Nuclear and Particle Physics - Lecture 12
The weak force

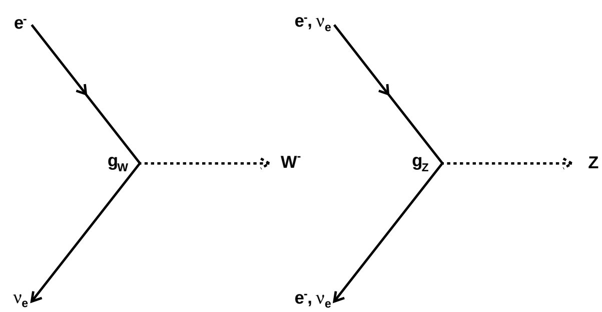
1 Introduction

When we started looking at QCD, we went through the properties of the gluons and quarks. We must now do the same for the weak force. In some ways, it is easier than for QCD, e.g. there is no confinement or asymptotic freedom, but there are other complicating factors which make it more difficult.

All fundamental fermions carry weak charge; in particular it is the only charge on the neutrinos. Hence, all quarks, charged and neutral leptons feel this force although we will concentrate on electrons and electron neutrinos for now.

2 The weak force bosons

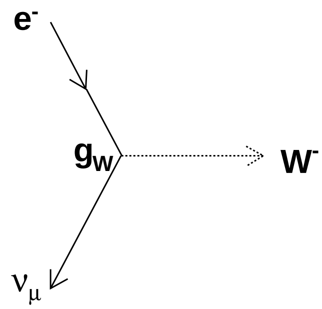
Firstly, let's look at the force bosons. There are three, namely the $W^{\pm}$ and the Z^0, where the W^+ and the W^- are antiparticles of each other and the Z (like the photon) is its own antiparticle. These are the equivalents of the eight gluons. However, the $W^{\pm}$ interactions have a critical difference from all the forces we have seen so far; they change the type of particle, e.g. an electron becomes an electron neutrino



This must in fact be true as the W carries off EM charge (which is always conserved) so the charges on the incoming and outgoing particles must always differ by one unit. The change of the type of particle therefore allows the fundamental particles to decay.

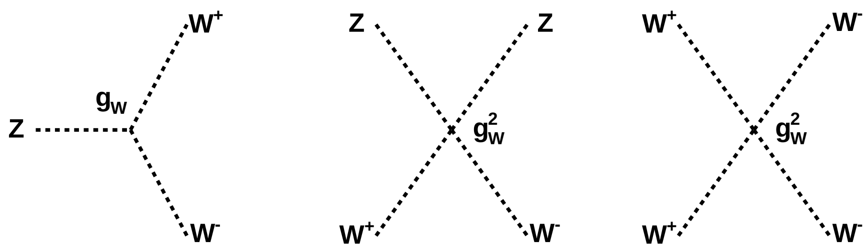
In contrast, the Z is uncharged and acts much more like the photon and gluons, in that it doesn't change the fermion type. Therefore the Z vertex has the same particles going in and out, e.g. electron goes to electron or electron neutrino goes to electron neutrino, as shown above. Note the weak charge for the Z is g_Z, not g_W; the weak force equivalent of charge is (apparently at least) different for the two bosons.

For completeness, the μ and τ and their neutrinos have identical interactions to the electron and the electron neutrino. They all carry the same universal weak charge strength g_W (or g_Z). Note that we never see the lepton type change, e.g. the following is forbidden
as is any other combination except $e \leftrightarrow \nu_e$, $\mu \leftrightarrow \nu_\mu$ and $\tau \leftrightarrow \nu_\tau$.



3 Self interactions

As for gluons, the $W^\pm$ and Z carry weak charge (and indeed the $W^\pm$ clearly also carry EM charge). Hence, there are again self-interaction vertices

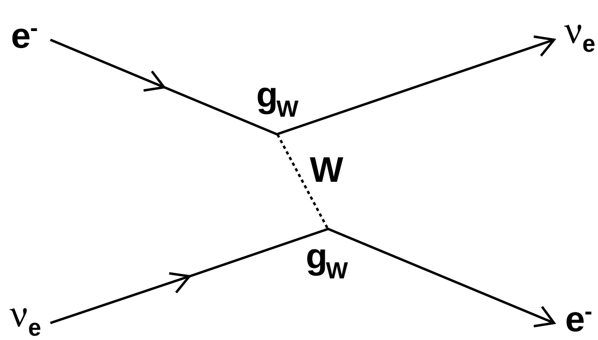


However, we see bare electrons so these cannot lead to confinement (or, it turns out, asymptotic freedom either). Why not? It is because, in contrast to the gluons, the $W^\pm$ and Z are *not* massless but are in fact very heavy; $m_W \approx 80$ GeV/c^2 and $m_Z \approx 91$ GeV/c^2. As we have seen, this restricts their range so preventing the field from remaining large at large distances. Thus, although self interacting, the weak force is not confining.

