Nuclear and Particle Physics - Lecture 7
Hadrons and Mesons

1 Introduction

The most obvious result of confinement is that free quarks have never been observed. We only every see hadrons, which are bound states of $q\overline{q}$ (mesons) or qqq (baryons). We don't know the potential between the quarks per se and Feynman diagram are perturbative calculations which are not useful for bound states. However, we can still understand a lot about the mass levels of these quark bound states.

2 Colour singlets

The hadrons themselves must be uncharged with respect to the strong force, or confinement would hold for them also and we would not observe the states as free particles. This means they must be the equivalent of neutral for the colour charge. Because this charge comes in three types (r, b and g) rather than one type as in electromagnetism, then we need the colourless combinations. In the last lecture, we saw the colourless combination is

$$C = 0 : \quad (r\overline{r} + b\overline{b} + g\overline{g})/\sqrt{3}$$

which does not appear on a gluon. This is one combination which is described as "uncoloured" or a "colour singlet", meaning the equivalent of colour neutral. This can clearly be formed using the $q\overline{q}$ combinations of mesons and so we would expect mesons to exist.

It also turns out that there is another way to make an uncoloured combination; specifically in triplets. Here there are $3 \times 3 \times 3 = 27$ combinations, but again only one is uncoloured; this is the combination rbg but in a totally antisymmetric combination

$$C = 0 : \quad (rbg - rgb + grb - gbr + bgr - brg)/\sqrt{6}$$

As an aside; this is why it is called "colour", as a red-blue-green combination gives something without colour, i.e. "white". Baryons, like the proton, contain three quarks and so again can form this colourless combination; hence baryons are expected also. There are no other colour singlet combinations so these map well onto the observed mesons and baryons and provides strong evidence for QCD. We'll now look in more detail at these states, starting with the mesons.

3 Mesons

We will restrict ourselves to the u, d and s quark "flavours" for now. There are then nine possible flavour combinations (a "nonet") of $q\overline{q}$. Each forms a meson with a ground state and excited states. All the nine ground states have the same quantum numbers as do all the first excited states; the wavefunctions only differ because of the different masses of the quarks which each meson contains. This is completely analogous to comparing the spectra of hydrogen and positronium when allowing for the different reduced mass.

These combinations and their ground and first excited states are listed in the table below, where the last column will be explained later. We call these ground states and first excited states separate (composite) particles, although in the analogy with the hydrogen atom, they correspond to just the excited states.

State	Meson	Measured Mass (GeV/c^2)	Quark Pair	Predicted Mass (GeV/c^2)
$J^P = 0^-$	$\pi^\pm$	0.1396	$u\overline{d}$, $d\overline{u}$	0.1395
	$K^\pm$	0.4937	$u\overline{s}$, $s\overline{u}$	0.4938
	K^0, $\overline{K}^0$	0.4977	$d\overline{s}$, $s\overline{d}$	0.4980
$J^{PC} = 0^{-+}$	π^0	0.1350	$u\overline{u}$	0.1340
	η	0.5475	$d\overline{d}$	0.1449
	η'	0.9578	$s\overline{s}$	0.7871
$J^P = 1^-$	$\rho^\pm$	0.7669	$u\overline{d}$, $d\overline{u}$	0.7702
	$K^{*\pm}$	0.8916	$u\overline{s}$, $s\overline{u}$	0.8918
	K^{*0}, $\overline{K}^{*0}$	0.8961	$d\overline{s}$, $s\overline{d}$	0.8932
$J^{PC} = 1^{--}$	ρ^0	0.7691	$u\overline{u}$	0.7692
	ω	0.7819	$d\overline{d}$	0.7713
	ϕ	1.0194	$s\overline{s}$	1.0365

To what extent can we understand the mass spectrum of these mesons? Normally for a bound state, we would say

$$m_{BS} = m_q + m_{\overline{q}} - \frac{E_B}{c^2}$$

where E_B is the binding energy which must be positive to obtain a stable state. However, there are some problems with this here; firstly, we cannot get free quarks so we don't know, per se, what their masses are. Secondly, as they cannot be free, then there is no requirement that the "binding energy" is positive. Hence, we will take all three terms are unknowns to be parametrised.

