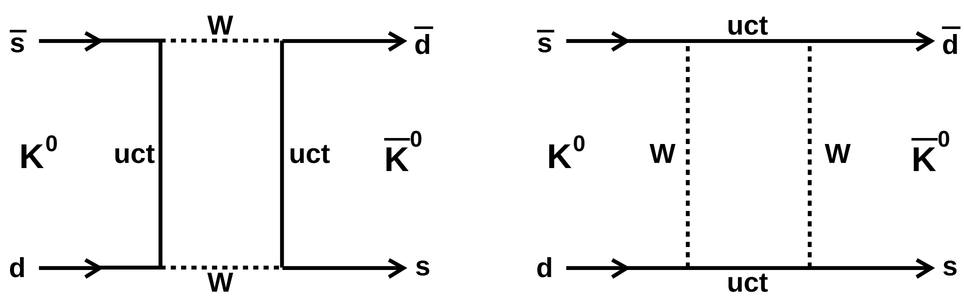
Nuclear and Particle Physics - Lecture 16
Neutral kaon decays and oscillations

1 Introduction

We have already seen that the neutral kaons will have sem-leptonic and hadronic decays. However, they also exhibit the phenomenon of mixing, in a very close analogy with the neutrinos.

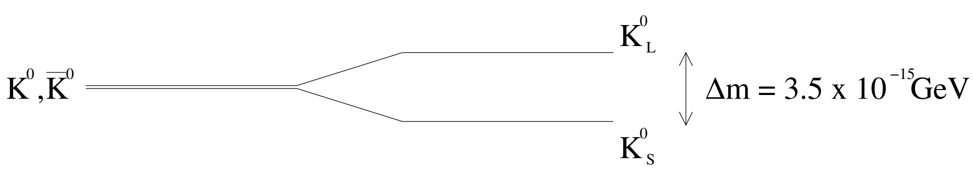
2 Neutral kaons

The cross-generation weak interactions result in the non-unit Cabibbo matrix and this allows the K^0 and $\overline{K}^0$ to oscillate; the $d\overline{s}$ and $s\overline{d}$ states can change into each other via a second order weak process.



Hence, unlike for the neutrino case, we understand the interaction connecting them and so can predict the value of the mixing angle (which for neutrinos we wrote as θ).

The physical states with well-defined masses are called K_S^0 and K_L^0 and these are mixtures of $d\overline{s}$ and $s\overline{d}$, just as for the neutrinos. Again, there is a mass splitting between the states



Because this is a low energy, second order weak effect, the mass splitting is small and is around 3.5×10^{-15} GeV/c^2, which is even less than for the neutrinos.

The kaon case has similarities and differences with neutrinos. The kaons decay, so the overall oscillation is damped with time by the exponential decay constant. We know the interaction which connects the states and so know (to a good approximation) what the mixing angle is; it turns out to be close to 45°. The kaon decays in several ways and these actually project out different states so particular decays tell us how much of particular states there are as a function of time. For the neutrinos, only the weak interaction states can be observed.

We will assume for the rest of this calculation that CP is a conserved quantity, which is not exactly true but is a good approximation. In this approximation, CP commutes with the Hamiltonian and so energy (or in the rest frame, mass) eigenstates are also CP eigenstates. Hence, these mixed states are states of definite CP. The K^0 and $\overline{K}^0$ are $J^P = 0^-$ states so we know that

$$\hat{P}|K^0\rangle = -|K^0\rangle, \qquad \hat{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

These two states are antiparticles of each other so we'll adopt the convention that

$$\hat{C}|K^0\rangle = -|\overline{K}^0\rangle$$

Reapplying the operator then must get us back to where we started, so

$$\hat{C}^2|K^0\rangle = -\hat{C}|\overline{K}^0\rangle = |K^0\rangle$$

so this means

$$\hat{C}|\overline{K}^0\rangle = -|K^0\rangle$$

also. Hence,

$$\hat{C}\hat{P}|K^0\rangle = |\overline{K}^0\rangle, \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = |K^0\rangle$$

We need to find CP eigenstates as these are the mass eigenstates in the limit of CP conserved, so consider the combinations

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right), \qquad |K_L^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$$

Applying the combined operator, these give

$$\hat{C}\hat{P}|K_S^0\rangle = +|K_S^0\rangle, \qquad \hat{C}\hat{P}|K_L^0\rangle = -|K_L^0\rangle$$

