Nuclear and Particle Physics - Lecture 10
Electron-proton and proton-antiproton reactions

1 Introduction

We have looked at the reaction $e^+e^- \to$ hadrons in the last lecture and seen how it is very similar to the annihilation reaction $e^+e^- \to \mu^+\mu^-$ at a fundamental level. Today, we will look at e^-p and $p\overline{p}$ reactions and, in a similar way, these will turn out to be very similar to basic QED reactions. These reaction clearly involve protons so we will need to understand how a proton interacts.

It is much easier to accelerate protons than electrons because they radiate less when they are accelerated; the power radiated off goes as $1/m^4$. Hence, while the highest energy electron and positron beams are around 100 GeV at the LEP collider at CERN, giving centre-of-mass energies of 200 GeV, protons have been accelerated to around 1 TeV, with the Tevatron $p\overline{p}$ collider at FNAL near Chicago having a centre-of-mass energy of 2 TeV. The highest energy ep collider, the Hera collider at the DESY laboratory in Hamburg, happens to have energies of 30 GeV electrons (or positrons) on 830 GeV protons.

In the future, the Large Hadron Collider (LHC) at CERN will run with 14 TeV in the centre of mass. This is scheduled to start in 2007 and is actually a proton-proton machine, rather than proton-antiproton.

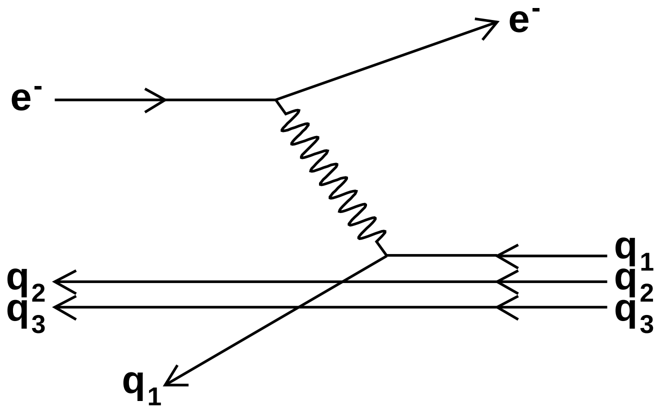
2 The basic interaction

At very low energies, electrons can elastically scatter off nuclei as a whole, with the nucleus not being excited in the collision. This is effectively the same as Rutherford scattering, although because the nuclear charge is not exactly at a point, then the cross section is modified by "form factors" which allow for the smeared charge distribution. These are effectively the Fourier transforms of the spatial charge distribution. These give a different angular distribution and indeed a measurement of the latter allows us to deduce the spatial charge distribution in a nucleus.

At somewhat higher energies, the nucleus is disrupted, i.e. the scattering becomes inelastic and the individual nucleons are knocked out. However, the scattering can still be easily described as the electrons are now colliding with individual protons i.e. $e^-p \to e^-p$. Again, this looks similar, but not identical, to the $e^-\mu^- \to e^-\mu^-$ scattering we looked at previously, with form factors to account for the proton charge distribution.

However, at high energies, even the proton is blown apart by the electron and many hadrons are observed emerging from the reaction, $e^-p \to e^-X$, where X refers to the total system of hadrons produced. This is called "deep inelastic scattering", where "deep" refers to cases where the mass of the X system is very large compared with the proton mass.

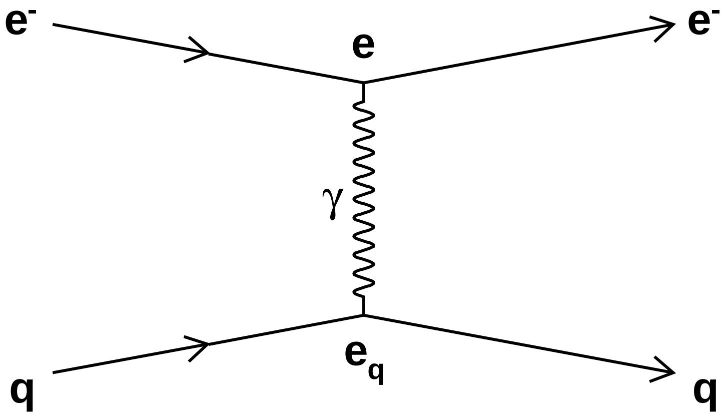
We know the proton is made of charged (uud) quarks so the fundamental underlying reaction here must be eq scattering, i.e. $e^-q \to e^-q$.



