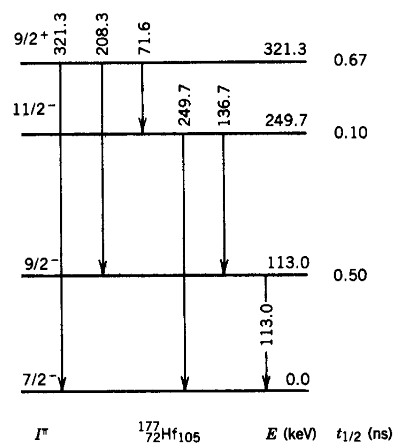
Nuclear and Particle Physics - Lecture 21
Gamma and beta decay

1 Introduction

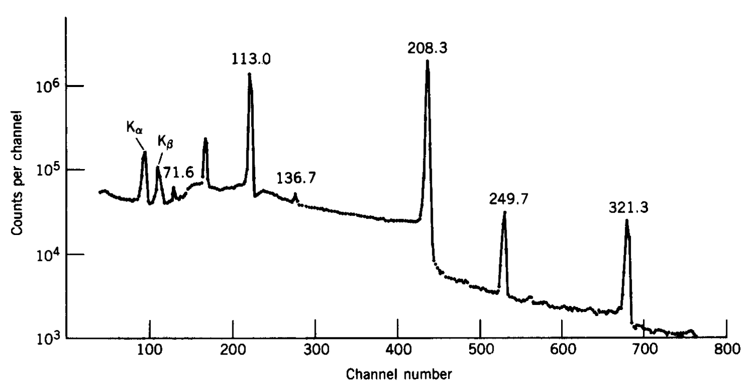
We want to look at how nuclei can decay. In general all decays result in what we consider as radioactivity and it is common knowledge that there is alpha, beta and gamma radiation. However, in fact, these decays are special cases of more general processes. We shall consider them in the reverse order; gamma and beta decay in this lecture and alpha decay in the next.

2 Gamma decay

We now know that the gamma particle observed in radiation is simply a high energy photon, so gamma decays are EM decays. We know the EM force does not allow us to change u to d quarks (or vice versa) so we cannot change the nucleus values of Z or N through these decays. Hence, there is a very limited type of decay possible, specifically from an excited state to a ground state of the same nucleus. This is therefore the nuclear equivalent of atomic emission of light, but at much higher energies. In the same way, the energy differences between levels can be deduced from the energy spectrum of the photons seen. An example for $^{177}_{72}$Hf is shown below; the state levels are



and the gamma spectrum observed is



Being an electromagnetic decay, the de-excitations tend to happen reasonably fast, as long as there is not too big a change of angular momentum between the initial and final state. For photons of order 1 MeV between similar states, lifetimes tend to be around 10^{-16} s, while for large changes in angular momentum $\Delta J \sim 4$ or 5, this can increase to 10^3 s. The supression is due to the radiation having to occur as a higher radiation multipole, just as in atoms. In all cases, it is extremely fast compared to the age of the Earth, so there are effectively no long-lived, naturally occuring isotopes which emit gamma radiation. This does not mean it is not seen naturally, but only because other types of radiation can leave the resulting nucleus in an excited state that then decays by gamma emission.

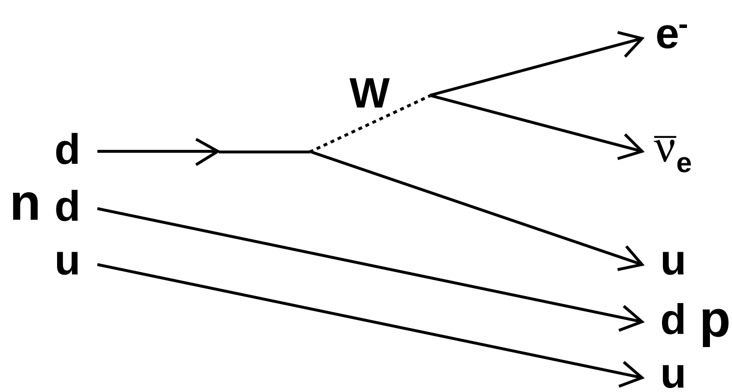
There is a related process to gamma decay called internal conversion. Here, instead of a real photon being seen, a virtual photon is emitted and absorbed by one of the atomic electrons. This energy is usually much higher than its binding energy so the electron is ejected from the atom with a fixed energy (neglecting its original binding energy).

