Nuclear and Particle Physics - Lecture 3
Particles, fields and forces

1 Introduction

You all know what we mean by particles and forces in a classical sense. However, we need to see how they connect with quantum fields and how this helps us consider matter and forces in very similar ways

2 Quantum fields and matter

The modern view of the basic way that particles come to exist is in terms of quantised fields, which is an extension of the quantum mechanics which you have done before, where you quantise particles. These fields have quantum equations for the field amplitude which are basically like the quantum simple harmonic oscillator but there are an infinite number of them, one for every possible frequency of a wave in the field. This means the amplitudes for waves of each frequency are therefore quantised into integer steps, just like the SHO. This is what we see as particles; the first excitation gives one particle of the frequency, a further excitation of the amplitude for the same frequency corresponds to two particles, etc. Clearly, we also could excite two different frequencies both to their first excited state to make two different frequency particles.

Hence, the concept of a quantum field, unlike normal quantum mechanics, allows an arbitrary and changable number of particles to exist. This is necessary since (as we shall see) we can create and annihilate particles in reactions and decays. The standard wavefunctions correspond to the equations of the particular frequency amplitude when it is excited.

3 Identical particles

Quantum field theory actually says there is only one electron quantum field for the whole Universe and every electron which exists is due to excitations of this field. Hence, all electrons are "identical" in the QM sense as they all arise from the same field. The theory says there are particular properties for the resulting wavefunctions, namely their symmetries under the exchange of these identical particles.

The actual symmetry depends on whether the particle is a fermion, meaning it has spin 1/2, 3/2, 5/2, ..., or a boson, meaning it has spin 0, 1, 2, ... For any two identical fermions, e.g. electrons, quantum field theory says their wavefunctions must obey the property of antisymmetry under interchange. This means that if we swap any two electrons (i.e. replace each by the other), the overall wavefunction must pick up a negative sign.

$$\psi(\boldsymbol{r}_2, \boldsymbol{r}_1) = -\psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

This property is not just for electrons, but for all fermions, which includes all the matter particles. It holds for composite as well as fundamental particles. This property of exchange antisymmetry leads to a well-known principle, namely the Pauli exclusion principle. If the two electrons were in the same state, then by definition, swapping them could not make any difference to the wavefunction. Hence, this would mean

$$\psi(\boldsymbol{r}_2, \boldsymbol{r}_1) = \psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

Clearly, this is incompatible with the previous equation, so what this says is that two electrons (or any fermions) cannot be in the same state, because they cannot then satisfy the exchange antisymmetry requirement.

It is actually quite easy to make a wavefunction with the right antisymmetry property; given any wavefunction solution for two electrons, then simply constructing a total wavefunction by

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) \propto \psi(\boldsymbol{r}_1, \boldsymbol{r}_2) - \psi(\boldsymbol{r}_2, \boldsymbol{r}_1)$$

gives the right property.

Antiparticles also obey this property with other identical antiparticles, e.g. positrons obey the Pauli exclusion principle with other positrons, as they are fermions. Howver, note that antiparticles, such as positrons, do *not* count as identical to particles, we can easily tell an electron and a positron apart (by their charge, for example). In terms of antiparticles being particles going backwards in time, this means the antiparticles have a different time dependence and so can never be in the same state as electrons, by definition.

Forces are made from boson particles and here, quantum field theory says identical particle exchange must be symmetric, rather than antisymmetric. This means there must be a positive sign under interchange

$$\psi(\boldsymbol{r}_2, \boldsymbol{r}_1) = \psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

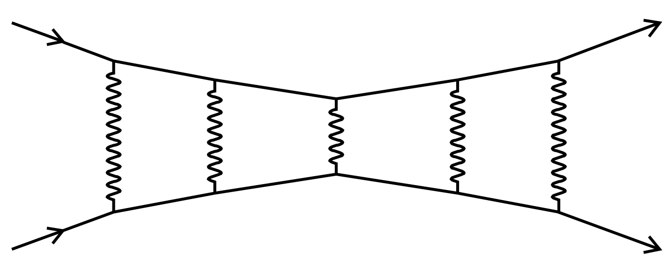
Hence, there is no problem with the two bosons being in the same state and so no Pauli exclusion principle for bosons. Again, to make a wavefunction of the right symmetry, simply do

