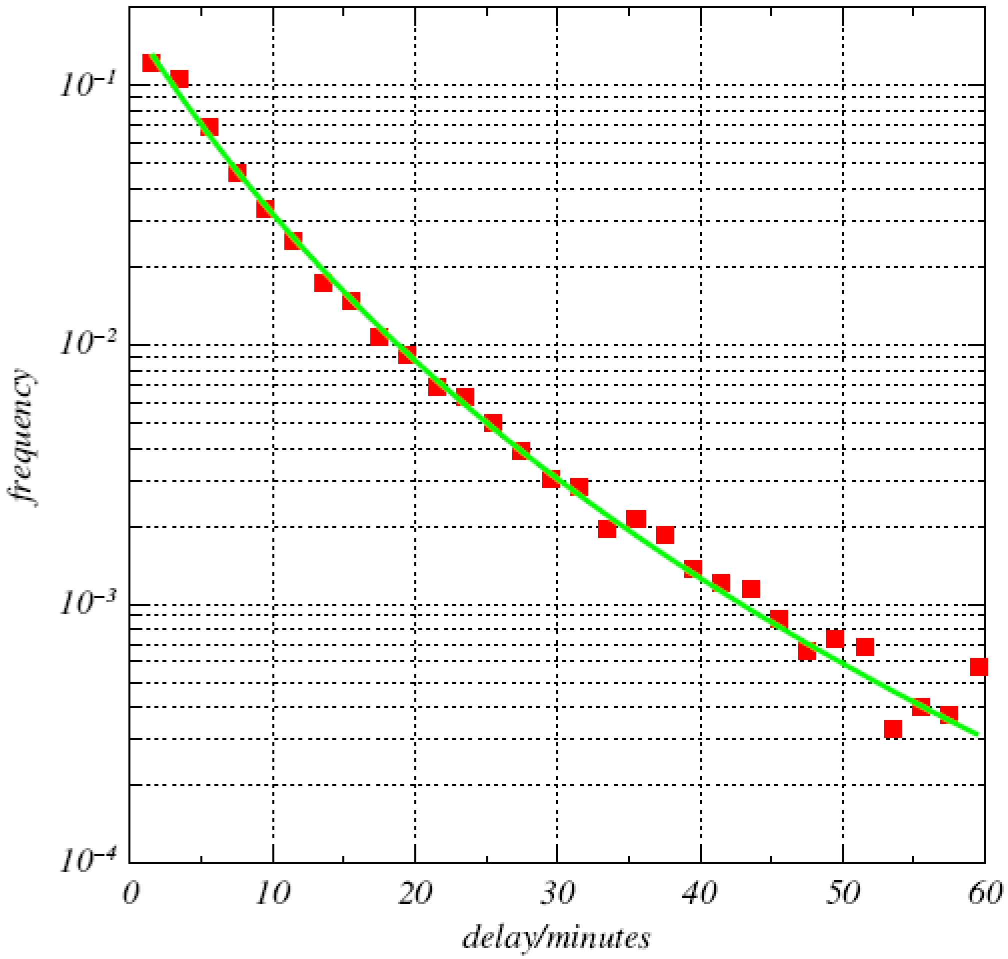
$$f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \left(\frac{n}{2\beta_0}\right)^{\frac{n}{2}} \beta^{\frac{n}{2}-1} \exp\left(-\frac{n\beta}{2\beta_0}\right)$$

which yields

$$p(t) \sim (1 + b(q-1)t)^{\frac{1}{1-q}}$$

$$\text{where } q = 1 + 2/(n+2) \text{ and } b = 2\beta_0/(2-q)$$

Applied to train times



Reading to London Paddington:

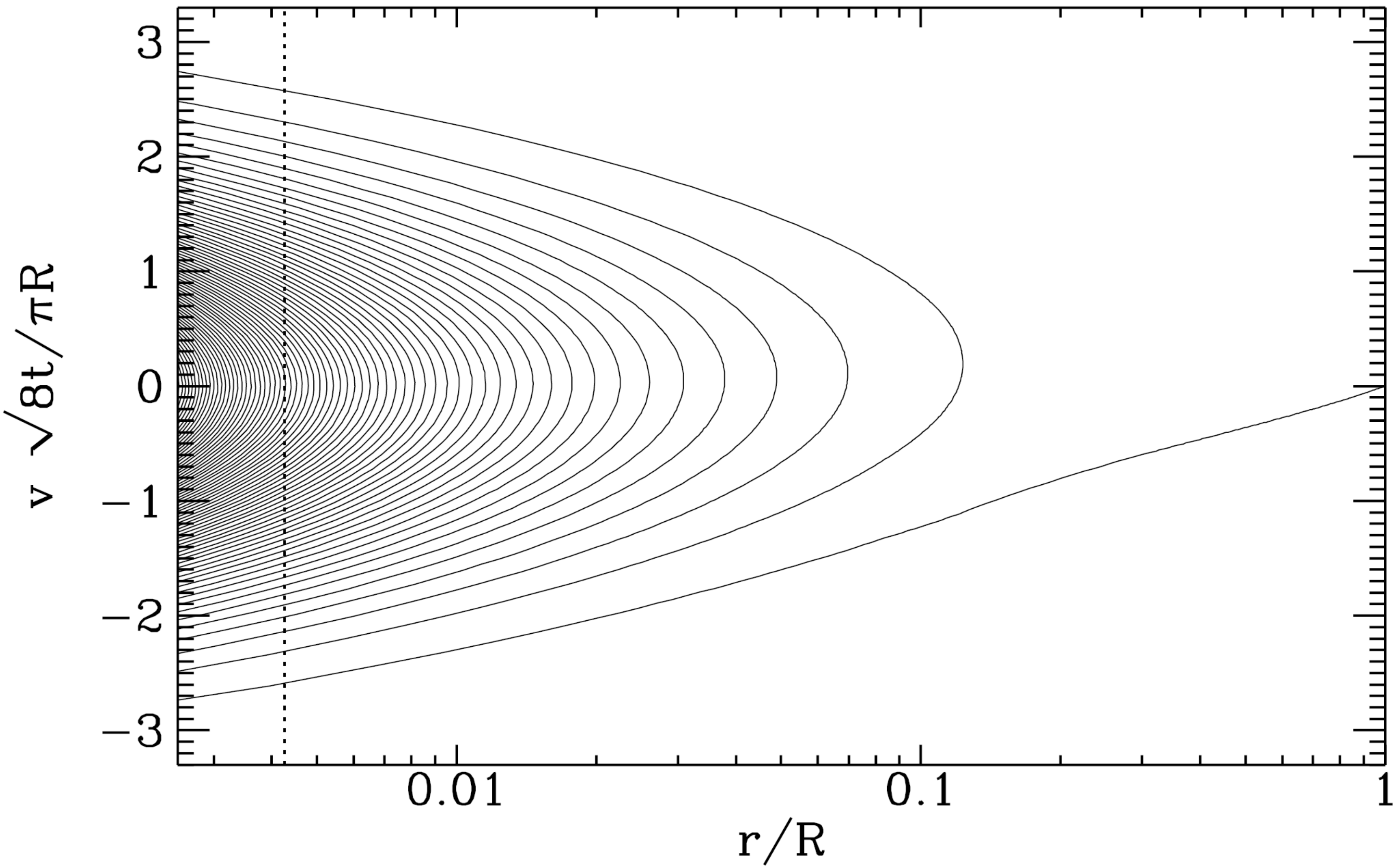
$$q = 1.183 \pm 0.0063, \; b = 0.202 \pm 2.7 \times 10^{-6}$$