What can we deduce about the quark masses? The table shows the pions are all very close in mass; these only differ in their u and d content so this implies $m_d \approx m_u$. The K^+ ($u\overline{s}$) and K^0 ($d\overline{s}$) are also very close in mass, so again this implies $m_d \approx m_u$. Both these kaon states are heavier than the pion states, so $m_s > m_{u,d}$. The same pattern appears in the excited states, so this makes sense there too.

What about the binding energy? In atoms, splitting between levels arise through $\boldsymbol{L}.\boldsymbol{S}$ couplings (the fine structure) and $\boldsymbol{S}_N.\boldsymbol{S}_e$ couplings (the hyperfine structure). As we will see later, there is no L here for either the ground or excited states, so let's consider what a "hyperfine" equivalent interaction would do. In atoms, it is due to the magnetic dipole moment of the circulating electron coupling to the nuclear magnetic dipole moment

$$\Delta E \propto \boldsymbol{\mu}_e.\boldsymbol{\mu}_N$$

This is the interaction in positronium which caused the $\sim 10^{-3}$ eV energy shift in the ground state, as mentioned previously.

Since the basic quark-gluon vertex for QCD is very similar to QED, then we might expect a magnetic-field equivalent in QCD and for particles to have QCD-magnetic moments. Since we know from the Dirac equation that for fundamental fermions

$$\boldsymbol{\mu} \propto \frac{\boldsymbol{s}}{m}$$

then we can say a QCD hyperfine equivalent interaction would have a term like

$$\propto \frac{\boldsymbol{s}_q.\boldsymbol{s}_{\overline{q}}}{m_q m_{\overline{q}}}$$

with the binding energy being the expectation value

$$\frac{E_B}{c^2} = -K\frac{\langle \boldsymbol{s}_q.\boldsymbol{s}_{\overline{q}}\rangle}{m_q m_{\overline{q}}}$$

for a constant of proportionality K and where the negative sign is chosen to make K positive. The $\boldsymbol{s}_q.\boldsymbol{s}_{\overline{q}}$ term might look difficult, but since $L = 0$, then

$$\boldsymbol{J} = \boldsymbol{s}_q + \boldsymbol{s}_{\overline{q}}$$

so

$$J^2 = s_q^2 + s_{\overline{q}}^2 + 2\boldsymbol{s}_q.\boldsymbol{s}_{\overline{q}}$$

which means

$$\langle \boldsymbol{s}_q.\boldsymbol{s}_{\overline{q}}\rangle = \frac{1}{2}\langle J^2 - s_q^2 - s_{\overline{q}}^2\rangle = \frac{1}{2}\left[J(J+1) - s_q(s_q+1) - s_{\overline{q}}(s_{\overline{q}}+1)\right]\hbar^2$$

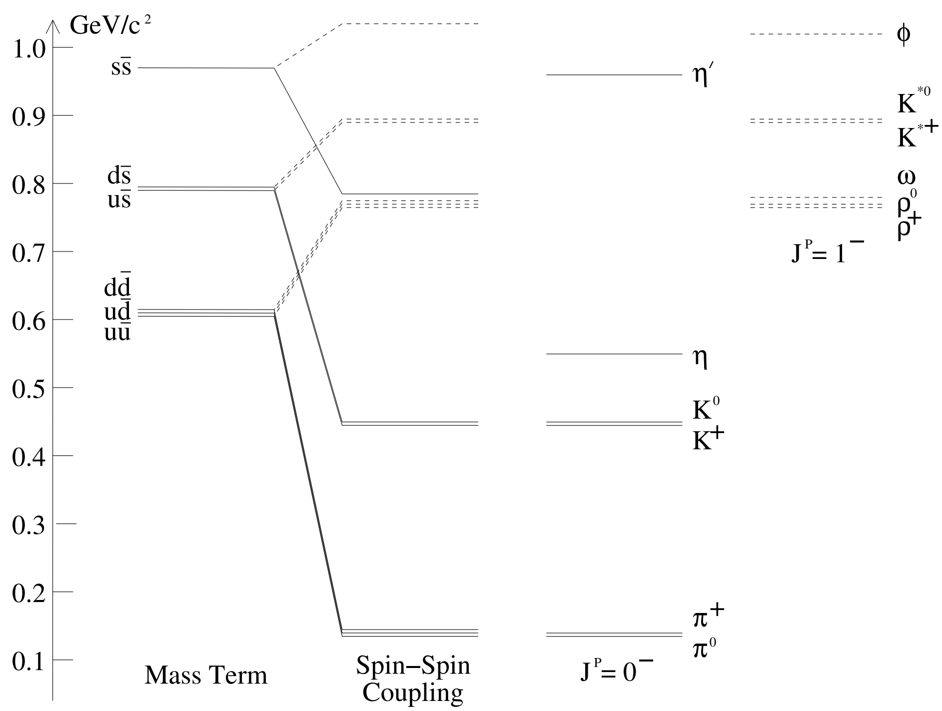
Clearly, $s_q = s_{\overline{q}} = 1/2$ and so for the ground states, with $J = 0$ we would get