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) \propto \psi(\boldsymbol{r}_1, \boldsymbol{r}_2) + \psi(\boldsymbol{r}_2, \boldsymbol{r}_1)$$

4 Forces and exchange of particles

What you probably conceive of as a force is something which pushes matter around and causes objects to change their motion, as per Newton's second law. In fact, this is true only in the classical limit of the systems which we call forces. When we combine the idea of a force (or potential) field with both relativity and quantum mechanics, we get a lot of other behaviour. We have to reconsider the concept of what a force can do.

The modern picture of how a force works under quantisation is that it is due to the emission and absorption of force particles, specifically bosons, between particles of matter. The bosons carry energy and momentum and so, to conserve these quantities, the energy and momentum of the matter must also change. This gives rise to the change of motion we see classically. An example is the scattering of an alpha particle (a helium nucleus) from a larger nucleus, so-called Rutherford scattering, which lead to the discovery of the structure of the atom. Both nuclei are positively charged and so classically we think of them repelling each other. At a fundamental level, this repulsion comes about because each is radiating off photons, the quantum bosons of the electromagnetic field. When a photon is radiated from one nucleus and absorbed by another, then they are pushed apart.



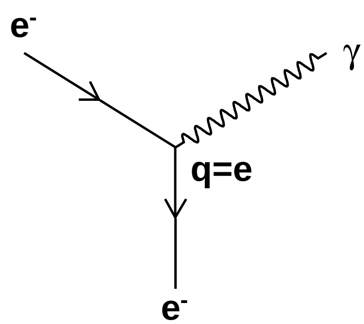
One loose analogue is ships firing cannonballs at each other; the recoil pushes the ship which is firing in the opposite direction (because the cannonball carries away momentum, which must be balanced to conserve it) and the impact pushes the other ship in the opposite direction as it absorbs the cannonball and its momentum. However, note that forces can also be attractive, as would be the case for a positive and a negative charge; there is no easy classical picture for this case, but the momentum of the exchanged boson is directed in the opposite direction.

However, we will also see that this emission of bosons actually allows particles not only to change their motion, but also their nature, i.e. they can decay or react because of the same forces. Sometimes the cannonballs split them apart! This clearly has no classical analogue.

5 Feynman diagrams

Feynman diagrams arise from perturbative calculations of the amplitudes for reactions like the above scattering process. It turns out that each of the mathematical terms in the perturbation series can be represented as a diagram, where each part of the diagram indicates a particular factor in the calculation. The derivation and calculation of Feynman diagrams is beyond the scope of this course, but they are still extremely useful in understanding the interactions.

Lines in the diagrams represent particles with well-defined energy and momentum, i.e. in a defined quantum eigenstate. These can meet at points ("vertices") where the actual interactions take place. The mathematical expressions for the forces, from which the diagrams arise, define precisely what sorts of vertices are allowed. For example, in quantum electrodynamics (QED), the relativistic quantum theory of electromagnetism, charged particles and photons, the only allowed vertex is one with two charged fermion lines (e.g. electrons) and one photon line, e.g.