3 Beta decay

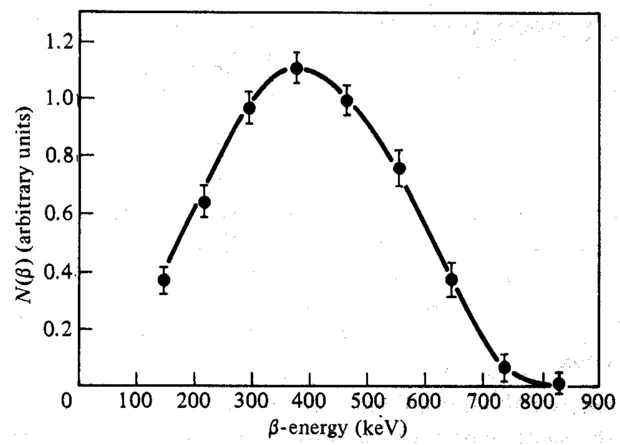
Beta decays have the ability to produce much more interesting behaviour as they are a weak process. The beta particle which is emitted, as we now know, is actually just an electron which is produced from a virtual W along with a (normally unobserved) electron antineutrino. The prototypical beta decay is that of a single neutron

$$n \rightarrow p + e^- + \overline{\nu}_e$$

which we understand at a fundamental level as having a Feynman diagram



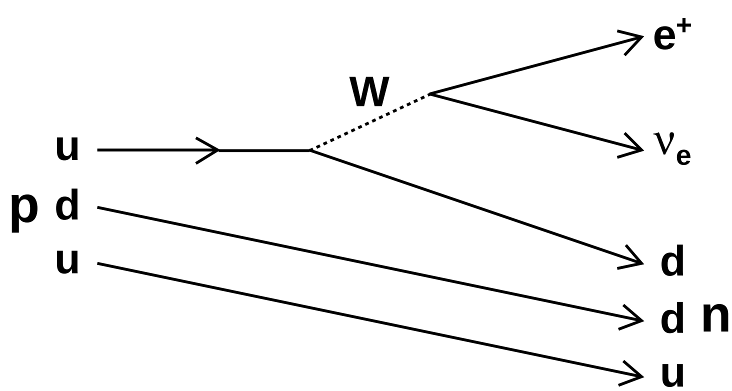
This is only just energetically allowed; the mass of the neutron is 939.57 MeV/c^2 and the proton is 938.28 MeV/c^2 while the electron is 0.51 MeV/c^2, so the energy released, often labelled by the variable Q, is $Q = 0.78$ MeV. (The neutrino masses are completely negligible at this level.) This means the neutron has a long lifetime compared with other hadron weak decays, such as the pion; the neutron has $\tau_n = 900$ s. Note the decay is three-body. As we saw before, this means the electron (and indeed the antineutrino) does not have a fixed energy but is emitted with energies over a spectrum. Indeed, it was that apparent violation of energy conservation in beta decays which led Pauli to first postulate the existance of an unseen particle, which we now know to be the neutrino.