station	q	b
Bath Spa	1.195	0.209
Birmingham	1.257	0.271
Cambridge	1.270	0.396
Canterbury East	1.298	0.400
Canterbury West	1.267	0.402
City Thameslink	1.124	0.277
Colchester	1.222	0.272
Coventry	1.291	0.330
Doncaster	1.289	0.332
Edinburgh	1.228	0.401
Ely	1.316	0.393
Ipswich	1.291	0.333
Leeds	1.247	0.273
Leicester	1.231	0.337
Manchester Piccadilly	1.231	0.332
Newcastle	1.378	0.330
Nottingham	1.166	0.209
Oxford	1.046	0.141
Peterborough	1.232	0.201
Reading	1.251	0.268
Sheffield	1.316	0.335
Swindon	1.226	0.253
York	1.311	0.259

Reasons why Non Extensive Statistics may be relevant for dark matter

1. Long range forces (gravity)
2. Lack of thermalisation (Non-self interacting)
3. Multiple populations of dark matter

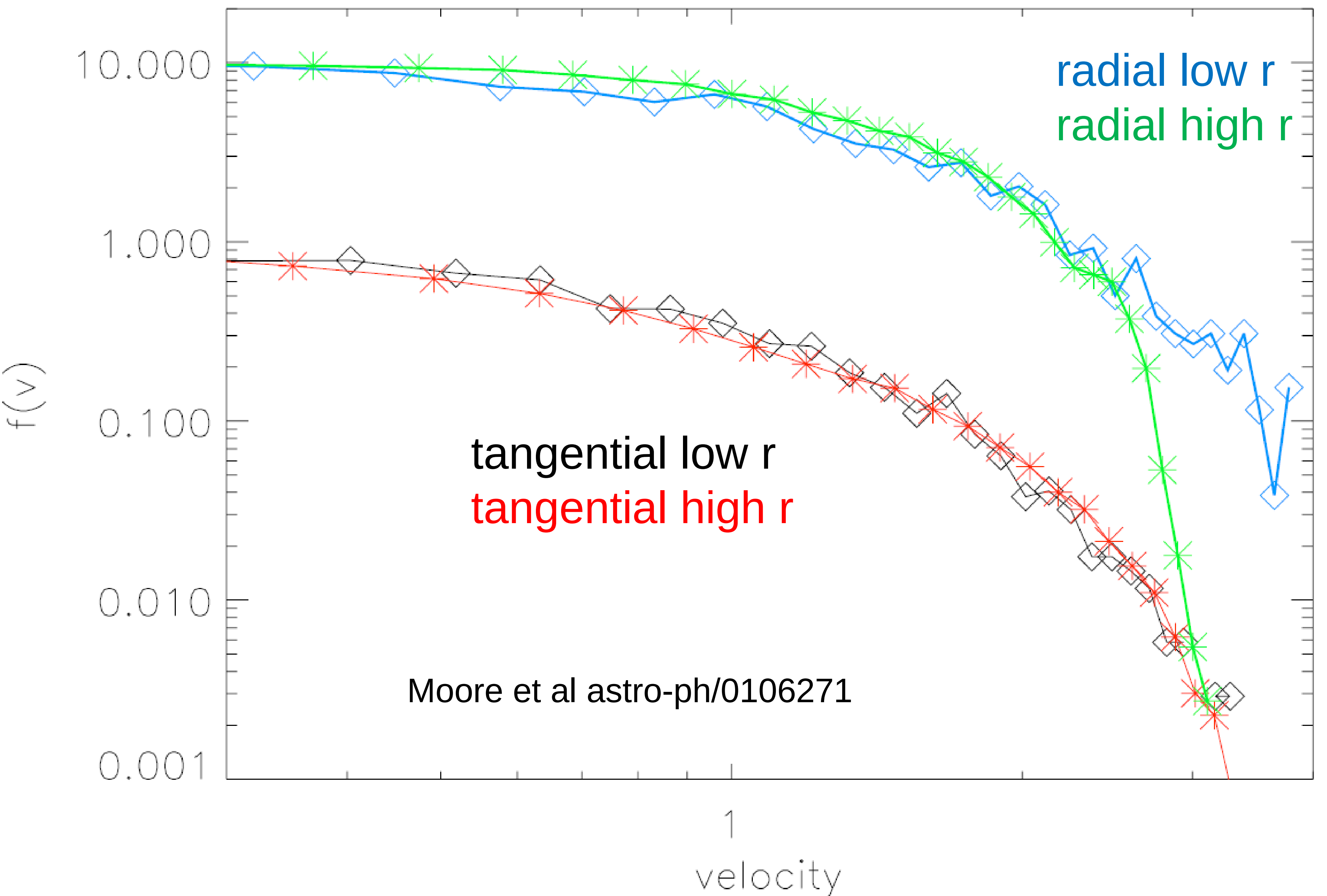
Intuition from spherical infall?