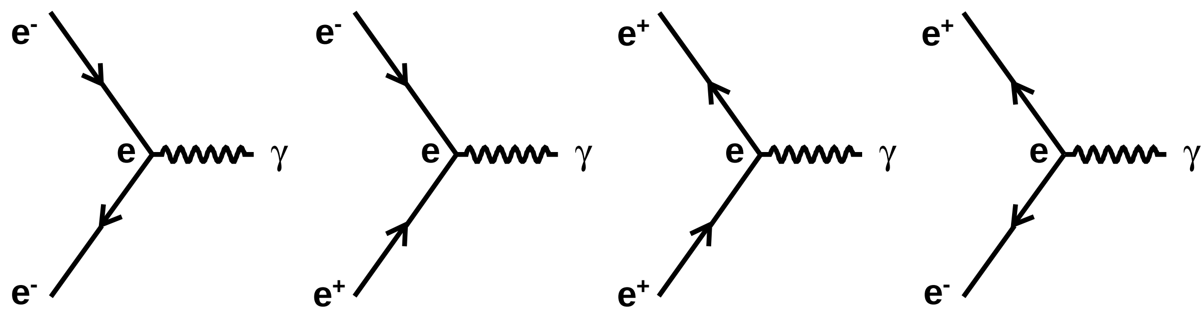


which represents an electron radiating (or absorbing) a photon and changing to another state. The first diagram above is constructed purely from these basic vertex units. Note the factor of the charge q (which is equal to e here) written by the vertex; it is the fact that the electron is charged which enables it to interact with the photon. The amplitude for this interaction to happen is directly proportional to the charge. Hence, diagrams with n vertices have a factor of e^n in the amplitude and hence e^{2n} in the probability. The dimensionless number which gives some idea of whether the electric charge is large or not is the fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$$

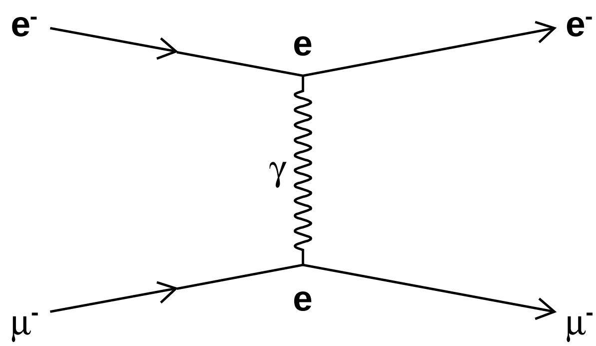
Hence, for n vertices, there will be a factor of α^n in the probability and so large n will have a smaller probability; diagrams with low numbers of vertices are more important.

I previously said that antiparticles act like particles going backwards in time; this statement only really makes sense in the context of Feynman diagrams. Explicitly, the same vertex can represent various other combinations of electrons and/or positrons by reversing their directions and replacing them with their antiparticles



i.e. radiation (or absorption) of a photon by an electron, e^+e^- annihilation, radiation (or absorption) of a photon by a positron and e^+e^- pair creation, respectively.

A reaction with only one vertex in the diagram, e.g. $e \to e\gamma$ cannot occur as it cannot conserve energy and momentum (as is easily seen by considering the initial electron rest frame). Hence, as actual reaction needs more than one vertex. For example, in $e^-\mu^-$ scattering, the lowest order diagram, which has two vertices, would be



This one diagram covers both emission by the electron followed be absorption by the muon and vice versa. Note, sometimes time and space axes are added to these diagrams to remove any ambiguity about what reaction is represented. Only the particles at the edges of the diagram (the original and final e^- and μ^- in this case) are observed experimentally. The photon which causes the scatter is never seen directly; it is said to be "virtual". It is not a free particle and so does not obey $E^2 = p^2c^2 + m^2c^4$.

The above diagram has two vertices and so gives an amplitude which is proportional to e^2. Hence, the probability for this reaction is proportional to e^4 or α^2.

6 Ranges of forces

It turns out that the range of a force is directly related to the mass of the bosons being exchanged. The most commonly known example of a force boson, the photon, is of course massless and this results in the electromagnetic force being said to have an infinite range. This does not mean the force or potential does not reduce with distance, of course. You all know that the electromagnetic potential due to a point charge goes as

$$\phi = \frac{Q}{4\pi\epsilon_0 r}$$

This actually arises as the solution of the time-independent potential equation resulting from Maxwell's equations, namely

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

for a charge density ρ. For the case of a point charge, then for all space outside the origin, the charge density is zero, so the potential outside satisfies

$$\nabla^2 \phi = 0$$

for which the above equation is the solution. How would this be modified if the photon had a non-zero mass? To answer this, we need to generalise the above equation in two steps. Firstly, how would it look if we were using the time-dependent Maxwell equations? Ignoring some mathematical complications, we would find it was

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0$$

which is the EM wave equation, allowing propagation of light.