Hence, K_S^0 and K_L^0 are P, C and CP eigenstates, i.e. they are each their own antiparticle and have $J^{PC} = 0^{--}$ and 0^{-+}, respectively, which give CP values of $+1$ and -1, respectively. These form the physical (i.e. mass) eigenstates. These are clearly equivalent to the neutrino mixed states with $\theta = 45^\circ$, so $\cos\theta = \sin\theta = 1/\sqrt{2}$. Note, we can also invert the equations to give

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right), \qquad |\overline{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle - |K_L^0\rangle\right)$$

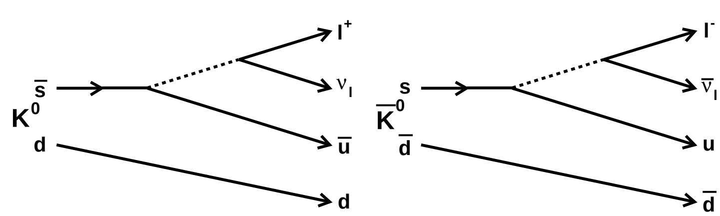
3 Neutral kaon decays

One of the interesting things about these states is how they decay. Like charged kaons, they can decay semi-leptonically (meaning to l, ν_l and pions) or hadronically (meaning to pions).

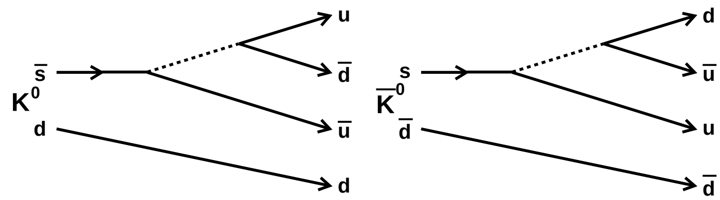
The semileptonic decays are different for the K^0 and $\overline{K}^0$ because each only gives leptons of particular charges

$$K^0 \to l^+\nu_l\pi^-, \qquad \overline{K}^0 \to l^-\overline{\nu}_l\pi^+$$

This is obvious from the Feynman diagrams for these decays



Hence, semi-leptonic decays project out the K^0 and $\overline{K}^0$ states. These would be expected to have equal partial widths and since the K_S^0 and K_L^0 are equal mixtures of these states, they should therefore decay equally to each charge type.



The situation is very different for the hadronic decays, where either two or three pions are kinematically possible. Both the K^0 and $\overline{K}^0$ give the same final state quarks so they cannot be distinguished from their decays.
These will then interfere and we need to think in terms of the K_S^0 and K_L^0 directly. We saw in previous lectures that the parity of a pion is -1 and that of an orbital angular momentum state is $(-1)^L$. Hence a state of n pions has parity $P = (-1)^n(-1)^L$. For spinless kaon decays $L = 0$, so two pions have $P = +1$ and three pions have $P = -1$.

What about C? The π^0 is $J^{PC} = 0^{-+}$, so

$$\hat{C}|n\pi^0\rangle = (C_{\pi^0})^n|n\pi^0\rangle = +|n\pi^0\rangle$$

For $\pi^+\pi^-$ in $L = 0$, $\hat{C}$ exchanges them; to swap them back gives another $(-1)^L = +1$ factor, so

$$\hat{C}|\pi^+\pi^-\rangle = +|\pi^+\pi^-\rangle$$

also. Finally, for $\pi^+\pi^-\pi^0$, this is similar to the above but with an extra π^0 which gives an extra $C_{\pi^0} = +1$, so for all cases of two or three pions

$$\hat{C}|n\pi\rangle = +|n\pi\rangle$$

Hence, combining the two operations, then

$$\hat{C}\hat{P}|2\pi\rangle = +|2\pi\rangle, \qquad \hat{C}\hat{P}|3\pi\rangle = -|3\pi\rangle$$

We are assuming CP is conserved; hence the only decays allowed are

$$K_S^0 \to \pi\pi, \qquad K_L^0 \to \pi\pi\pi$$

Hence, the hadronic decays project out the K_S^0 and K_L^0 states. Because the mass of the three pions is close to that of the kaon, the phase available for the two pion decay is much bigger and the partial widths are therefore very different

$$\Gamma(K_S^0 \to \pi\pi) = 7.4 \times 10^{-15}\ \text{GeV}, \qquad \Gamma(K_L^0 \to \pi\pi\pi) = 4.3 \times 10^{-18}\ \text{GeV}$$

In fact, the two pion decay dominates the K_S^0 decays and so the semi-leptonic decays are well below 1%. In contrast, the three pion decays of the K_L^0 are comparable with the semi-leptonic decays. This also results in the K_S^0 lifetime being much shorter than the K_L^0 lifetime

$$\tau_{K_S^0} = 8.9 \times 10^{-11}\ \text{s}, \qquad \tau_{K_L^0} = 5.2 \times 10^{-8}\ \text{s}$$

which are different by a factor of around 600. Hence a pure K_S^0 would decay away much quicker than a pure K_L^0 beam.