Sikivie, Tkachev and Wang astro-ph/9609022

Tangential tails vs. Radial Tails

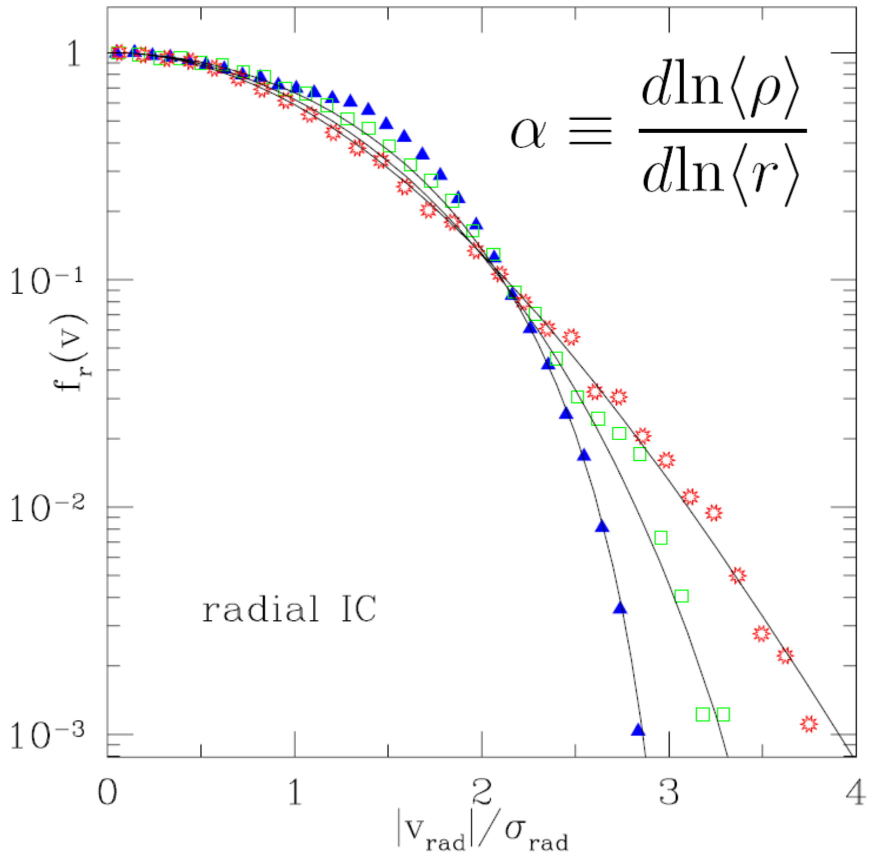
Earlier simulations also claim tangential distribution changes less

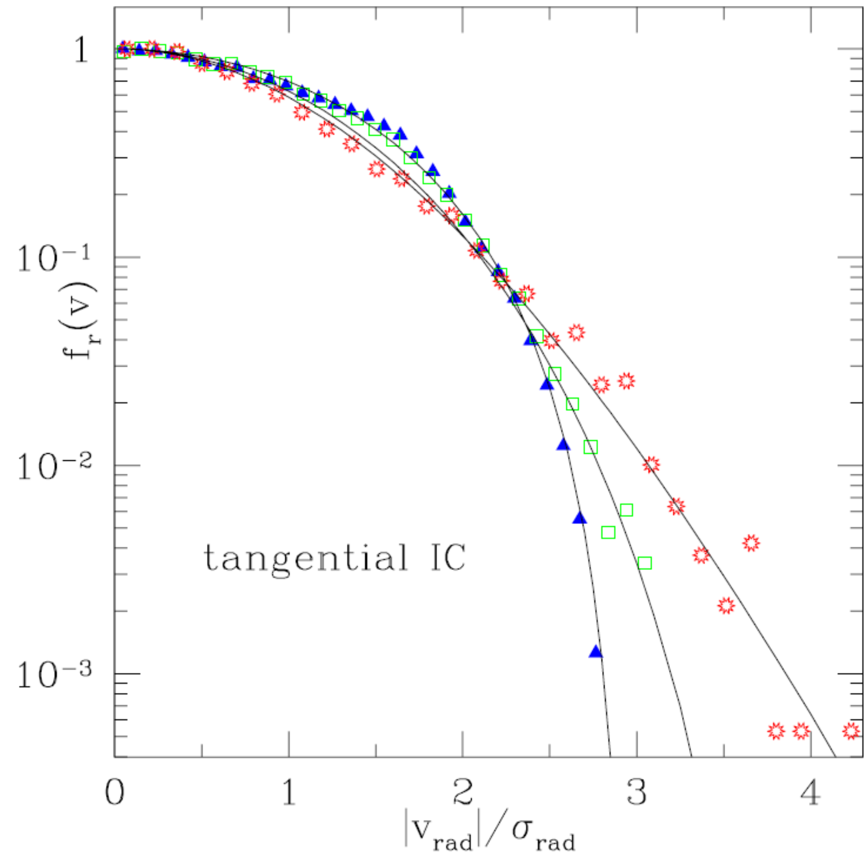


Universal Velocity Distribution?

Hansen, Moore, Zemp
and Stadel astro-ph/0505420

$$f_r(v) = \left(1 - (1-q)\left(\frac{v}{\kappa_1\,\sigma_{\rm rad}}\right)^2\right)^{\frac{q}{1-q}}$$