As the second step, we need to add a mass term. To see how to do this, then we need to consider the basic equation satisfied by all particles

$$E^2 = p^2 c^2 + m^2 c^4$$

What happens if we use this to form a Schrödinger-like equation? As usual, the energy operator is $\hat{E} = i\hbar\partial/\partial t$ and the momentum operator is $\hat{\boldsymbol{p}} = -i\hbar\boldsymbol{\nabla}$. Hence we would get

$$-\hbar^2 \frac{\partial^2 \phi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi + m^2 c^4 \phi$$

or

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi + \frac{m^2 c^2}{\hbar^2}\phi = 0$$

This is called the Klein-Gordon equation and is obeyed by the relativistic wavefunctions for all free particles. Note, it is *not* the only equation they obey; e.g. fermion wavefunctions satisfy the Dirac equation but that is beyond the scope of this course.

It is seen that the time-dependent Maxwell equation is just this equation with $m = 0$, as would be expected for a photon. Hence, it is clear that the time-independent potential equation for a point charge including a mass term is

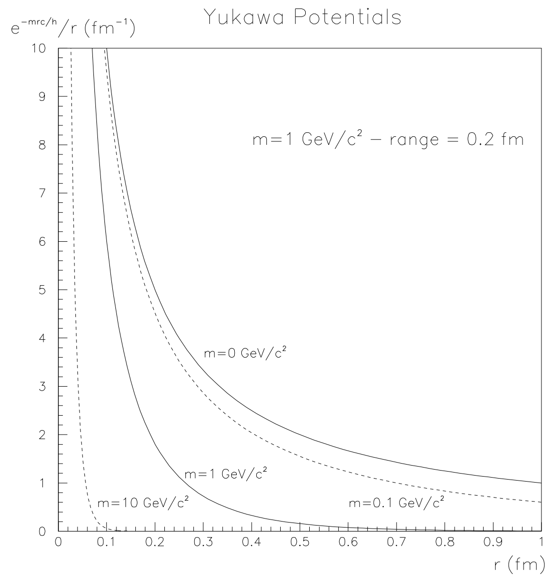
$$\nabla^2 \phi - \frac{m_\gamma^2 c^2}{\hbar^2}\phi = 0$$

for which the solution is modified to become

$$\phi = \frac{Q}{4\pi\epsilon_0 r} e^{-m_\gamma c r/\hbar}$$

This is called the Yukawa potential. How does this compare with the massless case? For $r \ll \hbar/mc$, they look very similar, while for larger values of the radius, then the potential is much reduced. Hence, the potential cuts off for radii greater than the "range" given by $\hbar/mc$; the larger the mass, the shorter the range of the force. To set the scale, a mass of 1 GeV/c^2 gives a range of 0.197 fm.

While the photon is thought to be exactly massless and hence has no cutoff range, this is not the case for the weak interaction bosons, the $W^\pm$ and the Z^0. These have masses of around 80 GeV/c^2 and 91 GeV/c^2, respectively, which correspond to ranges of around 0.002



fm $= 2 \times 10^{-18}$ m. This extremely short range is what makes the weak interaction appear weak; the actual equivalent of charge for the weak interactions is in fact a little bigger than for electromagnetism but it is masked by the mass effect at energies less than $M_{W,Z}c^2$. Note, most processes we will consider on this course are at much smaller energies that this, so you should think of the weak force as being very weak, i.e.

$$\text{Strong} \gg \text{Electromagnetic} \gg \text{Weak}$$

for reaction or decay rates.

The strong force bosons are called the gluons and they are massless, like photons. This means the strong force has, in principle, an infinite range. However, the gluons themselves carry the equivalent of the charge for the strong force. This means gluons can radiate and absorb other gluons and this complicates the picture. In fact, the force range is effectively limited to ~ 1 fm, i.e. nuclear sizes. Also, we never see "bare" colour charges but only the equivalent of uncharged combinations, namely the hadrons mentioned previously.