What do we actually see when this occurs? The non-reacting qq (usually called the "proton remnant") continues in its original direction and is seen as a jet of hadrons, just as for e^+e^- reactions. The struck quark forms another jet, so we should see

$$e^- p \to e^- + 2 \text{ jets}$$

However, in almost all cases, there is a hole in the detector to allow through the non-interacting protons, which is therefore in the direction of the proton remnant jet, so this is normally not actually fully detected in experiments. Clearly, the scattering (and hence physics) is actually reflected in the directions and energies of the outgoing electron and the other quark jet, so this is what we will concentrate on. We can draw the Feynman diagram for the basic reaction, as before.



It is identical to the QED reaction $e^-\mu^- \to e^-\mu^-$ except that we have the quark charge rather than the muon charge at one vertex. Hence

$$\text{Amplitude}(e\mu) \propto e^2, \qquad \text{Amplitude}(eq) \propto e e_q$$

and the cross section goes as

$$\sigma_{e\mu} \propto e^4, \qquad \sigma_{eq} \propto e^2 e_q^2 = e^4 \left(\frac{e_q}{e}\right)^2$$

so we would expect the cross section for eq scattering to be

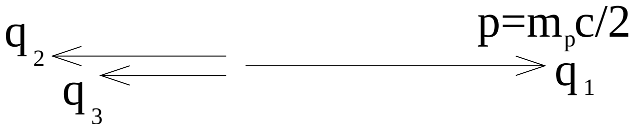
$$\sigma_{eq} = \sigma_{e\mu} \left(\frac{e_q}{e}\right)^2$$

In particular, it will have the same strong $1/\sin^4(\theta/2)$ dependence. There is, however, one additional complication here, which is not present in e^+e^- annihilation; we don't have a beam of quarks at a well-defined energy. While asymptotic freedom says the quarks look free in the proton for high energy interactions, they are not at a fixed energy.

3 Proton structure

For a moving proton, we might naively expect the quarks each to have 1/3 of the proton momentum if they are stationary in the proton rest frame and hence the basic interaction to have an energy of $E_{\rm cm}/3$. However, the quarks cannot be absolutely at rest within the proton as the Heisenberg uncertainty principle says that is impossible within a finite volume. This quark motion in the proton rest frame is called the "Fermi motion" and, since the proton has a size of order 1 fm, then the quark momenta are around 200 MeV/c in the rest frame.

This momentum might sound very small compared to 1 TeV but the Lorentz transformation to the boosted proton magnifies the effect. Neglecting the quark masses, the two extreme cases for the quark in the proton rest frame are $p_z = \pm m_p c/2$ and hence $E = m_p c^2/2$



since any more momentum than this could not be balanced by the other quarks without them having more energy than $m_p c^2$. The protons in a 1 TeV beam have an extremely high velocity, very close to c. In this frame, the quark momentum becomes

$$p'_z = \gamma(p_z + vE/c^2) = \gamma(\pm m_p c/2 + m_p c/2) = 0 \text{ or } \gamma m_p c$$

In this very boosted frame, the proton momentum is $\gamma m_p v \approx \gamma m_p c$ so this says the Fermi motion results in the quarks having momentum values anywhere from zero to the total proton momentum. We can defined the quark momentum relative to the proton momentum using the Feynman x variable

$$x = \frac{p_q}{p_p} = 0 \to 1$$

The variable x should be considered as the fraction of the proton momentum carried by the quark in the boosted frame. The centre-of-mass energy of the eq is related to that of the ep by x also, specifically

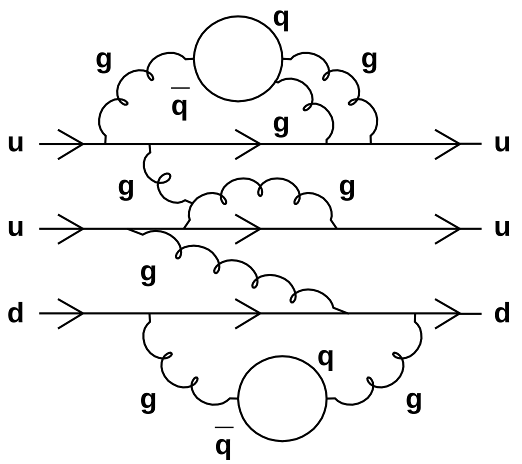
$$E_{{\rm cm}\ eq} = \sqrt{x} E_{{\rm cm}\ ep} \quad \text{or} \quad s_{eq} = x s_{ep}$$

We must consider the probability of reacting with a quark of a give value of x; this function $P(x)$ is what is actually measured in ep experiments. Since all values of x will happen with some probability, the total cross section is given by

$$\sigma = \int P(x)\sigma(x)dx$$

4 Valence and sea quarks

We have assumed the proton was simply a combination of two u quarks and a d quark. However, we know these "valence" quarks have a mess of other virtual quarks and gluons surrounding them, as their constituent mass is much higher than their real (current) mass, so a proton is really more complicated.