However, whichever way we make kaons, we cannot make pure K_S^0 or K_L^0 beams. We always make them via strong or EM interactions, such as

$$\pi^- p \to \Lambda K^0$$

where the Λ definitely contains an s quark so the $\bar{s}$ must have formed a K^0, not a $\overline{K}^0$. Hence, when produced, the kaons are initially in either a K^0 or a $\overline{K}^0$ state. We need to look at what happens with time. For a simple decaying particle, the wavefunction in its rest frame goes as

$$\psi \sim e^{-imc^2 t/\hbar} e^{-\Gamma t/2\hbar} = f(t)$$

so that

$$|\psi|^2 \sim e^{-\Gamma t/\hbar} = e^{-t/\tau}$$

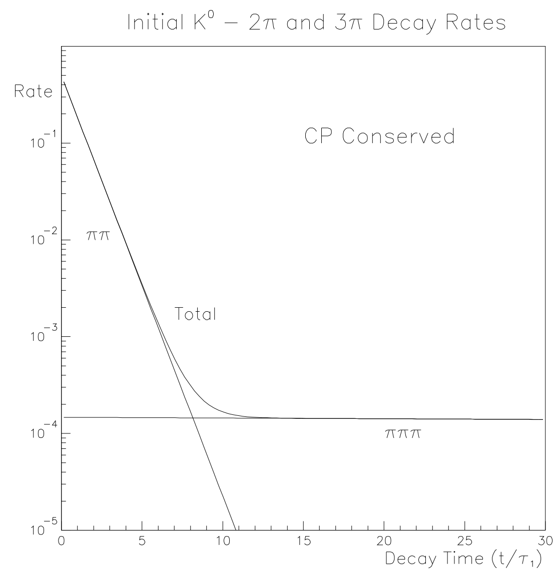
as required. An initial K^0 can be decomposed as

$$\psi(0) = |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right)$$

The K_S^0 and K_L^0 are the states with definite masses and widths, so the state at a later time is

$$\psi(t) = \frac{1}{\sqrt{2}}\left(f_S(t)|K_S^0\rangle + f_L(t)|K_L^0\rangle\right)$$

Consider the decays to pions. At time t, the intensity of K_S^0 in the beam is simply $|f_S(t)|^2 = e^{-\Gamma_1 t/\hbar}/2$ and similarly for K_L^0, so we see half the produced particles decay rapidly to two pions and some fraction of the rest decay slowly to three pions.

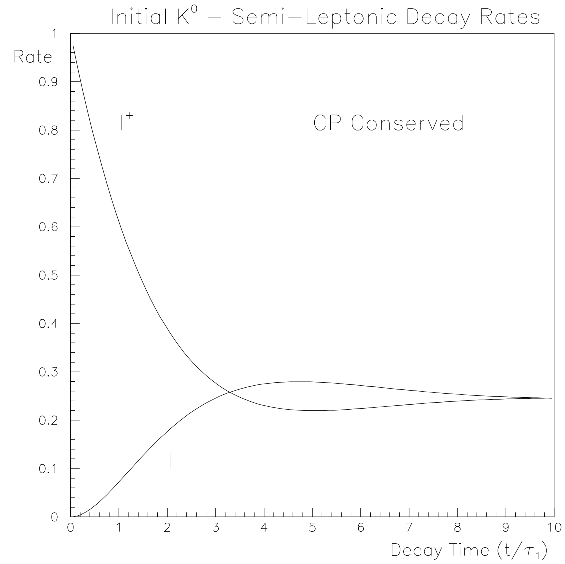


The decays to leptons are trickier; we need to project out the K^0 and $\overline{K}^0$ states as these decay to the different semi-leptonic modes. This is more like what we did for neutrinos. Hence, we write

