Nuclear and Particle Physics - Lecture 23
Nuclear Fission

1 Introduction

We have seen alpha decay can be understood as tunnelling through a Coulomb barrier when the alpha has less energy than the barrier height. Fission is the more general process of splitting a nucleus into two or more nuclear fragments. Although this clearly includes alpha decay, the term fission is more normally used to describe processes where the resulting nuclei are much more even in size.

2 Fission energetics

We have seen the maximum binding energy per nucleon is around ${}^{56}_{26}$Fe so we expect that fission is energetically possible for nuclei larger than around twice this size. It turns out that splitting a nucleus into two equal nuclei normally gives the largest energy release Q (as shown on the problem sheet). This is then

$$Q = m(Z,N) - 2m(Z/2,N/2) = 2B_E(Z/2,N/2) - B_E(Z,N)$$

This can be estimated from the semi-empirical mass formula. The volume terms clearly cancel, so ignoring the pairing term and approximating $Z(Z-1)$ to Z^2, then this is

$$Q = 2\left[-a_s\left(\frac{A}{2}\right)^{2/3} - a_c\frac{Z^2/4}{(A/2)^{1/3}} - a_a\frac{(N/2-Z/2)^2}{A/2}\right] + a_sA^{2/3} + a_c\frac{Z^2}{A^{1/3}} + a_a\frac{(N-Z)^2}{A}$$

This gives

$$\begin{aligned} Q &= a_sA^{2/3}\left[1-2\left(\frac{1}{2^{2/3}}\right)\right] + a_c\frac{Z^2}{A^{1/3}}\left[1-2\left(\frac{1}{2^{5/3}}\right)\right] + a_a\frac{(N-Z)^2}{A}\left[1-2\left(\frac{1}{2}\right)\right] \\ &= a_sA^{2/3}(1-2^{1/3}) + a_c\frac{Z^2}{A^{1/3}}(1-2^{-2/3}) \end{aligned}$$

Energy is released when $Q > 0$, meaning

$$a_c\frac{Z^2}{A^{1/3}}(1-2^{-2/3}) > a_sA^{2/3}(2^{1/3}-1)$$

or

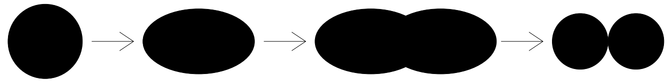
$$\frac{Z^2}{A} > \frac{a_s(2^{1/3}-1)}{a_c(1-2^{-2/3})} = 0.702\frac{a_s}{a_c} \approx 18$$

For nuclei on the beta-stability curve this is actually satisfied for $A > 100$, which gives $Z > 42$. Hence, like alpha decay, fission is only energetically possible for heavy nuclei.

Note, fission involves a much bigger change to A than alpha decay and so gives a much bigger shift closer to ${}^{56}_{26}$Fe in the binding energy per nucleon plot. Hence, the energy releases in fission are significantly higher. We saw typical alpha decay energies are less than 10 MeV. Fission decays tend to release hundreds of MeV. This is one of the reasons why practical applications of nuclear energy use fission not alpha decay.

3 Spontaneous fission

We can classically picture the process as the nucleus deforming and breaking up as follows



One critical issue is whether it takes energy or not to deform the nucleus in this way. The two terms which influence this are again the surface term and the Coulomb term. We have seen that nucleons on the surface are less strongly bound, due to missing nearest neighbours. For a given volume, a sphere has the smallest surface area and hence the biggest binding energy. Hence, deforming from a sphere to an ellipsoid with the same volume increases the suface area and so requires energy. Explicitly, an ellipsoid can be defined in terms of a small deformation parameter δ, where the major axis is $r(1+\delta)$ and the two minor axes are $r/\sqrt{1+\delta}$. The surface area of the ellipsoid is then

$$4\pi r^2\left(1+\frac{2\delta^2}{5}\right)$$

and so the change to the nucleus energy is $a_s(A^{2/3})(2\delta^2/5)$. However, a deformed nucleus has the protons on average further away from each other and so the Coulomb energy is decreased. Hence, as a nucleus deforms, Coulomb energy is released. The approximate electrostatic energy of an ellipsoid for small deformations is

$$\frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 r}\left(1-\frac{\delta^2}{5}\right)$$

and so the change to the nucleus energy from this effect is $-a_c(Z^2/A^{1/3})(\delta^2/5)$. For the Coloumb term to be bigger, then