Note that as far as the weak interaction is concerned, proton decay through

$$p \to n + e^+ + \nu_e$$

with a very similar Feynman diagram

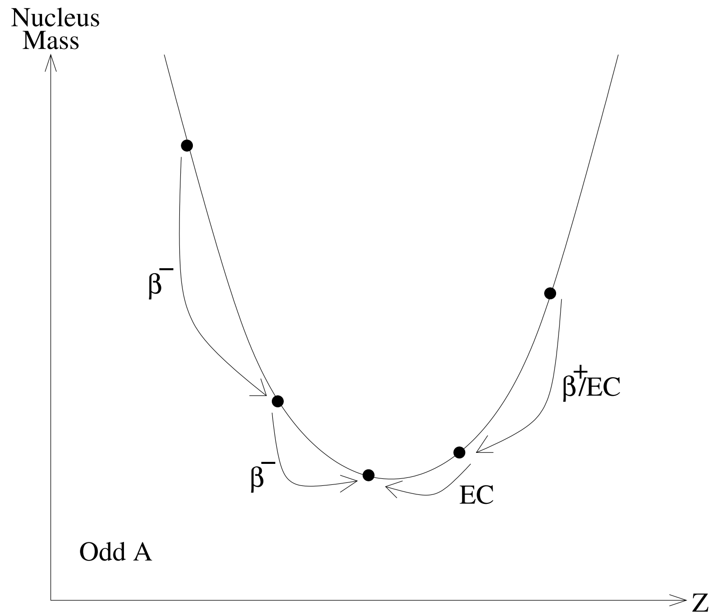


is possible. This does not happen purely because the mass of the proton is smaller than the mass of the neutron plus the mass of the electron.

The obvious question then is why we can have nuclei with neutrons which survive more than 900 s? It is because once a neutron is bound into a nucleus, then changing it into a proton is basically changing $N \to N-1$ and $Z \to Z+1$ for fixed A. We have seen this can change the nuclear binding energy quite substantially. Hence, this decay can actually increase the mass of the nucleus as a whole so neutron decay within a nucleus can become energetically not allowed.

4 Odd A nuclei

To see when this happens, let's look in more detail as odd A nuclei. Previously, we were interested in the binding energy, but now we need to know the total nuclear mass including the neutron-proton mass difference. Hence, plotting the mass looks inverted compared to the previous plots and gives a minimum in mass rather than a maximum in binding energy. The masses of nuclei for fixed A as Z varies are

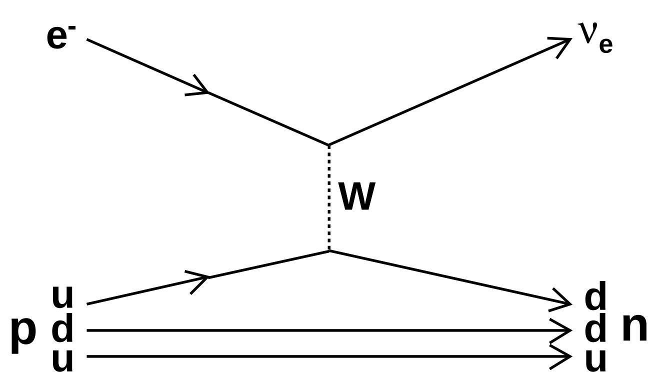


Nuclei with too many neutrons (and hence values of Z less than the value which gives the minimum) can beta decay and achieve better stability, just as a free neutron can.

What about nuclei with too many protons? Here, the nuclear mass would be increased even further from the minimum if a neutron decayed to a proton so this is not energetically allowed and the neutrons are stable. However, in the same way that nuclear masses become the important factor in determining whether neutrons can be stable even though free neutrons are not, then protons in nuclei can be energetically able to decay, even though free protons cannot. Hence, nuclei with Z greater than the minimum value can undergo positron emission as above, $p \to n + e^+ + \nu_e$, and so also obtain better stability. This process is very similar conceptually to beta decay and again the three-body decay results in a spectrum for the positron (and neutrino). However, there is a competing process to positron emission, which can also happen for these nuclei. Atomic electrons in an $l = 0$ state have a non-zero probability of being at the origin, i.e. $|\psi(0)|^2 \neq 0$, and hence being inside the nucleus. This means the process of electron capture can occur

$$e^- + p \to n + \nu_e$$

which has an identical diagram to positron emission but with the positron reversed to make it an electron



In fact, this process can occur even when positron emission is energetically not allowed. Positron emission requires the nuclear masses to be $m(Z,N) > m(Z-1,N+1) + m_e$, which electron capture requires $m_e + m(Z,N) > m(Z-1,N+1)$, i.e. $m(Z,N) > m(Z-1,N+1) - m_e$. Hence, if the nuclei are different in mass by less than m_e, only electron capture is possible.