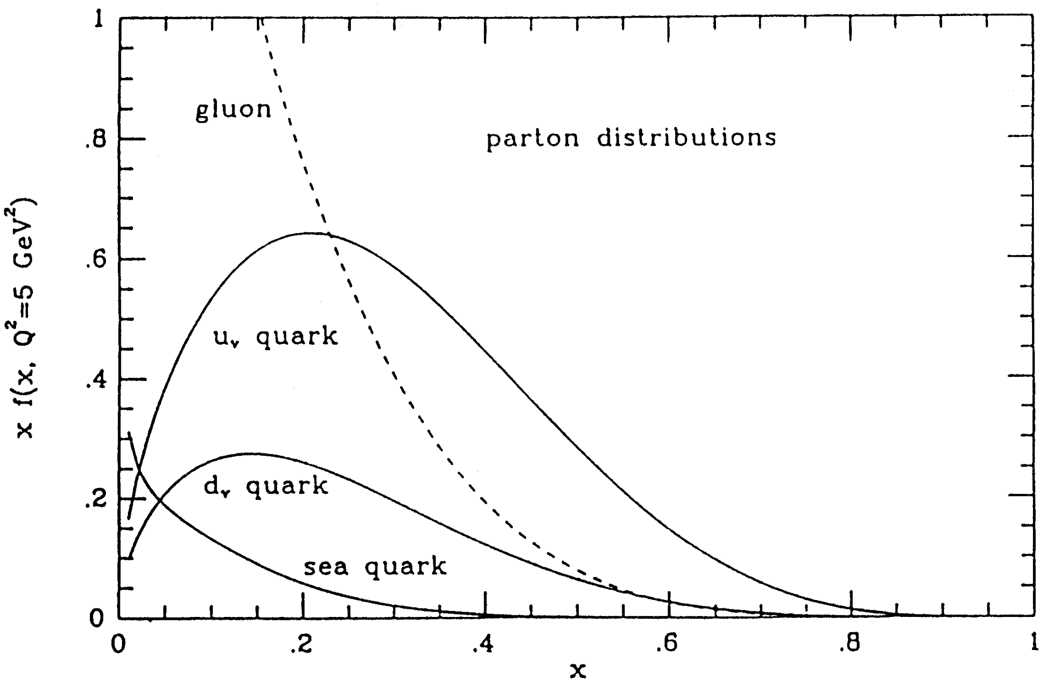


The photon could interact with one of the virtual "sea" quarks rather than the valence quark. The sea quarks are much more likely to have low momenta (due to Heisenberg's uncertainty principle) and the observed x distribution of the quarks $P(x)$ is the sum of the valence and sea parts.

The photon is not sensitive to the gluons in the proton as they are not charged. It turns out around half of the proton momentum is carried by the gluons, so the integral of $P(x)$

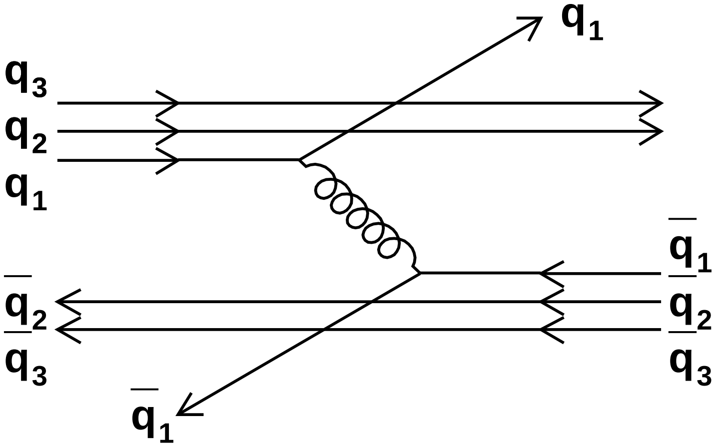
$$\int xP(x)\ dx \sim 0.5$$

not one, as might be expected. A typical calculation of the quark and gluon (collectively called partons) distributions is shown below; it is clear the average interaction energy is much less than the centre-of-mass energy, i.e. $\langle s_{eq} \rangle \ll s_{ep}$.