Just as for QED and QCD, we can form a dimensionless constant for W interactions

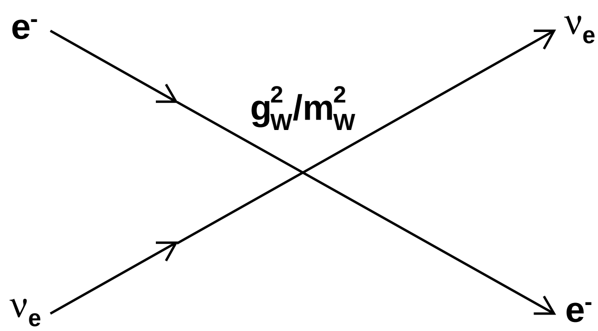
$$\alpha_W = \frac{g_W^2}{4\pi\hbar c}$$

The value of this was historically hard to measure. For interactions with energies well below $m_W c^2$, then only virtual W's could be made. This meant they were always restricted to be internal in the relevant Feynman diagrams and begin and end at two vertices, e.g. for $\nu_e e$ scattering



In the low energy limit, the term due to the W in these diagrams goes as $1/m_W^2$ so the amplitude was always proportional to g_W^2/m_W^2. Neither g_W nor m_W could be independently measured so only the value of this combination was known. This was parametrised as the Fermi coupling constant $G_F = \sqrt{2}/8\,\hbar^3 c^3 (g_W^2/m_W^2)$, where $G_F/(\hbar c)^3 = 1.166 \times 10^{-5}$ GeV^{-2}. The weak force was labelled "weak" as this value is so small; at low energies, it gives decay rates and cross sections much smaller than for EM interactions, let alone strong ones.

This low energy limit is effectively treating the vertices as being at a single point, i.e. an infinitely short range force (i.e. an infinitely massive boson) or equivalently as a four-point coupling



and indeed, it was Fermi's theory in terms of such a four-point coupling which originally gave rise to the constant G_F.

However, in the 1980's, real (as opposed to virtual) W particles were first observed and their mass of 80 GeV/c^2 was measured. This allowed g_W and hence α_W to be calculated from G_F; it turns out

$$\alpha_W \approx 0.034 \approx \frac{1}{29} > \alpha$$

i.e. it is substantially bigger than the fine structure constant. The weak force is really stronger than the EM force. For energies much greater than $m_W c^2$, the mass can be neglected and the weak force acts just like the EM force. At these energies, the weak force relative larger strength compared with the EM force becomes manifest.

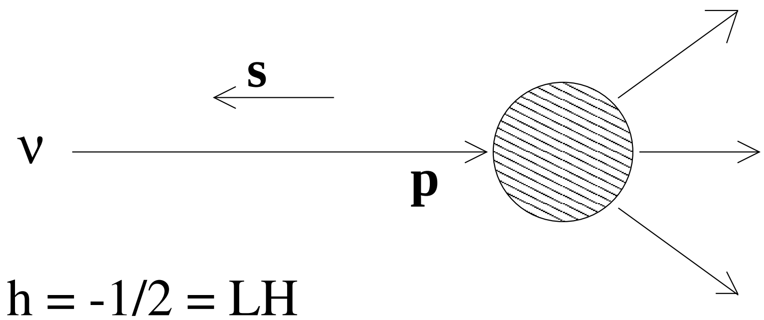
4 Helicity and handedness

Besides the bosons having mass, there is one more major difference between the weak force and the other two, and this is what leads to $\hat{P}$ and $\hat{C}$ violation. The weak vertex coupling strength actually depends on the helicity of the fermion.