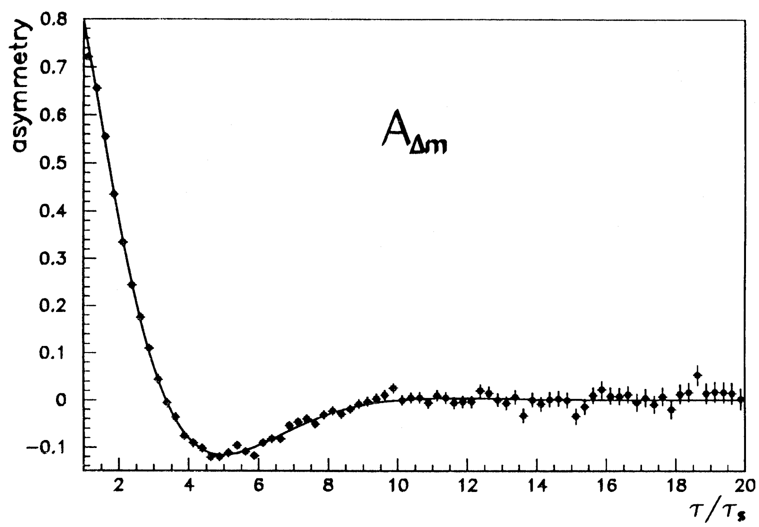
$$\begin{aligned}
\psi(t) &= \frac{1}{2}\left[f_S(t)\left(|K^0\rangle + |\overline{K}^0\rangle\right) + f_L(t)\left(|K^0\rangle - |\overline{K}^0\rangle\right)\right] \\
&= \frac{1}{2}\left[\left(f_S(t) + f_L(t)\right)|K^0\rangle + \left(f_S(t) - f_L(t)\right)|\overline{K}^0\rangle\right]
\end{aligned}$$

Therefore, the intensity of K^0, and hence the $l^+\nu_l\pi^-$ decays, is given by

$$\begin{aligned}
\frac{1}{4}|f_S(t) + f_L(t)|^2 &= \frac{1}{4}\left[|f_S(t)|^2 + |f_L(t)|^2 + |f_S(t)^* f_L(t)| + |f_L(t)^* f_S(t)|\right] \\
&= \frac{1}{4}\left[e^{-\Gamma_1 t/\hbar} + e^{-\Gamma_2 t/\hbar} + 2e^{-(\Gamma_1+\Gamma_2)t/2\hbar}\cos(\Delta m c^2 t/\hbar)\right]
\end{aligned}$$



where $\Delta m = m_2 - m_1$.
The intensity to $\overline{K}^0$ and hence the $l^- \overline{\nu}_l \pi^+$ decays, is similar but the sign of the cosine term changes. The interference changes as a function of time because the e^{-imt} terms are slightly different (by Δm) and so the separate parts slowly drift in and out of phase. The rates of each of these two decays actually go up and down with time. The usual measure is the asymmetry, since efficiencies, etc., cancel.



This allows Δm to be measured. Effectively, we are comparing it with Γ rather than m, so this is often expressed as

$$x = \frac{\Delta m}{\Gamma_1} = 0.4738 \pm 0.0009$$

4 CP violation

The assumption we made that CP is a symmetry is actually not true, because the CKM matrix is not purely real. The obvious place to look for such as effect is for the decays $K_S^0 \to \pi\pi\pi$ and $K_L^0 \to \pi\pi$. The former is hard to see as the K_S^0 is always produced with K_L^0 and the three pion decay is hard to see given the large background from the CP conserving K_L^0 decays. The same would be true for the $K_L^0 \to \pi\pi$ decays, except that the K_S^0 lifetime is much shorter. Hence, by waiting long enough, all the K_S^0 mesons decay away. It is then straightforward to study the decays of the K_L^0 and indeed, decays to two pions are seen with a branching fraction of 0.2%. This in fact was the first observation which demonstrated CP violation back in 1964.

In addition, CP violation also means the rates of

$$K_L^0 \to l^+\nu_l\pi^-, \qquad K_L^0 \to l^-\overline{\nu}_l\pi^+$$

are not exactly equal. The asymmetry plot above does not in fact go exactly to zero for long times, but has a value 0.3%. This means a K_L^0 is more likely to decay to an l^+ than an l^-; a clear example of a matter-antimatter difference. Such differences, which arise because of CP violation, are thought to be the reason why the Universe has more matter than antimatter.

Since then, the only other system which has shown CP violation is the B^0 system, which also oscillates like the K^0. CP violation in B^0 decays was only first observed in 2001.