4.1 Proton-(anti)proton colliders

Proton colliders have a distinguished history as discovery machines; the $W^{\pm}$, Z and top quark were all first observed at proton colliders. For both the Tevatron and the LHC, the main motivation for these experiments is to use the increased energy available to find particles which have not been able to be made previously as there was not enough energy to put into their mass. Specifically, they are both searching for the Higgs (the missing link of the SM) or some alternative we haven't thought of yet which is beyond the SM. However, their search range is not as high as the centre of mass energy might imply. Consider the fundamental processes occuring in proton collisions. Just as for DIS, the basic process is due to the quarks or gluons. One example is quark-quark scattering.

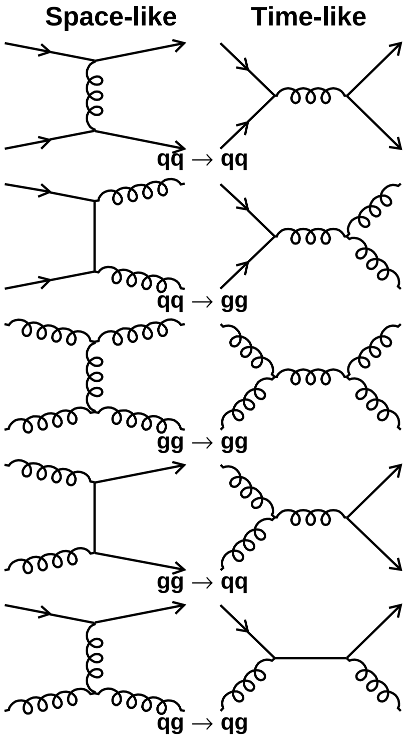


The Feynman diagram for this has a structure very similar to electron-muon scattering and indeed, we can use one to calculate the other. Of course, just as for DIS, we have to take into account the fact that the quarks do not have a well-defined energy, but have some fraction x of the proton momentum, where the probability distribution for x is the same structure function as was measured in electron-proton reactions. Using the structure functions from electron-proton scattering in proton-antiproton scattering gives consistent results, again good evidence for the quark model. Of course, in this case, both the incoming particles have this uncertainty and so there are two integrals to do, over the two x distributions. In fact, things are more complicated since there are many more combinations of quarks and gluons which can interact, even at lowest order.

Here, the gluons can interact as these are purely strong interactions so the probability distribution has to include them also. Hence, we have to work with the probability distributions for gluons as well as for quarks. The interaction centre-of-mass energy is $s_{qq} = x_1 x_2 s_{pp}$ and the total cross section is

$$\sigma = \int\int P'(x_1)P'(x_2)\sigma(x_1, x_2)dx_1\, dx_2$$

All these diagram produce two outgoing particles which give jets, and leave the two sets of proton remnants heading done the beampipe, just as for DIS. Note that of all these, only the second, third and fourth require a quark-antiquark annihilation and so for the rest, proton-proton is as good as proton-antiproton. In addition, we know that protons actually have sea quarks and antiquarks produced in virtual loops in addition to the three valence quarks. In fact, the number of sea quarks rises with energy as we can more finely resolve the structure with a higher energy (and hence shorter wavelength) probe. At the high energies of the LHC, then there are hundreds of sea quarks and antiquarks, compared with just the three valence quarks,



so again, proton-proton is just as good as proton-antiproton. Technically, antiprotons are much harder to made and handle in accelerators, so the LHC chose not to use them.

The structure functions themselves also fall off rapidly with x. To make a large centre-of-mass energy requires substantial energy in the incoming particles. The probability of getting two high x values is very small and gets smaller very quickly as x increases, so the probability of having large centre-of-mass energy in the fundamental interaction drops rapidly. Hence, the discovery range of the 14 TeV LHC proton collider is more limited that the centre of mass energy might imply. It is certainly not two orders of magnitude greater than the 200 GeV LEP collider; the 2 TeV Tevatron, which has a centre of mass energy ten times higher than LEP, is approximately comparible in discover potential. The relative benefit obviously depends on the ease of identifying the required signal but the LHC is expected to be a frontline discovery machine for the next decade or so.