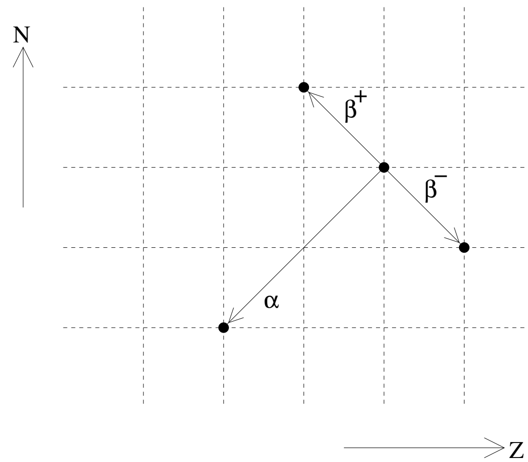
Nuclear and Particle Physics - Lecture 22
Alpha decay

1 Introduction

We have looked at gamma decays (due to the EM force) and beta decays (due to the weak force) and now will look at alpha decays, which are due to the strong/nuclear force. In contrast to the previous decays which do not change A, alpha decays happen by emission of some of the nucleons from the nucleus. Specifically, for alpha decay, an alpha particle, ${}^4_2\mathrm{He}$, is ejected so generically

$$ {}^A_Z X \to {}^{A-4}_{Z-2} Y + {}^4_2\mathrm{He} $$

for some X and Y. Y clearly has a different number of nucleons to X. Compare how alpha and beta decays move the nuclei around in Z,N plane



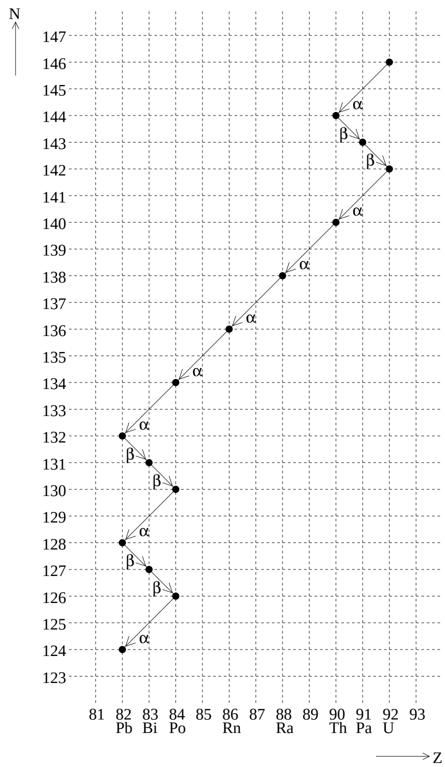
Note that gamma decays cannot change Z, N or A.

2 Fission and fusion

Alpha decay is in fact only one specific case of a whole range of processes which involve emission of nucleons. These range from single proton or neutron emission up to splitting the nucleus into two roughly equal parts. Particularly in the latter case, these are called fission decays.

Which nuclei would we expect to fission? The binding energy per nucleon curve shows that the maximum occurs around ${}^{56}_{26}\mathrm{Fe}$ and drops off to either side. Hence, both small A and large A nuclei are less strongly bound per nucleon than medium A. This means there will be some energy release if two small nuclei are combined into a larger one, a process called fusion which will be discussed later in the course. Similarly, there will be an energy release if a large nucleus is split into two smaller parts; this is fission and what concerns us here.

We are specifically looking at alpha decays in this lecture as this is the most common form of fission. Why is alpha decay the most common? The binding energy per nucleon curve does not fall very fast; it drops by only 10% over the whole A range above ${}^{56}_{26}\mathrm{Fe}$. The slope is roughly 0.01 MeV/A. Hence, the energy release is quite small unless the products are particularly strongly bound. This is the case for ${}^4_2\mathrm{He}$ which is doubly magic and has $B_E = 28.3$ MeV, which although it has an A well below ${}^{56}_{26}\mathrm{Fe}$, is still ~ 7 MeV per nucleon. Even for alpha decay, it is only energetically possible for nuclei with $A > 150$ and needs significantly higher values of A than this for a reasonable energy release. Hence, alpha decay is seen mainly in heavy nuclei with large $A > 200$. However, such nuclei after alpha decay will leave a daughter nucleus with $A - 4$ which will also normally be above 150 and so will itself be able to alpha decay. Hence a sequence of alpha decays is often seen and can be many decays long. They continue until they reach a nucleus which is stable (or semi-stable). One example is the decay sequence starting from $^{238}_{92}$U which ends at $^{206}_{82}$Pb. (Note that the Pb nucleus has Z at a magic number, hence being quite stable.)