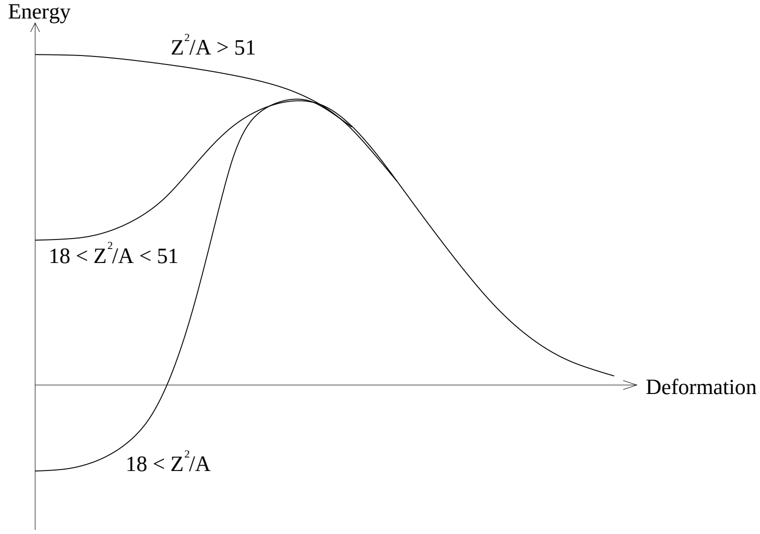
$$a_c\frac{Z^2}{A^{1/3}}\frac{\delta^2}{5} > a_s A^{2/3}\frac{2\delta^2}{5}$$

which means

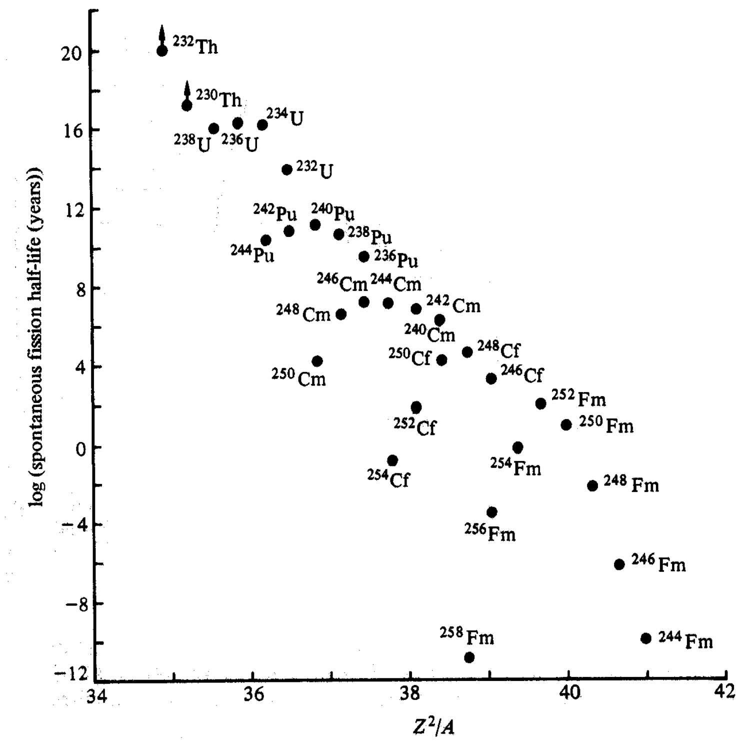
$$\frac{Z^2}{A} > \frac{2a_s}{a_c} \approx 51$$

For nuclei on the beta-stability curve, this corresponds to $A > 407$ which is $Z > 144$. For these very large nuclei, the deformation actually reduces the nucleus energy so there is no barrier to this happening. This means they can spontaneously decay through fission and will do so extremely fast, $\sim 10^{-20}$ s. This sets an absolute upper limit to the periodic table.

For nuclei intermediate between these two values, i.e. $18 < Z^2/A < 51$, then fission overall is energetically allowed but the deformation to start it requires energy and so, as for alpha decay, there is a potential barrier to tunnel through. In this case we cannot really consider one of the large daughter nuclei as forming and moving in a potential within the nucleus, so it is not so easy to draw such a potential well as for alpha decay. However a qualitative picture can be obtained by looking at how the energy needed to deform the nucleus depends on the deformation. We know that for nuclei in this range, it takes energy to deform the nucleus but when the daughter nuclei are separated enough that the nuclear force between them is small, then the Coulomb force dominates, as for alpha decay. Hence, a rough idea of the energy as it deforms is



The fact that nuclei in this intermediate range have to tunnel again gives a very strong lifetime range and exponential dependence on Z^2/A over 30 orders of magnitude.