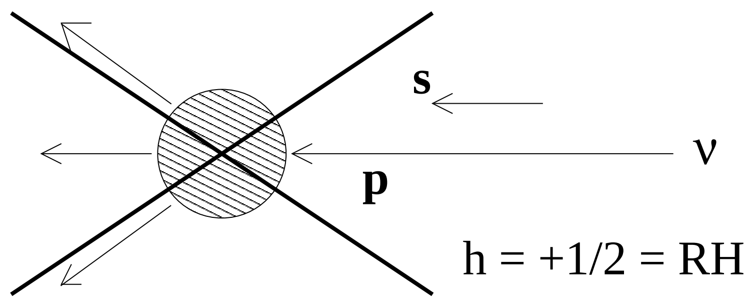
Helicity we defined as the spin resolved along the momentum direction and so for a spin 1/2 particle, the helicity can be $h = \pm 1/2$

Bizarrely, the W bosons only interact with leptons and quarks with $h = -1/2$ and not with ones with $h = +1/2$. Actually, strictly speaking, the exact quantity is something called handedness (which has no classical analogue) but in the limit of $v \to c$, handedness and helicity eigenstates correspond. Specifically, as $v \to c$, "right-handed" (RH) becomes helicity $+1/2$ and "left-handed" (LH) becomes helicity $-1/2$. The weak force is therefore often called a left-handed interaction. Conversely, it turns out that antifermions only interact if they are right-handed, i.e. $h = +1/2$ in the relativistic limit. The actual amount of RH and LH in a helicity state is given by $(1 \pm v/c)/2$. Note, for very slow particles, with $v \ll c$, then they are approximately equally RH and LH. As most reactions we consider are highly relativistic, then using helicity instead of handedness is a good approximation.

This behaviour of the fermions "losing their weak charge" if their spin flips over goes against all our intuition. However, it directly explains why $\hat{P}$ and $\hat{C}$ are violated. Consider any weak reaction which is started by an incoming neutrino, which is almost massless and so has a velocity very close to $v = c$

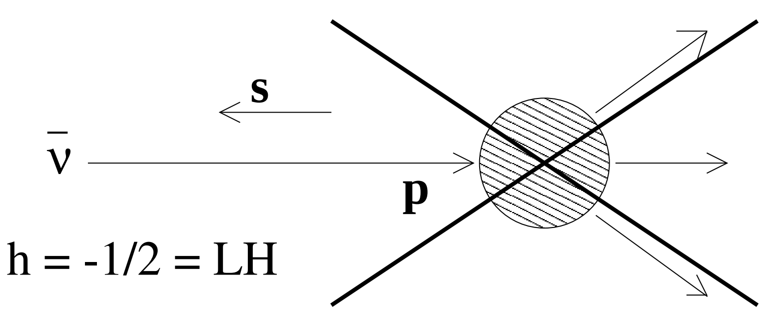


Under a parity operation, all momenta (being polar vectors) change sign but the spin (being an axial vector) does not. Hence, this system becomes

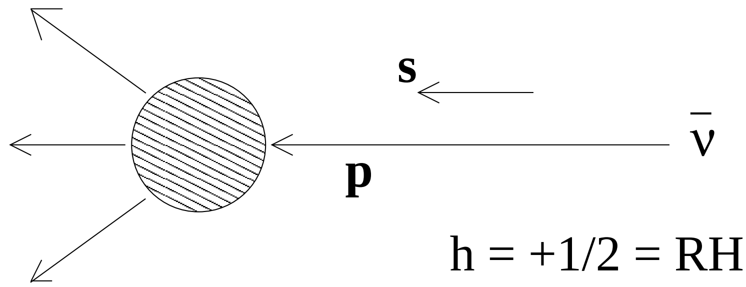


This reaction does not happen as the neutrino is now right-handed; the rates of any weak reaction and its parity inverted equivalent are not equal. Indeed, one of the two rates is always actually zero in the relativistic limit.

The same effect is responsible for $\hat{C}$ violation also; applying a $\hat{C}$ operation changes the neutrino to an antineutrino but does not effect the momentum or spin, so



which again does not happen as the antineutrino is left-handed. Note, applying a $\hat{P}$ operation to this $\hat{C}$ inverted system (or equivalently a $\hat{C}$ operation to the $\hat{P}$ inverted system above) gives



which works fine; it involves a right-handed antineutrino. In fact, this will have exactly the same rate as the original reaction. Hence, $\hat{P}$ and $\hat{C}$ are violated because of this wierd handedness behaviour, but the combined $\hat{P}\hat{C}$ (or $\hat{C}\hat{P}$) operation is not.