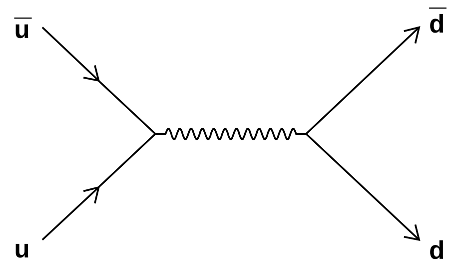
$$m_{GS} = m_q + m_{\overline{q}} - \frac{3}{4}\frac{K\hbar^2}{m_q m_{\overline{q}}}$$

and for the first excited state, with $J = 1$

$$m_{FES} = m_q + m_{\overline{q}} + \frac{1}{4}\frac{K\hbar^2}{m_q m_{\overline{q}}}$$

where the constant of proportionality K is to be determined. For this model, we have six useful values of the masses (ground and excited states of the different flavour pairs) and four parameters (m_u, m_d, m_s and K). Good agreement for all six is obtained with $m_u = 0.3052$ GeV/c^2, $m_d = 0.3075$ GeV/c^2, $m_s = 0.4871$ GeV/c^2 and $K = 0.0592$ GeV$^3/\hbar^2$c^6, This then gives predictions of the mesons containing the $u\overline{u}$, $d\overline{d}$ and $s\overline{s}$. The result is shown in the table above and graphically as follows.

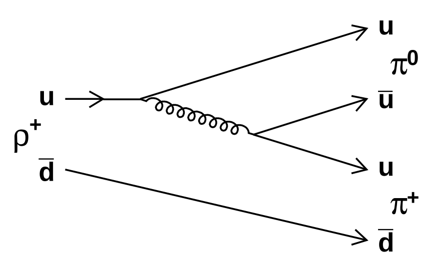




The π^0, ρ^0, ω and ϕ are seen to be in good agreement, but the η and η' are not. This can be understood by the fact that a pure (e.g.) $u\overline{u}$ state can change to (e.g.) a $d\overline{d}$ as follows so that the flavour eigenstates are not (necessarily) the mass eigenstates. The π^0, η and η' are in fact mixtures of $u\overline{u}$, $d\overline{d}$ and $s\overline{s}$ in varying proportions; the π^0 happens to have very little $s\overline{s}$ and so still agrees well with the basic model. The ρ^0, ω and ϕ will have the same effect; they all agree well because the ρ^0 and ω happen to be almost totally $u\overline{u}$ and $d\overline{d}$ while the ϕ is almost totally $s\overline{s}$. Hence, only the η and η' have significant amounts of all three flavours and so only they show significant disagreement with this model.

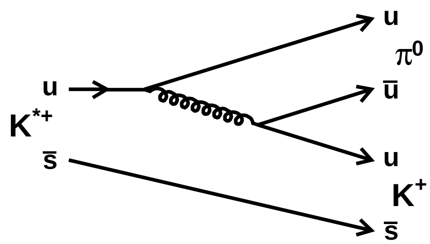
4 Excited state decays

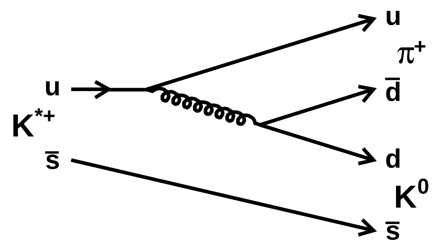
The excited states can decay to ground states, as would be expected. This is possible through photon emission in a similar way as in atoms, e.g. $\rho^+ \to \pi^+\gamma$ but is in fact very rare with a branching fraction 4×10^{-4}. This is because it is (clearly) an electromagnetic force decay and there are alternative decays which use the strong force and hence, since $\alpha_S \gg \alpha$, these will go much faster. As $m_\rho > 2m_\pi$, then in fact the ρ^+ mostly decays as $\rho^+ \to \pi^+\pi^0$. This requires a new $q\overline{q}$ pair to be created at a gluon vertex, e.g.