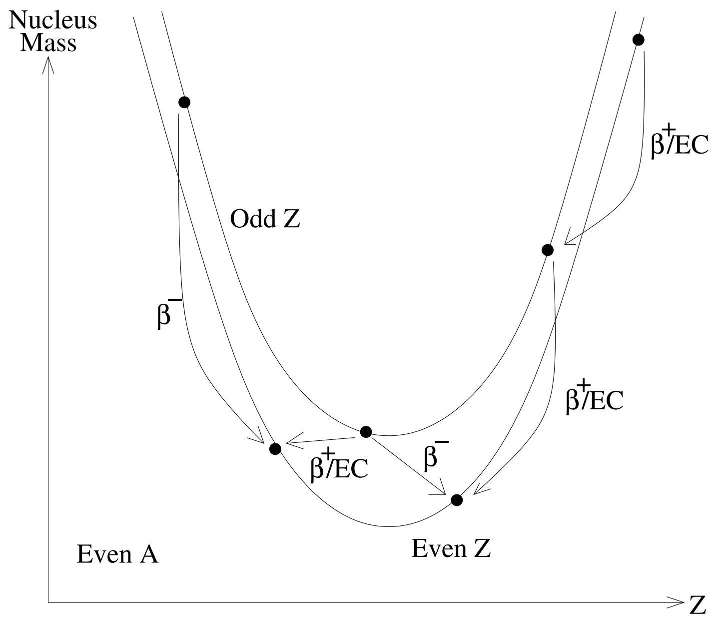
Clearly, this is possible because of the atomic electrons; the equivalent process which could in principle compete with beta decay, namely positron capture

$$e^+ + n \to p + \overline{\nu}_e$$

does not occur as there are no positrons circulating around the nucleus.

5 Even A nuclei

Things are similar for even A nuclei but the pairing term adds a twist



Beta decay from either side alternates between even-even (the lower mass curve) or odd-odd (the higher mass curve). Hence, it is perfectly possible to have an odd-odd nucleus which can decay both by beta decay, positron decay and electron capture. The resulting two nuclei are both unable to decay by those modes, even though one can be significantly heaver than the other. In fact, this raises the possibility of a process called double beta decay. If the heavier even-even nucleus can convert two neutrons to protons (or vice versa, depending on whether the lower Z nucleus is heavier or lighter) simultaneously, then it does not have to go via the heavier intermediate odd-odd state, but can go directly to the lower mass even-even nucleus

$${}^{A}_{Z}X \to {}_{Z+2}^{\ \ A} Y + 2e^- + 2\overline{\nu}_e$$

An example is ${}^{106}_{48}$Cd which can decay to ${}^{106}_{46}$Pd through double β^+ decay. Of course, it is very unlikely for two weak decays to happen at the same time so the rate is minute and the lifetimes correspondingly very long, up to 10^{22} years.

6 Neutrino masses

Note, electron capture is a two-body final state so the neutrino (and nucleus) have a single fixed energy of emission. In the other decays, the electron and positron energies are not unique.

The spectrum goes from zero right up to the difference in mass of the initial and final nucleus, ignoring the neutrino mass. If, however, the neutrino has a non-zero mass, then the maximum energy of the electron or positron is more limited. This has been used to try to measure the neutrino mass; the upper portion of the electron spectrum should be truncated by an amount which depends on the neutrino mass. The most studied beta decay for this purpose is tritium, which decays to helium-3, ${}^3_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + e^- + \overline{\nu}_e$, with a very small energy release of only $Q = 18.6$ keV. The small energy release makes the observation of the effects of a non-zero neutrino mass easier to see. However, it is still a very hard experiment and while neutrino masses of more than ~ 15 eV/c^2 have been excluded it is very hard to rule out neutrino masses at the level implied by oscillations.