# Nuclear and Particle Physics - Lecture 4
# Decays and reactions

## 1 Introduction

In this lecture, we will discuss a few of the aspects and limitations of what we can and cannot expect to learn from decays and reactions.

## 2 How to measure the properties of the forces

The idea of the study of particle physics is to determine the form and nature of the forces between the particles. This raises the question of what measurements we can do to study the forces? There are three basic properties which can be experimentally determined;

1. The masses (or energies) of bound states, i.e. the mass spectrum. This is only useful in practise for the strong force as bound states of the weak interaction are too feebly bound to be seen and we already know the EM force law. To form a bound state involves many boson exchanges.

2. The decay rates or widths of unstable particles. QM relates the lifetime $\tau$ to the width $\Gamma$ by $\Gamma = \hbar/\tau$, as discussed below. Decays usually involved only a few boson exchanges.

3. The reaction rates, usually expressed as cross sections (again discussed below). As for decays, these usually involve only a few exchanges.

There are also several properties of the interactions which tell us about the forces, particularly their conservation laws.

## 3 Decays

A decay is basically the change from one quantum state (the initial particle) to another (the final or "daughter" particles). Until it is observed to decay, the initial state is not changing in any way. Hence, if the particle is known to exist at some time, then the probability it will decay within a short time $dt$ has to be independent of how long it has lived for previously, i.e. it cannot depend on $t$. Hence, the probability of decay with $dt$ must be $\lambda dt$, where the constant $\lambda$ summarises all the QM state information about the decay. $\lambda$ is called the decay rate (or transition) constant and has units of $s^{-1}$. It gives the probability per unit time for the state to decay. Note, the above equation says the decay is as likely to happen in any $dt$ as any other if the particle is known to exist, but *not* that the decay is as likely at any $t$ as any other. To have a long decay time requires it to have not decayed in all the $dt$'s up to that time. We can actually calculate the probability distribution of the total time taken to decay from the above. Let $S(t)$ be the probability the particle will survive at least until time $t$ after it is known to exist at time $t = 0$. Hence, the chance of it surviving until $t$ and then decaying in the next $dt$ is clearly $S(t)\lambda dt$. Alternatively, the chance of it not decaying in the next $dt$ is $S(t)(1 - \lambda dt)$. However, by definition, this must be $S(t + dt)$, so

$$S(t)(1 - \lambda dt) = S(t + dt) = S(t) + \frac{dS}{dt}dt$$

Therefore

$$\frac{dS}{dt} = -S\lambda$$

so

$$\frac{dS}{S} = -\lambda dt$$

and integrating gives

$$\ln S = k - \lambda t$$

for some integration constant $k$. This is

$$S = e^k e^{-\lambda t}$$

and since the particle exists at $t = 0$, then $S(0) = 1$, so $k = 0$ and

$$S = e^{-\lambda t}$$

the famous exponential decay law. As stated above, the probability of living until time $t$ and then decaying within $dt$ is given by $S\lambda dt = \lambda e^{-\lambda t} dt$. Hence, the average time the particle lives can be calculated by

$$\tau = \langle t \rangle = \int_0^\infty t\lambda e^{-\lambda t}\, dt$$

Using integration by parts, this gives

$$\tau = \left[-te^{-\lambda t}\right]_0^\infty + \int_0^\infty e^{-\lambda t}\, dt = \left[-\frac{1}{\lambda}e^{-\lambda t}\right]_0^\infty = \frac{1}{\lambda}$$

This average is called the lifetime of the particle. In terms of the lifetime

$$S(t) = e^{-t/\tau}$$

Sometimes we consider many particles (so we can ignor statistical fluctuations) and want to known how many remain at time $t$. With very many initial decaying particles (e.g. a lump of material containing many nuclei), initial number $N_0$, then clearly the number which exist at any later time is given by

$$N(t) = N_0 S(t) = N_0 e^{-\lambda t}$$

Hence, the number is reduced by $1/e = 1/2.718$ after one lifetime. Note that

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$$

showing that $\lambda$ is indeed the rate of decay per nucleus.