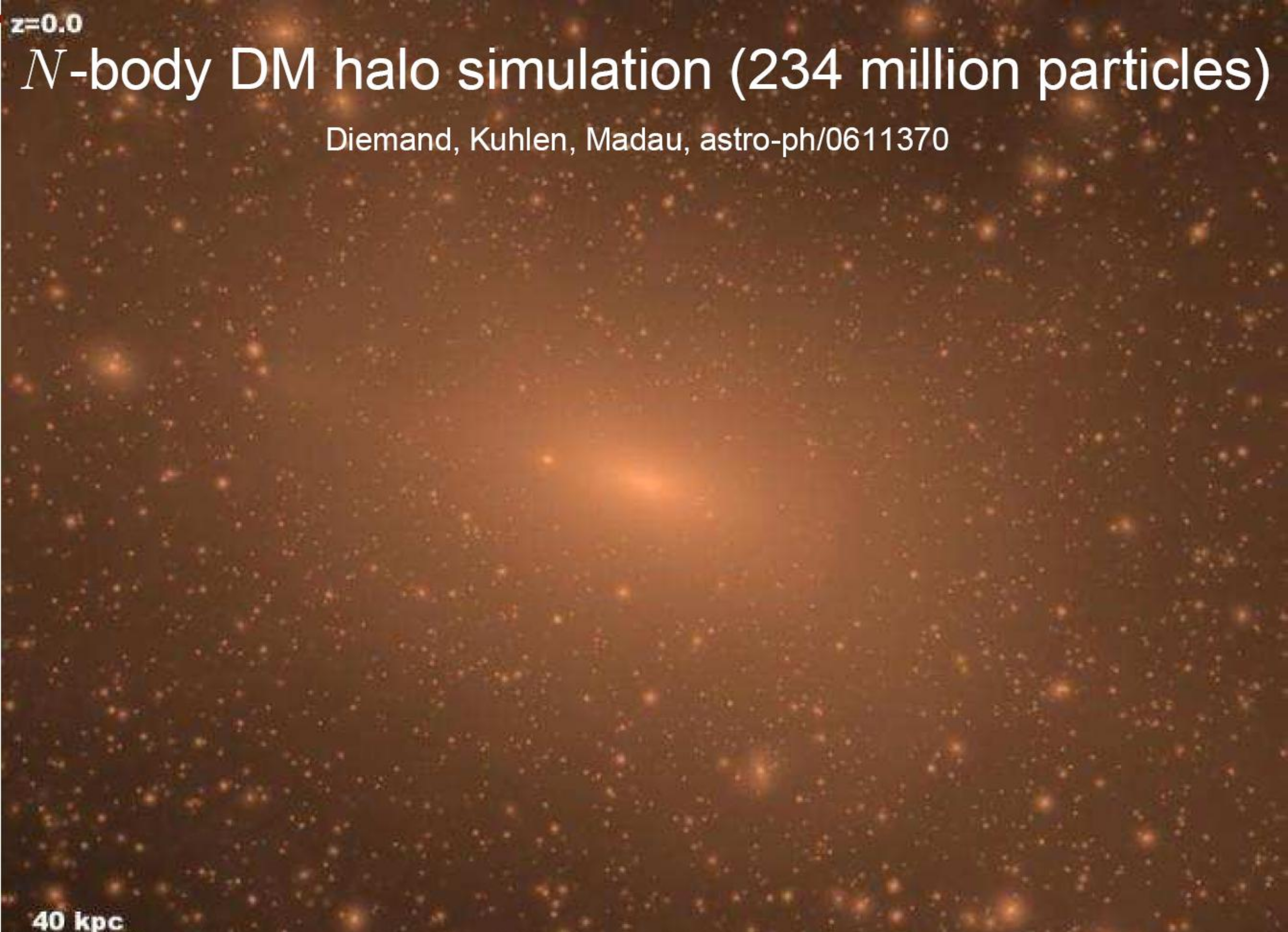




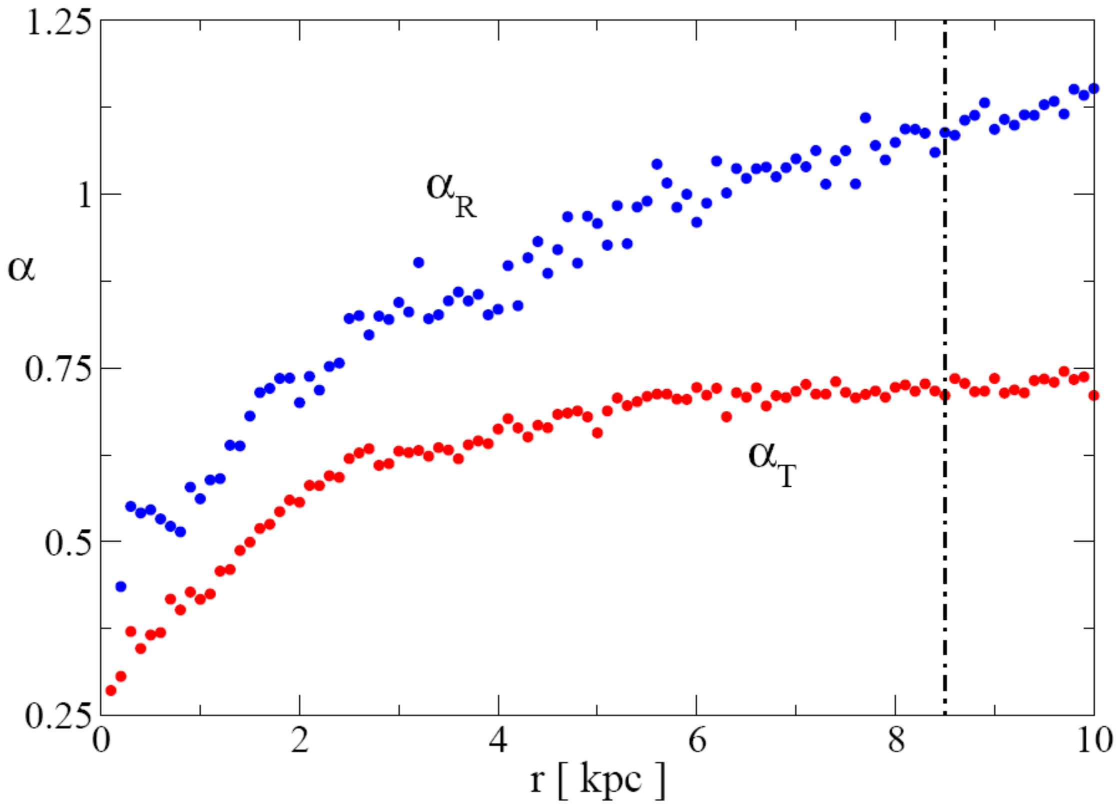
α = -1.1	q=0.79	α = -1.1	q=0.75
α =-2	q=0.91	α =-2	q=0.89
α =-2.6	q=1.05	α =-2.6	q=1.045

N-body DM halo simulation (234 million particles)