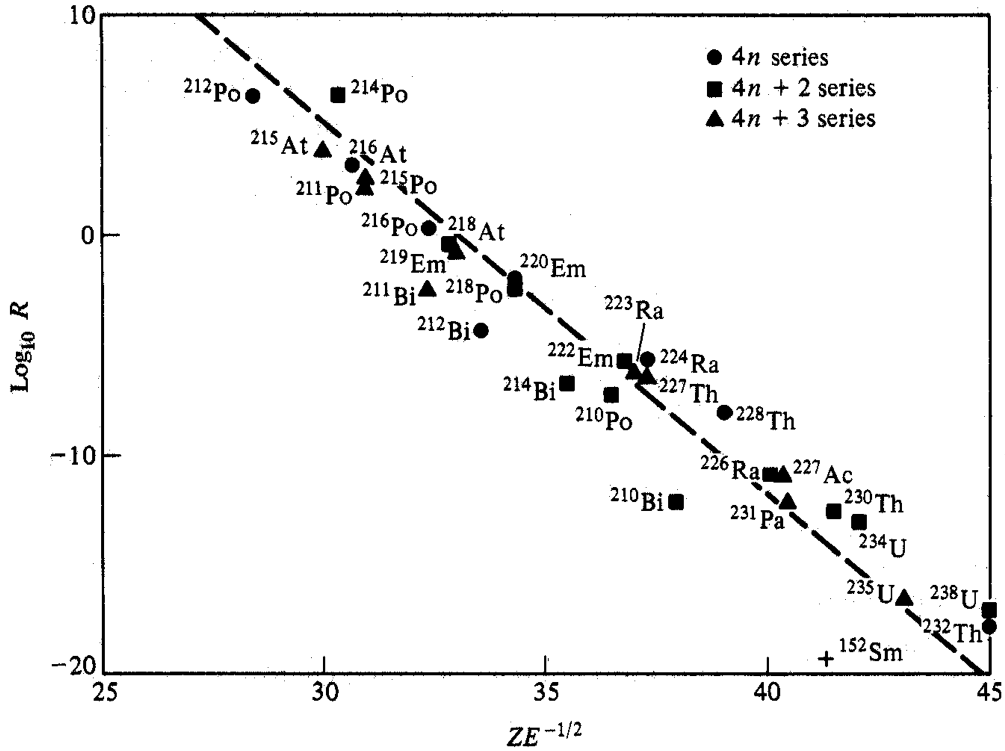


Note there are several beta decay steps here too. Why? Alpha decays reduce Z and N equally, specifically reducing them each by two per decay. However, the heavy, large A starting nucleus will have $N > Z$ as that is what is needed to be near the beta-stability curve. Hence, a pure alpha decay sequence would leave a lower A nucleus with a higher and higher fraction of neutrons, whereas the beta-stability curve requires the fraction of neutrons to become lower as A is reduced. Hence, the beta decays bring the intermediate nuclei back closer to the beta-stability curve. Note, there will always be too many neutrons, not too few, so positron emission or electron capture are basically not seen in these sequences. This is why, although radioactivity was discovered in the 19^{th} century, antimatter (specifically the positron) was not found until 1932.

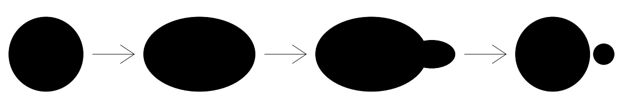
One final point; beta decay does not change A but alpha decay changes it by 4. Hence, all heavy nuclei with the same $A/4$ remainder (A modulo 4, or writing $A = 4n + m$ then the value of m) will end up at the same nucleus. Hence, there are effectively only four such decay sequences.

3 Alpha decay rates

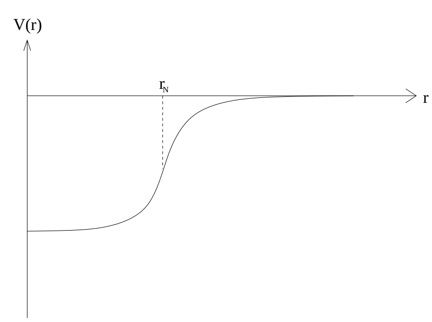
One obvious question is why do we see any of these sequences at all? This is a strong force decay and they have had around 5 billion years to decay. The answer is that some of the lifetimes for these decays are billions of years, despite being due to the strong force. The range of lifetimes for alpha decay is found to vary by over 25 orders of magnitude and this is effectively totally determined by the Z value and the amount of energy release, Q.