Another quantity often used in nuclear physics is the half-life, $\tau_{1/2}$, which is defined to be the time taken for 1/2 the initial sample to have decayed, for which

$$N(\tau_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda \tau_{1/2}}$$

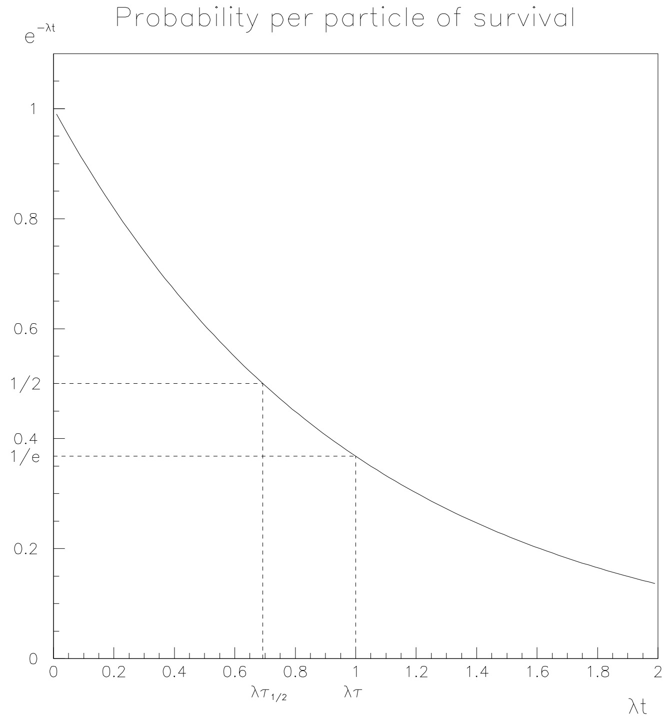
Hence

$$e^{\lambda \tau_{1/2}} = 2$$

so

$$\tau_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 = 0.693\tau$$

Hence, the half-life is shorter than the average lifetime by $\sim 31\%$.
Note, these quantities are all independent of the “real” start time of the material, i.e. when it was actually created. If we observe the particle is alive at $t = 0$, everything after that is independent of anything that happened previously.




Another aspect of decays arises due to a fundamental property of QM. Any state which is unstable, i.e. has a finite lifetime, does not have an exact energy (or equivalently mass if you consider the rest frame of the decaying particle). You can think of this as another application of Heisenberg's uncertainty principle

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

In this case

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

Hence, for a particle with a finite lifetime, the mass is uncertain and this is quantised by the width $\Gamma$

$$\Delta mc^2 = \Gamma = \frac{\hbar}{\tau} = \hbar\lambda$$

where $\Gamma$ is called with width of the particle. Note that very short lifetimes have large widths while very long-lived particles have small widths. Stable particles, such as the proton and electron, have zero width.

A complication on the above is that it is often the case that particles can decay in several different ways. Each of these decay modes will happen independently of the others, so we can consider each to have a separate decay rate constant $\lambda_i$ and hence we can define *partial widths*

$$\Gamma_i = \hbar\lambda_i$$

The total decay rate is clearly the sum of the separate rates so

$$\lambda = \sum_i \lambda_i$$

and the total width, which is the actual physical uncertainty on the mass, is then

$$\Gamma = \hbar\lambda = \sum_i \hbar\lambda_i = \sum_i \Gamma_i$$

The proportion of decays to a particular mode is called the branching fraction

$$\mathcal{B}_i = \frac{\Gamma_i}{\Gamma} = \frac{\lambda_i}{\lambda}$$

and clearly

$$\sum_i \mathcal{B}_i = 1$$

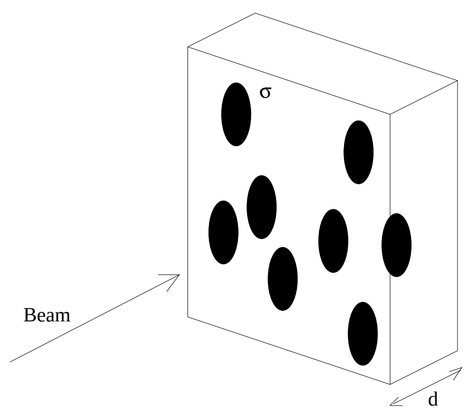
The lifetime is as before

$$\tau = \frac{\hbar}{\Gamma}$$

as there is no physical meaning to a "partial lifetime".