Diemand, Kuhlen, Madau, astro-ph/0611370



Via Lactea non-Gaussianity and anisotropy

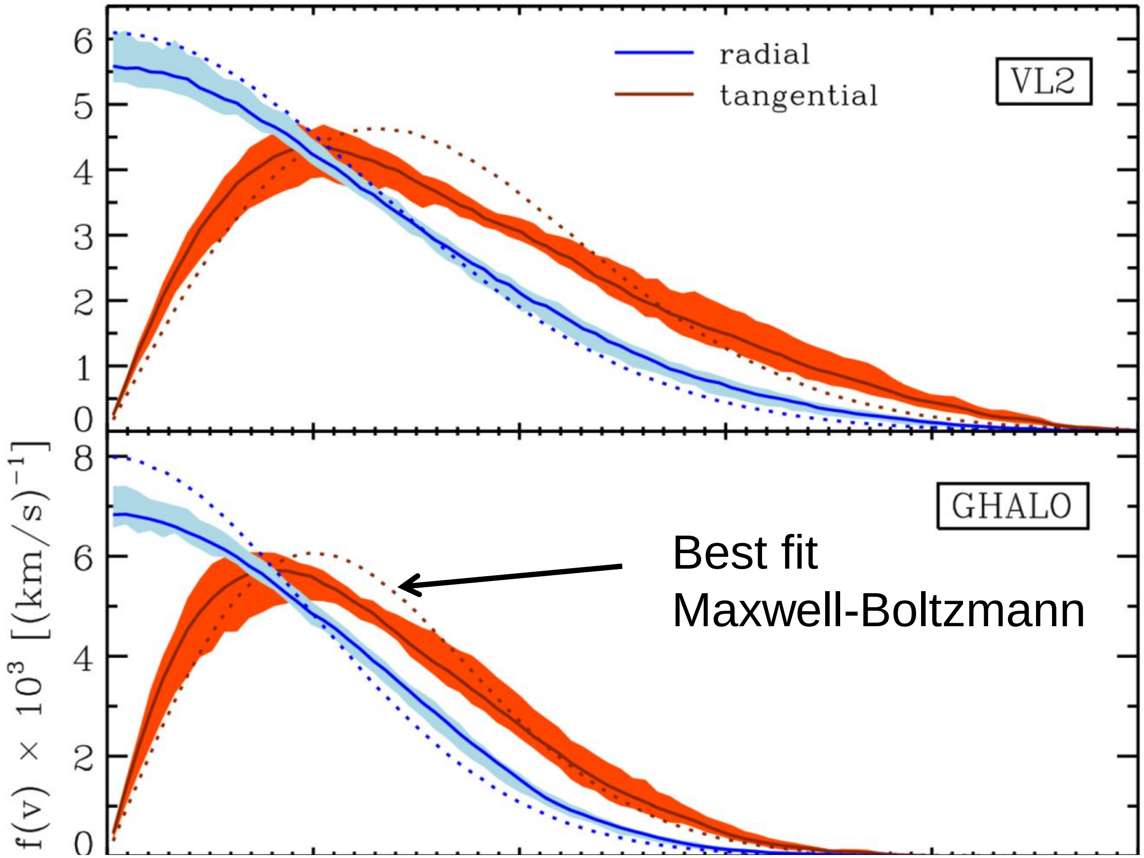


$$\frac{1}{N_R}\exp\left[-\left(\frac{\tilde{v}_R^2}{f_R^2}\right)^{\alpha_R}\right] \qquad \frac{2\pi v_T}{N_T}\exp\left[-\left(\frac{\tilde{v}_T^2}{f_T^2}\right)^{\alpha_T}\right]$$

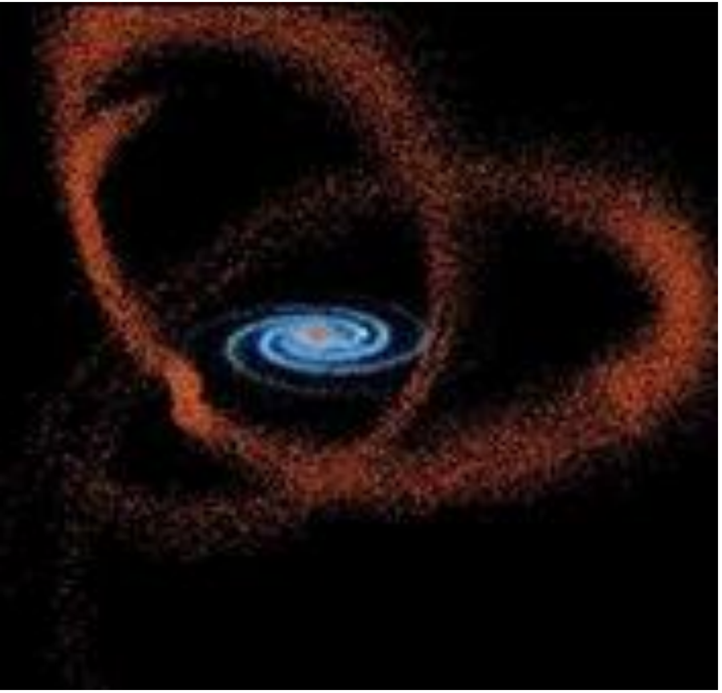
MF and Schwetz : arXiv 0808.0704 :

Velocity distributions around 8.5 kpc.