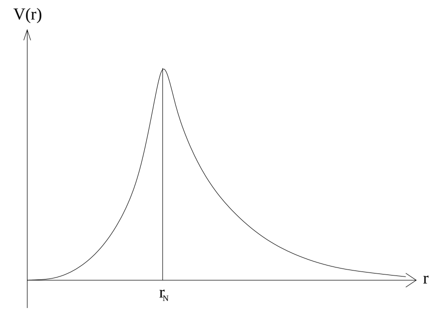
This extremely strong exponential dependence on $Z/\sqrt{Q}$ can be understood by considering the alpha decay process in more detail. To be emitted, the alpha particle has to form outside the nucleus



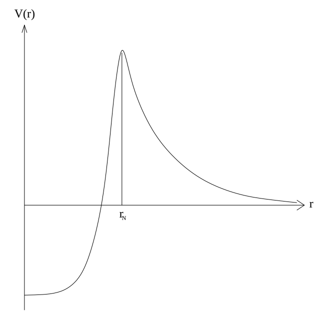
However, we can take a simple model where we consider it to have an independent existance within the nucleus before the decay. What potential would it then feel? There are two forces, the nuclear and the EM force. The nuclear potential for the alpha will be effectively the same as for the nucleons, i.e. something like the Saxon-Woods shape which we discussed previously.



There is also a large Coulomb repulsion due to the positive charges on both the nucleus and alpha so the EM contribution to the potential looks like



The total is then

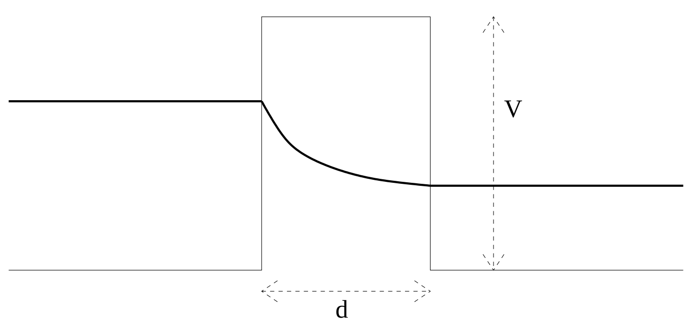


which shows the alpha particle has a large potential barrier to overcome in its decay. If the energy release Q is large, then its energy will be above the maximum of the potential and it can decay very fast. However, the maximum of the potential will be

$$V_{\max} \sim \frac{2(Z-2)e^2}{4\pi\epsilon_0 r_n} \sim \frac{2Ze^2}{4\pi\epsilon_0 r_0 A^{1/3}} = 2.4\frac{Z}{A^{1/3}} \text{ MeV}$$

For the nuclei which alpha decay, $Z/A^{1/3} \sim 15$ and so this barrier is several 10's of MeV, much larger than most observed alpha decay Q values.

The usual case is that the energy release means it is well below this maximum so it can only exist and subsequently get through by QM tunnelling. As you will know, any QM tunnelling process drops exponentially as the barrier width or height increases. For a square barrier



the amplitude goes as e^{-kr}, where $k = \sqrt{2m(V_0 - E)}/\hbar$. This means the probability of transmission through the whole barrier is proportional to e^{-2kd}. Hence, a large d or small E give a very small probability.

The above is for a constant V_0. However, the Coloumb barrier is a function of r and so in this case the probability for an alpha getting through is proportional to

$$e^{-2\int k(r)\,dr} = e^{-2G}$$

where the Gamow factor G is

$$G = \int k(r)\,dr = \frac{\sqrt{2m}}{\hbar}\int \sqrt{V(r) - Q}\,dr = \frac{\sqrt{2mQ}}{\hbar}\int \sqrt{\frac{2Ze^2}{4\pi\epsilon_0 Qr} - 1}\,dr$$

The integral is over the range of radii for which the alpha energy is below the barrier, i.e. from the nucleus radius out to the radius where the alpha energy is greater than the potential. For small Q, this latter radius is much bigger than the nuclear radius. This integral is not trivial (but can be found in most text books) but in the approximation of the upper limit being much bigger than the lower, the integral is

$$\int \sqrt{\frac{2Ze^2}{4\pi\epsilon_0 Qr} - 1}\,dr \approx \frac{2Ze^2}{4\pi\epsilon_0 Q}\frac{\pi}{2} = \frac{Ze^2}{4\epsilon_0 Q}$$

This means the Gamow factor is

$$G = \frac{\sqrt{2mQ}}{\hbar}\frac{Ze^2}{4\epsilon_0 Q} = \frac{e^2\sqrt{2m}}{4\epsilon_0\hbar}\frac{Z}{\sqrt{Q}} = 2.0\frac{Z}{\sqrt{Q}}\ \mathrm{MeV}^{1/2}$$

The rate per nucleus R (equal to the inverse of the lifetime) is then expected to be

$$R = \frac{1}{\tau} = ae^{-2G}$$

so

$$\log R = \log a - (2\log e)G = \log a - 1.7\frac{Z}{\sqrt{Q}}$$

The plot shows a slope of -1.7 which agrees well with most of the measured values. Hence, tunneling is what causes the huge variation in lifetimes; for Q values which only differ by 1 MeV, a difference of 10^5 is seen.