## 4 Reactions

What about reactions? We can clearly measure the rate of a reaction if we fire a beam at some material containing the target particles. However, the rate will depend on the rate at which the beam particles enter the material and the density of the target particles in the material, as well as the fundamental force properties we are trying to measure. We need a basic property which is independent of all the other factors which might vary from one experiment to another. This property is given by the *cross section*, which is the effective target area presented to the incoming particle for it to cause the reaction. Consider a thin piece of material, thickness $d$, containing target particles with number density $n$



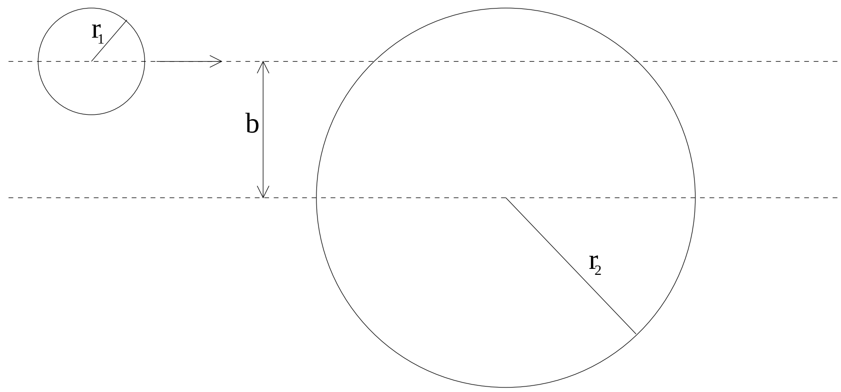

Each target particle has an area, the cross section $\sigma$, for the reaction. For a front surface area $A$, there are $Adn$ targets, which therefore have a total target area $Adn\sigma$. Hence, the probability of an incoming particle hitting one of the targets is $Adn\sigma/A = dn\sigma$. Hence, the reaction rate is simply the beam rate times $dn\sigma$, where $d$ and $n$ can be different for different experiments, but $\sigma$ is the physical property we compare.

Note, this only works for thin material, meaning $d$ is small enough that the probability of a reaction $dn\sigma \ll 1$. Otherwise, every incoming particle will see several targets and have multiple interactions. In this situation, a more appropriate measure is the mean free path of the particle in the material. Clearly, on average the particle will move a distance before interacting corresponding to a probability of 1, so the mean free path $l$ is given by $ln\sigma = 1$ or

$$l = \frac{1}{n\sigma}$$

Another measure is the mean time between collisions; since $l = vt$, this is clearly $1/vn\sigma$.

We can calculate very simply models for cross sections easily. Consider a (not very physical) classical model for nuclei as being hard spheres, like billiard balls. (We are ignoring any electromagnetic interaction here.) Consider a light incoming nucleus of radius $r_1$ and a heavy target nucleus of radius $r_2$, where $m_2 \gg m_1$ so we can ignor the motion of the target nucleus. The impact parameter $b$ is defined as the distance of closest approach of the centres of the nuclei if no interaction happened.




Clearly, a collision happens whenever $b < r_1+r_2$ so the cross section is simply given by $\pi(r_1+r_2)^2$. Note, as is generally the case, the cross section depends on both the particles taking part in the reaction.

We might also be interested in the angular distribution of the scattered nuclei after the reaction, in which case we want the cross section to scatter to a particle angle. This hard scattering can be shown to have a distribution

$$\frac{d\sigma}{d\theta} = \frac{\pi}{2}(r_1 + r_2)^2 \sin\theta$$

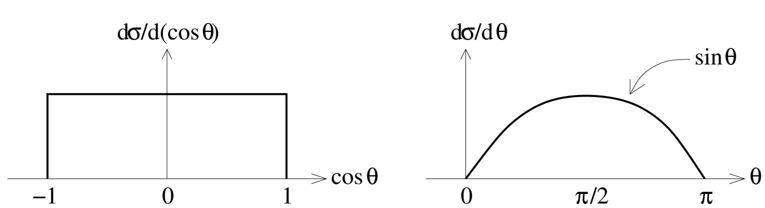
or equivalently, since it is uniform in azimuth

$$\frac{d\sigma}{d\theta d\phi} = \frac{1}{4}(r_1 + r_2)^2 \sin\theta$$

In terms of the solid angle element $d\Omega = \sin\theta\, d\theta\, d\phi$, which can also be written as $d(\cos\theta)\, d\phi$, then this becomes

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d(\cos\theta)\, d\phi} = \frac{(r_1 + r_2)^2}{4}$$

This shows the cross section per solid angle is independent of $\theta$ or $\phi$. By definition, this means it is *isotropic*, i.e. the scattered particles are emitted equally in all directions. Note this is why we use $d/d(\cos\theta)$; in terms of $d/d\theta$ we have a $\sin\theta$ dependence but this is what is needed for an isotropic distribution as $\theta$ does not map trivially onto an isotropic distribution.




A more physical cross section calculation, for EM scattering of a charged particle from a nucleus, which is known as *Rutherford scattering*, is given on the problem sheet.