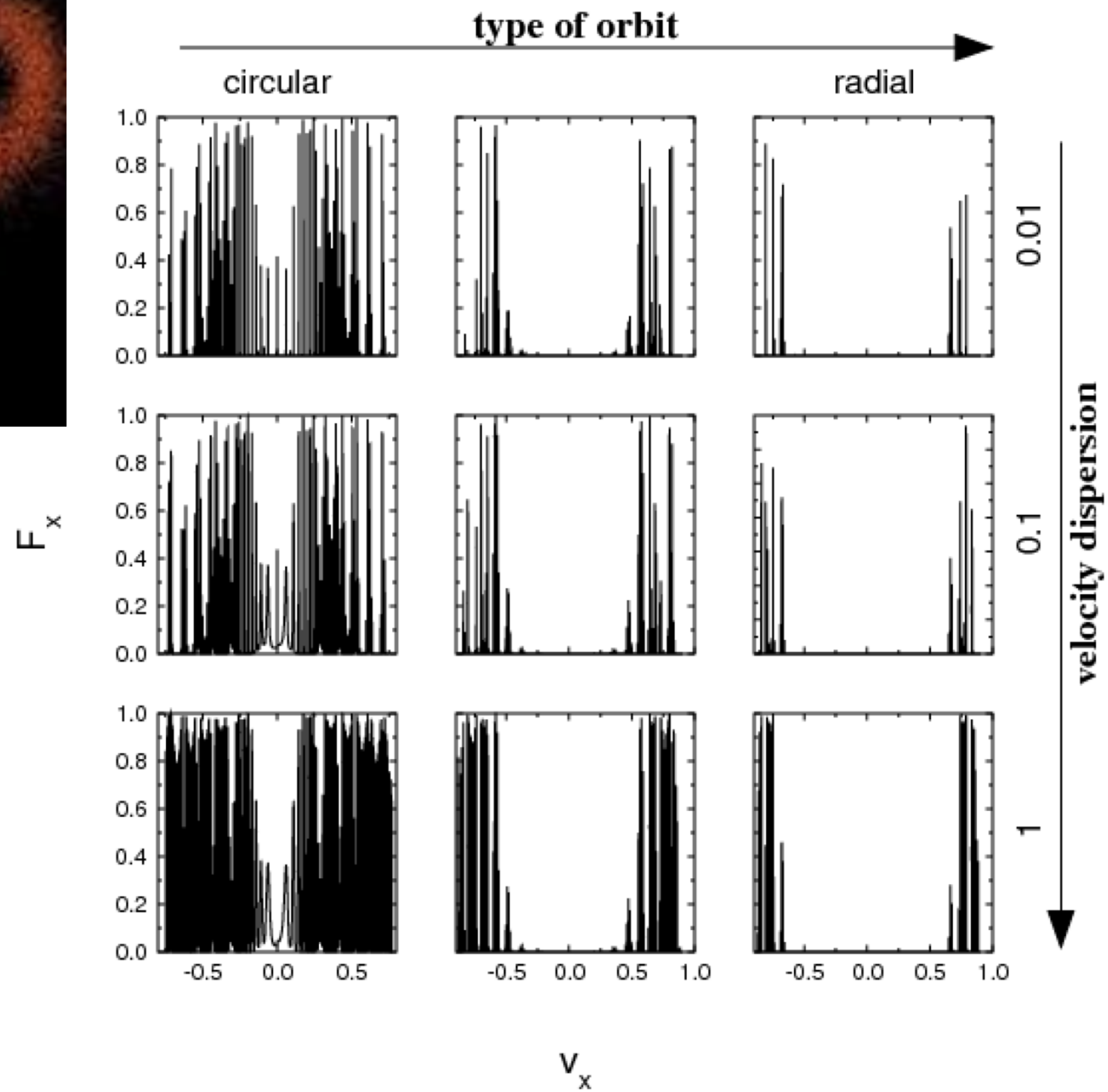
Kuhlen et al. arXiv: 0912.2358



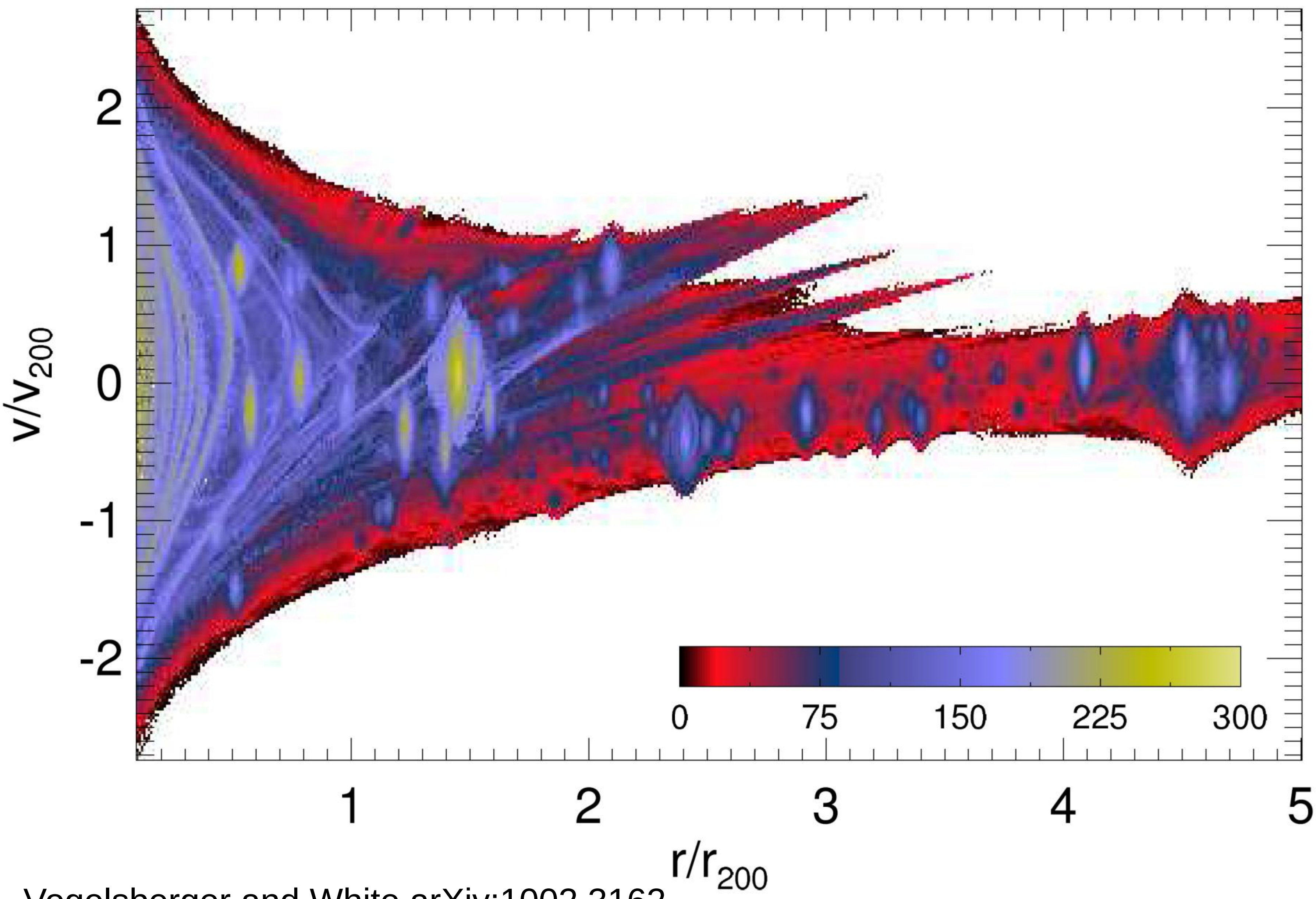
		radial				tangential			
		shell	median	16^{th}	84^{th}	shell	median	16^{th}	84^{th}
VL2	$\bar{v}_{r,t}$ [km/s]	202.4	199.9	185.5	212.7	128.9	135.1	124.2	148.9
	$\alpha_{r,t}$	0.934	0.941	0.877	0.985	0.642	0.657	0.638	0.674
GHALO	$\bar{v}_{r,t}$ [km/s]	167.9	163.6	156.4	173.0	103.1	114.3	93.21	137.0
	$\alpha_{r,t}$	1.12	1.11	1.02	1.20	0.685	0.719	0.666	0.819
$GHALO_s$	$\bar{v}_{r,t}$ [km/s]	217.9	213.8	202.3	226.6	138.2	162.2	125.1	183.1
	$\alpha_{r,t}$	1.11	1.11	1.01	1.18	0.687	0.759	0.664	0.842