Because this is a strong force decay with only two vertices, then it goes very fast and the lifetime of the ρ^+ is around 10^{-23} s. This gives a width of 150 MeV, which is not negligible compared with its mass of 770 MeV/c^2. The ρ^0 decays as $\rho^0 \to \pi^+\pi^-$ (but not to $\pi^0\pi^0$ due to boson exchange symmetry, see Problem Sheet 1) and again has a similar lifetime and width. These are typical orders of magnitude for strong decay particle lifetimes.

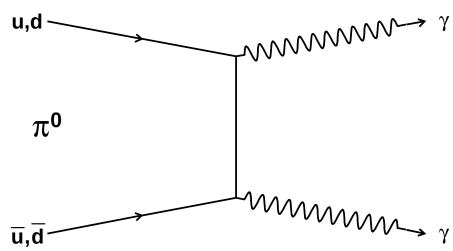
Because the strong force does not change flavour, the K^* mesons must decay to produce a K meson so as to preserve the s quark, e.g. $K^{*+} \to K^+\pi^0$ or $K^{*+} \to K^0\pi^+$
The decay rates are again of the same order and the K^* widths are around 50 MeV. Slightly smaller widths are found for the ω and ϕ but only because of a more restricted set of final states which they can decay to; they all decay strongly.





5 Ground state decays

In contrast, the ground states cannot decay via the strong force as they are the ground state eigenstates of this force. The six mesons $u\overline{d}$, $u\overline{s}$ and $d\overline{s}$ and their antimesons, i.e. $\pi^\pm$, $K^\pm$, K^0 and $\overline{K}^0$, must also be stable to EM decays as these do not change the quark flavour and there are no lighter states with these flavours. Hence these decay only weakly. The other three ground states, i.e. π^0, η and η', can decay via the EM force, e.g. the main decay mode for the π^0 is $\pi^0 \to \gamma\gamma$



which is entirely analogous to the positronium decay.

We used the fact above that there is no orbital angular momentum in these mesons. The three which decay via the EM force are all observed to have $J^P = 0^-$ from the assumption of angular momentum and parity conservation in their decays and we assume the wavefunctions for all states are equivalent, but have various combinations of different quarks in them. The parity of orbital angular momentum states is

$$P_L = (-1)^L$$

The Dirac equation tells us the intrinsic parity of fermions and antifermions are opposite and the convention is that

$$P_f = +1, \qquad P_{\overline{f}} = -1$$

so that an $q\overline{q}$ state will have a total parity of

$$P_{f\overline{f}} = P_f P_{\overline{f}} P_L = (-1)^{L+1}$$

The spin of two spin 1/2 particles is $S = 0$ or 1 so for a total spin of $J = 0$, we can have either $S = 0$, $L = 0$ or $S = 1$, $L = 1$; only the former gives the observed parity, so we know the ground states are $S = 0$, $L = 0$. Similarly, the first excited states are observed to be $J^P = 1^-$ and this is thought to be due to $S = 1$, $L = 0$.

The three unflavoured states in each case are also eigenstates of the charge conjugation operator C; the equivalent formula is

$$C_{f\overline{f}} = (-1)^{L+S}$$

and so would be $C = +1$ for the ground states and $C = -1$ for the excited states. These values are given in the table above. We will discuss P and C further in future lectures.

6 Heavy quarks

There are also three other flavours of quarks, charm c, bottom b and top t. Of these, top quarks decay too fast to form hadrons, but mesons containing charm ($m_c \sim 1.6$ GeV/c^2) and bottom ($m_b \sim 4.9$ GeV/c^2) quarks are also seen. For example, singly-charmed mesons such as $c\overline{u}$ (D^0 and D^{*0} for the ground and first excited states) and $c\overline{d}$ (D^+ and D^{*+}) have been observed, as well as "hidden-charm" (i.e. no net charm) mesons containing $c\overline{c}$ (η_c and J/ψ). Similar combinations of bottom quarks in mesons have also been observed.