Spontaneous fission does not normally have a rate competative with alpha decay; for example $^{238}_{92}$U has a partial half life for alpha decay of around 10^9 years and for fission of 10^{16} years.

As is the case for alpha decay, the daughter nuclei tend to have too many neutrons to lie on the beta-stability curve. For alpha decays, where the changes are small, then beta decay is usual way the nuclei return to the beta-stability curve. However, in fission, the daughter nuclei are usually highly excited and in any case often so far away from the curve that they often spontaneously emit neutrons, typically between one and four per fission. In fact, this subsequent neutron emission can itself be considered to be a further fission process, although as neutrons have no charge, there is no Coulomb barrier to overcome.

4 Induced fission

It is clear that fission for most of the heavy nuclei is difficult and hence slow. It can be speeded up enormously by exciting the nuclei. For practical uses of the fission energy, this is clearly essential. If enough energy can be added to a nucleus to raise its energy higher than the fission barrier, then it will fission within 10^{-20} s, the very fast rate associated with the strong force, as happens for the nuclei above the barrier, i.e. those with $Z^2/A > 51$.

There are several ways to excite nuclei to higher levels. The most obvious would be to bombard them with gamma radiation, as that is how excited nuclei can decay and indeed it is perfectly possible to excite them in this way. However, this is an EM reaction and so does not have as large a cross-section as a strong force reaction. In addition, there are no simple ways to make intense gamma sources in practise.

To have a high fission rate requires a strongly interacting particle, such as a proton, neutron or alpha. Of these, neutrons are special; they are neutral and so, unlike protons and alphas, are not repelled from the nucleus by their EM charge. To react via the nuclear force, protons and alphas would have to either have a high enough energy to overcome the Coulomb barrier or would have to tunnel through the barrier, drastically reducing the rate. Neutrons have no such problem and so can react at any energy effectively down to zero. Cross sections tend to be largest at low energies as, roughly speaking, there is more time to react. Neutron absorption dominates at such energies, leaving an excited nucleus. Clearly, any excited nucleus has some probability of gamma decaying to the ground state and this competes with the fission decay. The ratio of gamma to fission decays depends strongly on how far above the fission barrier the excited nucleus is.

When a nucleus absorbs a neutron, the N and A values increase by one. Hence, if we have a nucleus ${}^{A}_{Z}X$ in which we want to induce fission, then we have to start with its isotope ${}^{A-1}_{Z}X$

$${}^{A-1}_{Z}X + n \rightarrow {}^{A}_{Z}X^* \rightarrow \text{ fission}$$

Of course, the neutron has to have enough energy to excite the nucleus over the fission barrier. This depends on the relative binding energies of the two isotopes. The total input energy of the reaction is

$$m(Z, N-1)c^2 + E_n = Zm_pc^2 + (N-1)m_nc^2 - B_{Ei} + m_nc^2 + T_n = Zm_pc^2 + Nm_nc^2 - B_{Ei} + T_n$$

where T_n is the neutron kinetic energy. The final nucleus has a energy in its ground state of

$$m(Z, N)c^2 = Zm_pc^2 + Nm_nc^2 - B_{Ef}$$

Hence, neglecting the small nucleus recoil energy, the excited nucleus will be formed at an energy above the ground state given by the difference of these

$$\Delta E = B_{Ef} - B_{Ei} + T_n$$

To fission, ΔE has to be greater than the fission barrier height. If the initial isotope is weakly bound and the final is strongly bound, so $B_{Ef} - B_{Ei}$ is large, then T_n can be small and even zero. Conversely, a small $B_{Ef} - B_{Ei}$ would need a large neutron energy with a subsequent reduction in cross section. The binding energy tends to be vary quite smoothly (except near magic numbers) in the semi-empirical mass formula with the exception of the pairing term. Clearly as A changes by one in neutron absorption, then one of the isotopes is even A and the other odd A. Hence, depending on whether the even A is even-even or odd-odd, the major part of the difference will be given by $a_p/A^{1/2} \sim 2$ MeV. Examples of nuclei which can fission after absorbing even a zero energy neutron are the odd A nuclei $^{233}_{92}$U, $^{235}_{92}$U, $^{239}_{94}$Pu and $^{241}_{94}$Pu. These all have even Z and odd N; the extra neutron then makes these nuclei even-even and so have a large B_{Ef}. Even A nuclei which require a fast neutron include $^{232}_{90}$Th, $^{238}_{92}$U, $^{240}_{94}$Pu and $^{242}_{94}$Pu. In these cases, they are all even-even and so have a large B_{Ei} to start with.