Fantin, Merrifield and Green
arXiv:0808.1050

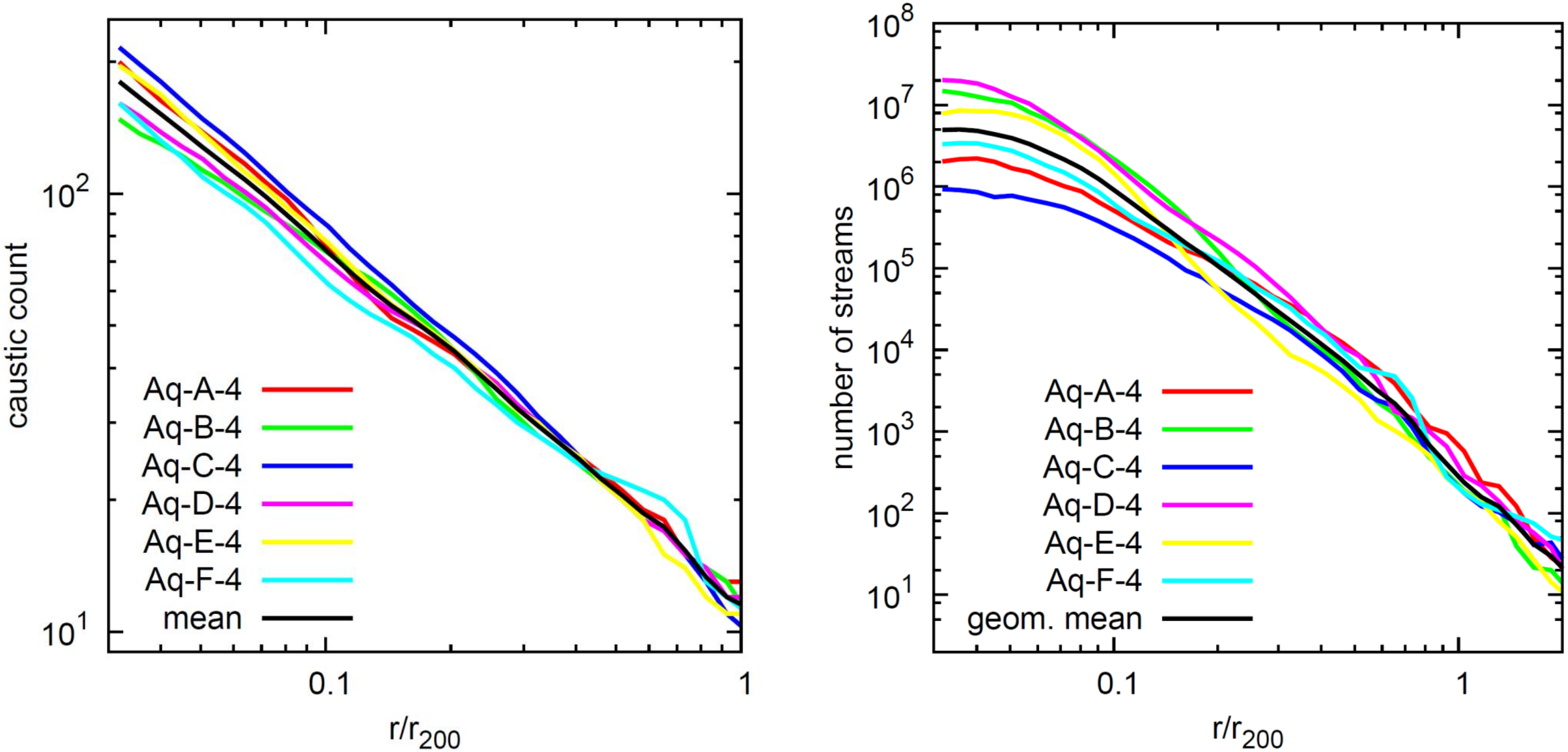


Detailed resimulation of Aquarius Halo



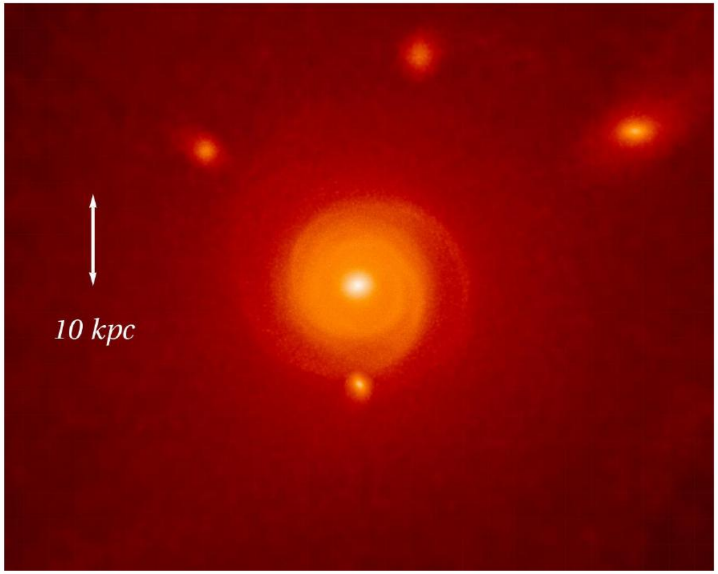
Vogelsberger and White arXiv:1002.3162

Large Number of Fine Streams Contributing to Dispersion
- No sharp features

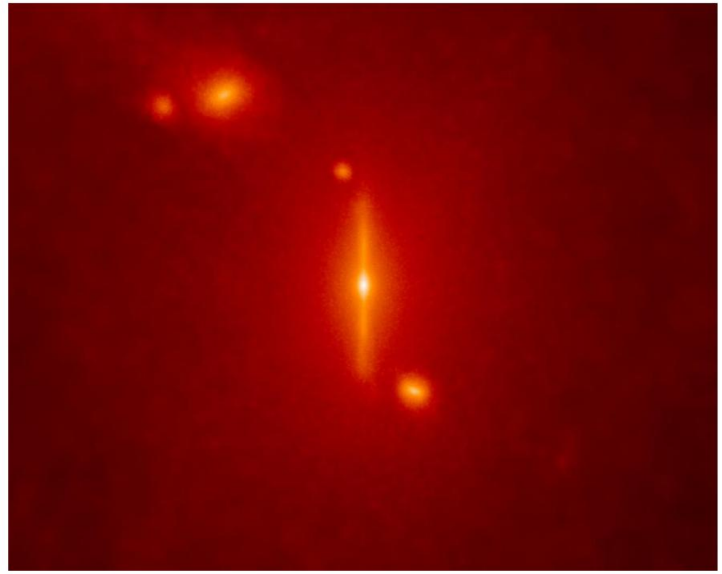


Vogelsberger and White arXiv:1002.3162

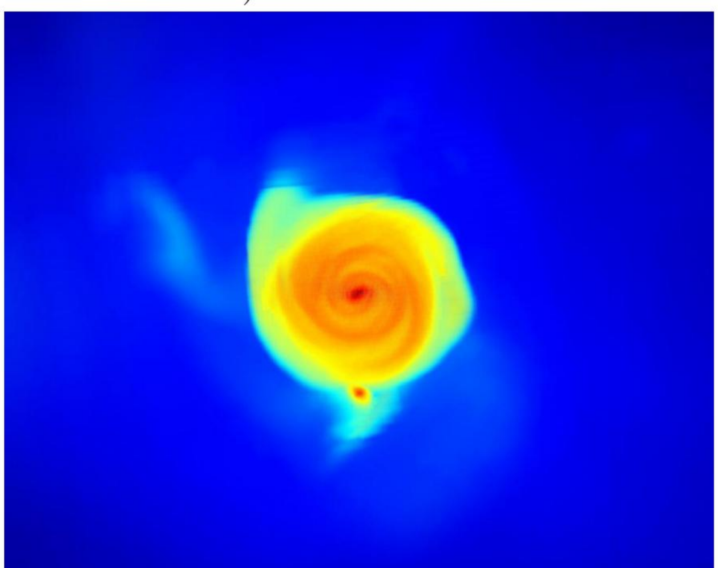
Simulations with Baryons



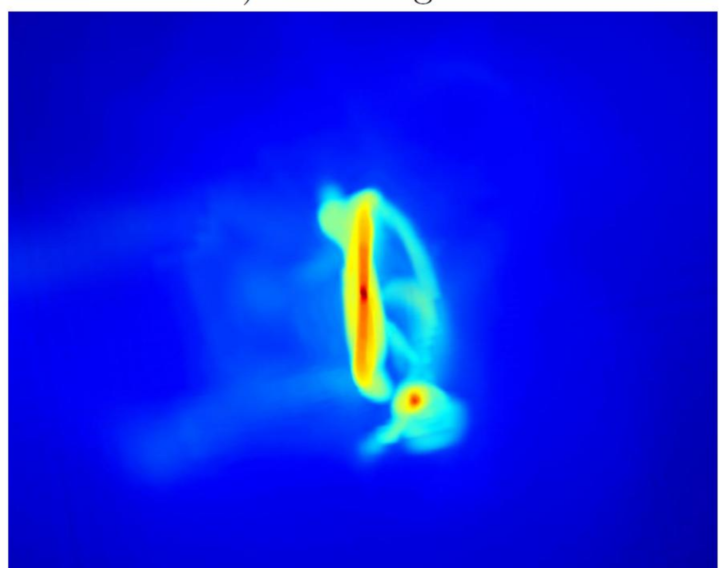
a) Stars face on



b) Stars edge on



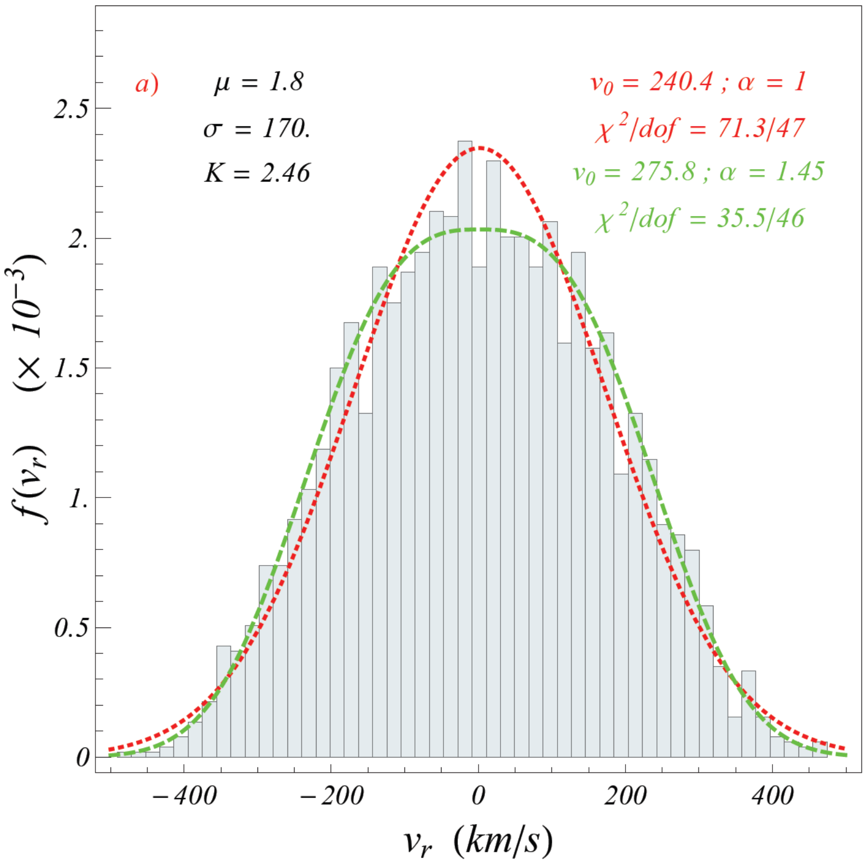
c) Gas face on

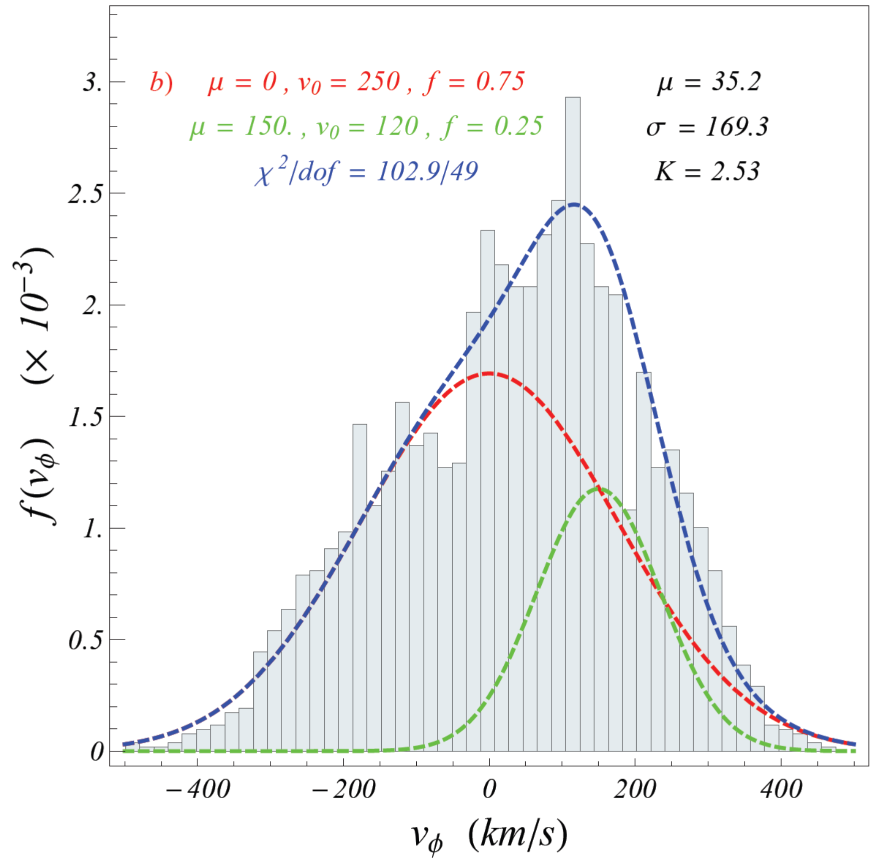


d) Gas edge on

Ling et al.
arXiv:
0909.2028

Simulations with baryons





Non-gaussianity

Evidence for co-rotating dark disk?

Ling et al. arXiv:0909.2028

Conclusions

1. Dark Matter velocity distribution non-Gaussian.
2. Dark Matter velocity anisotropic.
3. Dark Matter velocity badly understood.
4. Normally will not effect discovery of dark matter in direct experiment.
5. Critical for comparing different experiments with each other,
6. Critical for comparing direct detection with colliders.
7. Will affect interpretation of positive signal from any detector.


damn anti-gravity cat
always disproving ma theorem